# AP® CALCULUS BC
# 2008 SCORING GUIDELINES (Form B)

## Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel $x$ kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

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| Solution | Scoring |
|---|---|
| (a) $\int_0^2 r(t)\,dt = 206.370$ kilometers | $2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$ |
| (b) $\dfrac{dg}{dt} = \dfrac{dg}{dx} \cdot \dfrac{dx}{dt}; \quad \dfrac{dx}{dt} = r(t)$ <br> $\left.\dfrac{dg}{dt}\right\rvert_{t=2} = \left.\dfrac{dg}{dx}\right\rvert_{x=206.370} \cdot r(2)$ <br> $= (0.050)(120) = 6$ liters/hour | $3: \begin{cases} 2: \text{uses chain rule} \\ 1: \text{answer with units} \end{cases}$ |
| (c) Let $T$ be the time at which the car's speed reaches 80 kilometers per hour. <br><br> Then, $r(T) = 80$ or $T = 0.331453$ hours. <br><br> At time $T$, the car has gone <br> $x(T) = \int_0^T r(t)\,dt = 10.794097$ kilometers <br> and has consumed $g(x(T)) = 0.537$ liters of gasoline. | $4: \begin{cases} 1: \text{equation } r(t) = 80 \\ 2: \text{distance integral} \\ 1: \text{answer} \end{cases}$ |

Form B
AB/BC 2
2A₁

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Work for problem 2(a)

Distance travelled by car during first 2 hours $= \int_0^2 r(t)\,dt$

$$= \int_0^2 120(1 - e^{-10t^2})\,dt$$

$$= 206.370 \text{ km}$$

Work for problem 2(b)

$g(x) = 0.05x(1 - e^{-x/2})$

Rate of change w.r.t. time of $g(x) = \frac{dg(x)}{dt} = 0.05(1 - e^{-x/2})\,r(t) + 0.05x(\frac{1}{2}e^{-x/2})\,r(t)$

∴ At time $t = 2$ hours, Rate of change w.r.t time of $g(x)$

$$= 0.05(1 - e^{(206.370/2)})\,r(2) + 0.05(206.370)(\tfrac{1}{2}e^{(-206.370/2)})\,r(2)$$

$$= 6 \text{ L/hr.}$$

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Continue problem 2 on page 7.

Work for problem 2(c)

$r(t) = 80$ km/hr $\Rightarrow 120(1 - e^{-10t^2}) = 80$

$\therefore t = 0.331$ hr

$\therefore$ Distance travelled $= \int_0^{0.331} r(t)\, dt$

$= \int_0^{0.331} 120(1 - e^{-10t^2})\, dt$

$= 10.794$ km

$\therefore$ Liters of gasoline used $= g(10.794)$

$= 0.05(10.794)(1 - e^{(10.794/2)})$

$= 0.537$ L

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Form B.
AB/BC 2
2B1

Work for problem 2(a)

$r(t) = 120(1-e^{-10t^2})$

$\int_0^2 r(t) = 206.37$

$\boxed{A: 206.37 \text{ km}}$

Work for problem 2(b)

$r(t) = \frac{dr}{dt} = 120(1-e^{-10t^2})$

$R(t) = \int 120(1-e^{-10t^2}) \text{. km}$

$g(R(t)) = 0.05x(1-e^{-x/2})$

$g'(R(t)) = 0.05e^{-\frac{x}{2}} \cdot (e^{\frac{x}{2}} + 0.5(x-2))$

$\left\{ \begin{array}{l} \text{when } t=2, \\ x = 206.37 \text{ km} \end{array} \right.$

$g'(x) = 0.05$

$\boxed{A: 0.05 \text{ liters per km}}$

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Continue problem 2 on page 7.

Work for problem 2(c)

$r(t) = 120\left(1-e^{-10t^2}\right) = 80$

$t = \cancel{-0.331453}$ or $0.331453$

$R(t) = \int_0^{0.331453} r(t)$

$= 10.7941$ km

$\rightarrow g(10.7941) = 0.53726$

A: 0.537 liters

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Form B
AB/BC 2
2C1

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Work for problem 2(a)

the distance traveled = $\int_0^2 r(t)\,dt$

$= \int_0^2 120(1-e^{-10t^2})\,dt = 206.37$ km

Work for problem 2(b)

$\frac{g'(x)}{r'(t)} =$

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Continue problem 2 on page 7.

2C2

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Work for problem 2(c)

at the speed of 80

$$80 = 120(1 - e^{-10t^2})$$

we find (t) and subsatet it in the result found in part ~~#~~ (b)

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## Question 2

**Sample: 2A**
**Score: 9**

The student earned all 9 points.

**Sample: 2B**
**Score: 6**

The student earned 6 points: 2 points in part (a), no points in part (b), and 4 points in part (c). The student presents correct work in parts (a) and (c). In part (b) the student attempts to use the chain rule but does not put together the correct pieces necessary to answer the question.

**Sample: 2C**
**Score: 3**

The student earned 3 points: 2 points in part (a), no points in part (b), and 1 point in part (c). The student presents correct work in part (a). No points were earned in part (b). In part (c) the student sets $r(t) = 80$ and earned the first point. Since the student does not solve the equation for $t$, the response did not earn the remaining points.