AP® Calculus BC
2003 Sample Student Responses
Form B

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A,

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} + (x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$\Rightarrow f(x) = 1 + \frac{2}{3}(x-2) + \frac{3}{9}(x-2)^2 + \frac{4}{27}(x-2)^3 + \dots + \frac{n+1}{3^n}(x-2)^n + \dots$$

Work for problem 6(b)

Using the ratio test:

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n+2}{3^{n+1}}(x-2)^{n+1}\cdot\frac{3^n}{n+1}\frac{1}{(x-2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{n+2}{n+1}\cdot\frac{x-2}{3}\right| = \frac{x-2}{3}$$

$\Rightarrow$ Since $|L|<1$ for series to converge.

$$-1 < \frac{x-2}{3} < 1 \quad \rightarrow \quad -1 < x < 5$$

$\Rightarrow$ The radius of convergence is 3.

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g(2) = 3$, $g^{(n)}(2) = \frac{n!}{3^{n-1}}$.

$\Rightarrow \quad g(x) = 3 + (x-2) + \frac{1}{3}(x-2)^2 + \frac{1}{9}(x-2)^3 + \ldots + \frac{1}{3^{n-1}}(x-2)^n + \ldots$

Work for problem 6(d)

Since $T_n = \frac{(-4)^n}{3^{n-1}}$ if $x = -2$, Then,

$T_n = (-1)^n \cdot 3\left(\frac{4}{3}\right)^n$

$\Rightarrow$ alternating geometric sequence with $r = \frac{4}{3} > 1$; it is neither absolutely convergent or conditionally convergent, hence g does not converge at $x = -2$.

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
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Work for problem 6(a)

$$f^{(n)}(2) = \frac{(n+1)!}{3^n}$$

$$f(x) = 1 + \frac{2!}{3}(x-2) + \frac{3!}{2!3^2}(x-2)^2 + \frac{4!}{3!3^3}(x-2)^3 + \cdots + \frac{n+1}{3^n}(x-2)^n$$

Work for problem 6(b)

$$f(x) = \sum_{0}^{\infty} \frac{n}{3^n}(x-2)^n$$

by ratio test, $\lim_{n\to\infty}\left|\frac{(n+2)\left(\frac{x-2}{3}\right)^{n+1}}{n+1\left(\frac{x-2}{3}\right)^{n}}\right| = \lim_{n\to\infty}\left|\left(\frac{n+2}{n+1}\right)\left(\frac{x-2}{3}\right)\right| \to \left|\frac{x-2}{3}\right|$

$f(x)$ converges if $\left|\frac{x-2}{3}\right| < 1$

$|x-2| < 3$

∴ the radius of convergence of f about $x = 2$ is 2 //

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g(2)=3$; $g'=f(x)$

$$g=\int f(x)\,dx = x+C+\frac{1}{3}(x-2)^2+\frac{1}{3^2}(x-2)^3+\frac{1}{3^3}(x-2)^4+\cdots+\frac{1}{3^n}(x-2)^{n+1}$$

Work for problem 6(d)

by ratio test, $\lim_{n\to\infty}\left|\frac{\left(\frac{x-2}{3}\right)^{n+2}}{\left(\frac{x-2}{3}\right)^{n+1}}\right| \to \left|\frac{x-2}{3}\right|$

$\therefore g$ converges if $\left|\frac{x-2}{3}\right|<1$

$-1<x-2<3$

$1<x<5$

$x=-2$ is outside of the range of convergence

$\therefore g$ is not converge at $x=-2$

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