AP® Calculus BC
2003 Scoring Commentary
Form B

Question 1

This problem presented two regions. Region R was bounded by a graph and the x-axis. Region S was bounded by the graph, the line tangent to the graph at one point, and the x-axis. Explicit formulas for the function whose graph was shown and for the line were given. The questions involved integration to find an area and a volume, as well as differentiation to find the slope of the tangent line at a given point. Part (a) asked students to show that the line was, in fact, tangent to the graph at $x = 3$. Students needed to verify that the line and the graph intersected at $x = 3$ and that the slope of the line was the slope of the line tangent to the graph at that point. Part (b) asked to find the area of the region S. This could be done in several ways. One of the easiest was to find the area under the graph from $x = 3$ to $x = 4$ and then subtract that area from the area of the triangle bounded by the line, the vertical line $x = 3$, and the x-axis. Part (c) asked to find the volume of a solid of revolution whose cross sections were discs.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample B</u> (Score 7)

The student earned 7 points: 1 point in part (a), 4 points in part (b), and 2 points in part (c). The student lost 1 point in part (a) since $f'(3)$ was not given. In part (c), the student lost 1 point since the answer was rounded to 2 decimal places, instead of the required 3 places.

Question 2

The problem presented a region R bounded by two circles and the x-axis. It asked for the setup of definite integrals to find the area of R using three different approaches: part (a) asked for integration with respect to x, part (b) asked for integration with respect to y, and part (c) asked for integration using polar coordinates. While students were not required to find the area of R, evaluating all three integrals and verifying that they gave the same numerical value may have been helpful to them. In fact, the area of R could also be found without calculus: the portion of R below the line $y = x$ is one-eighth of the area of a circle of radius $\sqrt{2}$ and thus has area $\frac{\pi}{4}$. The portion of R to the left of the line $x = 1$ is one-quarter of the area of a circle of radius 1 and thus has area $\frac{\pi}{4}$. The overlap of these two pieces is a triangle of area $\frac{1}{2}$, so the total area is $\frac{\pi - 1}{2}$. For this reason, the emphasis in this problem was on the setup of the integrals and not the evaluation. In parts (a) and (c), students needed either to set up more than one integral, or to set up one integral and add to it the area of a region that could be calculated without using a definite integral.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample B</u> (Score 7)

The student earned 7 points: 2 points in part (a), 3 points in part (b), and 2 points in part (c). In part (a), the student lost 1 point for an incorrect integrand. In part (c), the student lost 1 point for incorrect limits on the definite integral.

Question 3

This problem presented a function, given in tabular form, which represented the diameter of a circular blood vessel at certain distances from one end. This problem tested the ability to approximate, use, and interpret integrals. It also required knowledge of, and the ability to use, the Mean Value Theorem. Part (a) required that students know how to set up an integral that represented the average value of the function. While the given function represents the diameter, the question asks about the average value of the radius. This was intentional. Part (b) required knowledge of how to set up the midpoint Riemann sum as an approximation to a definite integral. Part (c) presented an integral and tested whether students could recognize that this is π times the definite integral of the square of the radius from distance 125 mm to 275 mm, and thus represents the volume of the circular blood vessel taken over this interval. References to the specific interval and to the units, mm^3, were important. Part (d) could be done in several ways. One approach was for students to recognize that if there are two values of x at which B' takes on the same value, then the Mean Value Theorem guarantees a value of x at which $B''(x) = 0.$ There were several ways of showing that there were two values of x at which B' took on the same value. One was to observe that, by the Mean Value Theorem, there is a value on $[60, 180]$ at which B' is zero and another on $[240, 360]$ at which B' is again zero. Another was to observe that B must have a local maximum between $x = 0$ and $x = 120$ and a local minimum between $x = 60$ and $x = 180,$ and B' is zero at both of these local extrema. A third would be to invoke the Mean Value Theorem to show that there is a value of x between 120 and 180 at which B' is $\frac{2}{60},$ and another value of x between 300 and 360 at which B' is again $\frac{2}{60}.$ However, it was incorrect to assume that the function B attained a local extremum at any of the given points.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample B</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (d), the student earned 1 point for correctly stating that $B'(x) = 0$ at some point x in the interval $(0, 360)$.

Question 4

This problem involved particle motion in the plane. The x- and y-coordinates of the point at time t were given as explicit functions. The problem asked several questions that required finding and using the derivatives of these functions. In part (a), students needed to know how to find the velocity vector and the speed. Part (b) required knowing the relationship between the derivatives of the coordinate functions and the value of $\frac{dy}{dx}$. The limit in question could be found by dividing numerator and denominator by e^{3t}, and then recognizing that for positive k and as t approaches $-\infty$, e^{-kt} approaches zero. Parts (c) and (d) tested whether students knew how to use the derivatives of the coordinate functions to determine where the path of the particle was horizontal and where it was vertical.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in each of parts (a), (b), and (c), and 1 point in part (d). The student wrote an incorrect formula for speed in part (a) and lost 1 point. In part (d), the student did not find the value of t and lost 1 point.

Question 5

This problem presented the graph of a function f and asked questions about the function g defined as the definite integral of f over the interval from 2 to x. While it was possible to find an explicit algebraic representation of f as a piecewise-linear function, the questions were designed to be more easily answered using the Fundamental Theorem of Calculus and the geometric meaning of the integral. Part (a) required the use of the Fundamental Theorem of Calculus, the recognition of integral as area, and the ability to read the derivative of a linear function from its graph. Part (b) asked for the average rate of change of the function g, which required translating the difference in the values of g into a definite integral of f, and evaluating that definite integral from the graph of the function. Students could use $g(3)$ from part (a) and compute $g(0)$ separately. Part (c) required solving $f(c) = \frac{7}{3}$ and recognizing that there are exactly two solutions. Although this looks like a problem involving the Mean Value Theorem, it actually is not, though it could be used to initiate classroom discussion of the Mean Value Theorem. Part (d) required students to find a point of inflection by reading the graph of the derivative.

Sample A (Score 9)

The student earned all 9 points.

Sample B (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 2 points in part (d). In part (a), the student did not find $g''(3)$, losing that point. In part (c), the student gave a correct justification, but used an incorrect equation, and therefore lost the answer point.

Question 6

This problem presented a Taylor series. Students were given the values of $f(2)$ and all derivatives of f at $x = 2$. Part (a) required students to use the given information to construct the Taylor series for f about $x = 2$. Part (b) asked for the radius of convergence of the power series. Part (c) tested whether students knew how to use the Taylor series for f to find the series for a function whose derivative is f. In part (d), students needed to recognize that the radius of convergence is measured from the point about which the series is expanded. Although this question did not ask for the sum of either Taylor series, $g(x)$ is a geometric series, and so the sum could be found. Since $f(x) = g'(x)$, a closed form for f could be found.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 3 points in part (a), 2 points in part (b), and 1 point in each of parts (c) and (d). In part (b), the student gave the incorrect radius of convergence and lost 1 point. In part (c), the student wrote the constant C in the expression for g and lost 1 point.