AP® Calculus BC
2003 Scoring Commentary

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Question 1

This problem presented a region bounded by two graphs and a vertical line. Students needed to use integration to find an area and two volumes. A graphing calculator was required to find the point of intersection of the two graphs. Part (a) required the use of a definite integral to find the area of the region. Part (b) required another application of integration to find the volume generated by revolving this region about a horizontal line. The resulting solid had cross sections in the shape of washers. Part (c) again required integration, this time to find the volume of a solid whose base was the region and whose cross sections were rectangles.

The mean score was 5.82.

Sample C (Score 9)

The student earned all 9 points.

Sample F (Score 7)

The student earned 7 points: 2 points in part (a), 3 points in part (b), 1 point in part (c), and the region point. In part (c), the student failed to earn the integrand point since there is an extraneous factor of π. The student also failed to earn the answer point since the stated answer is not correct for the integral.

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Question 2

This was a problem dealing with particle motion in the plane. It presented an explicit formula for $x'(t)$ only. A graph of the particle's motion was provided, giving some information on the slope of the tangent to the path of the particle at four points. Part (a) asked for the signs of $x'(t)$ and $y'(t)$ at one of these points. One approach was to use the relationship between the signs of x' and y' and the sign of the slope of the tangent line. A simpler approach was to recognize that both the x- and y-coordinates of the point are decreasing as the particle moves along the path through the point C. The sign of $x'(t)$ could also be read from the derivative. Part (b) required students to recognize that if x and y are differentiable functions of t, then the slope of the tangent can be undefined only when $x'(t) = 0.$ Students then needed to solve this equation. In part (c), students were required to know the relationship connecting $x'(t),$ $y'(t),$ and the slope of the tangent line. Furthermore, students needed to know how to use these values to find the velocity vector and the speed. Part (d) asked for the total change in the x-coordinate, requiring students to set up and evaluate a definite integral.

The mean score was 4.12.

<u>Sample EE</u> (Score 9)

The student earned all 9 points.

<u>Sample YY</u> (Score 7)

The student earned 7 points: 2 points in each of parts (a), (b), and (d), and 1 point in part (c). In part (c), the student computed $y'(8)$ incorrectly and did not compute the speed of the particle, losing 2 points.

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Question 3

This problem gave a region bounded by two graphs and the x-axis. Students needed to show knowledge of how to calculate this area using both rectangular and polar coordinates. Part (a) required students to find the coordinates of the point of intersection of the two graphs and to find the value of the derivative of x with respect to y on one of the curves at that point. In part (b), students were required to find the area by setting up and evaluating a definite integral with respect to y. Part (c) required students to demonstrate knowledge of how to transform an equation in rectangular coordinates into one in polar coordinates. Part (d) required students to know how to set up an integral in polar coordinates for the area of the region. While it was not necessary to evaluate this integral, such an evaluation, compared with the answer to part (b), would have provided a check on the answers to parts (b) and (d).

The mean score was 5.00.

Sample Q (Score 9)

The student earned all 9 points.

Sample F (Score 7)

The student earned 7 points: 1 point in each of parts (a) and (d), 3 points in part (b), and 2 points in part (c). In part (a), the student failed to earn full credit, since the value of $\frac{dx}{dy}$ at point P is not found. In part (d), the upper limit of the integral is incorrect.

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Question 4

This problem presented students with the graph of the derivative of f. It explored the students' understanding of the relationship between the behavior of the derivative and of the function f. In part (a), students needed to know how to use the sign of the derivative to determine when the function is increasing. Part (b) could be answered by knowing how to determine the concavity of f by observing where the graph of f' is increasing or decreasing. Part (c) required knowing the relationship between the values of f' and the slope of the tangent line of f. Part (d) required an application of the Fundamental Theorem of Calculus, recognizing that $f(b) - f(a)$ is the integral of f' over the closed interval $[a, b]$, and then using this and the fact that $f(0) = 3$ to calculate the values of f at $x = -3$ and $x = 4$. The value of f at $x = 4$ is most easily computed by finding the area between the graph of f' and the x-axis (a rectangle minus a semicircle) and subtracting this area from the value of f at $x = 0$.

The mean score was 4.14.

<u>Sample C</u> (Score 9)

The student earned all 9 points.

<u>Sample E</u> (Score 7)

The student earned 7 points: 2 points in each of parts (a), (b), and (d), and 1 point in part (c). In part (d), the student handled the initial condition incorrectly in finding $f(-3)$ and lost a point. Furthermore, in finding $f(4)$, the student evaluated the definite integral incorrectly, losing another point.

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Question 5

This problem presented a related rates setting, as well as a separable differential equation. The volume of coffee in a cylindrical pot was changing at a rate that was given as a function of the height. Part (a) asked students to find the rate of change of height as a function of the height. This required translating the information on the rate of change of the volume V into mathematical notation and showing how to use the chain rule to find the desired result. The solution was given so that all students could start part (b) on an equal footing. Part (b) asked students to solve the separable differential equation that was the solution to part (a). Part (c) used the answer to part (b) and required recognition that the coffeepot was empty precisely when the height was zero.

The mean score was 5.89.

Sample B (Score 9)

The student earned all 9 points.

Sample Y (Score 7)

The student earned 7 points: 3 points in part (a), 4 points in part (b), and 0 points in part (c). In part (b), the student failed to earn full credit since there is a sign error in solving for h. Since the student's solution in part (c) was not consistent with the work in part (b), the student failed to earn the point in this part.

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Question 6

This problem involved a power series. Students were given the power series expansion of a function and then asked questions that probed understanding of this function. Part (a) required that students know how to read or compute the values of the first and second derivative at $x = 0$ from the series. Students then needed to use this information to describe the nature of the critical point at $x = 0$. In part (b), students were required to know that with an alternating series whose terms are decreasing and approaching zero in absolute value, the error introduced by truncating the series has absolute value bounded by the absolute value of the next term. Part (c) presented a differential equation and asked students to show that the given power series satisfied that equation. Students could have done this by differentiating the power series, multiplying it by x, adding the original power series to this product, and then recognizing the result as the power series expansion for cosine. Alternatively, this part could be done by recognizing that x times the power series is the series for sine, the derivative of xy is $xy' + y$, and the derivative of sine is cosine.

The mean score was 2.67.

<u>Sample Z</u> (Score 9)

The student earned all 9 points.

<u>Sample A</u> (Score 7)

The student earned 7 points: 3 points in part (a), 1 point in part (b), and 3 points in part (c). In part (a), the student incorrectly argued that the function has a local maximum with a first derivative test, losing a point. In part (c), the student failed to earn full credit by only writing the first three terms of the series for $xy' + y$, from which the series for $\cos x$ cannot be definitively identified.

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