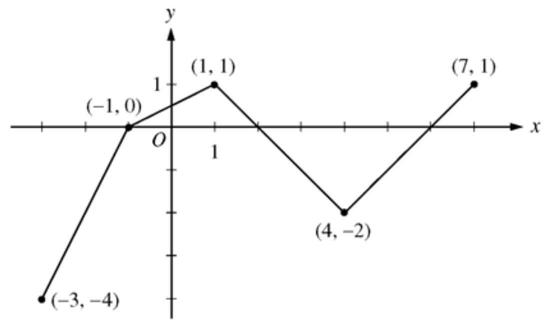
AP® CALCULUS BC
2008 SCORING GUIDELINES (Form B)

Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

(d) $$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$

Form B AB/BC 5
5A₁

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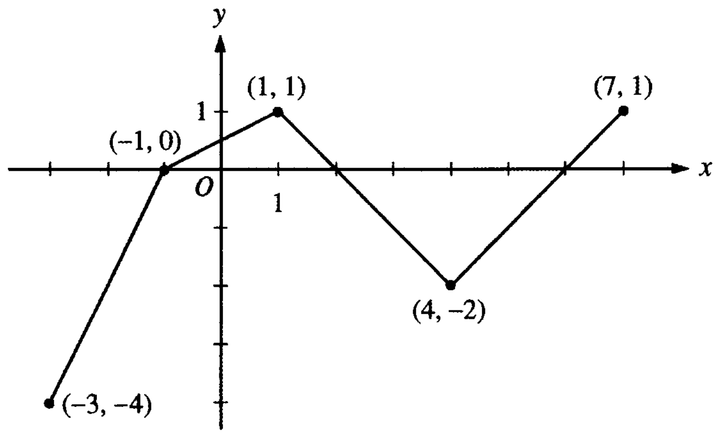

Graph of g'

Work for problem 5(a)

At points of inflection, g' should change from increasing to decreasing, or from decreasing to increasing.

Therefore, $x = 1, 4$

Work for problem 5(b)

$g'(x) = 0$ or the endpoints

$x=-3$: $\frac{7}{2} + 4 = \frac{15}{2}$

$x=-1$: $5 - 3 \times 1 \times \frac{1}{2} = \frac{7}{2}$

$x=2$: $g(2) = 5$

$x=6$: $g(6) = 5 - 4 \times 2 \times \frac{1}{2} = 1$

$x=7$: $g(7) = 1 + 1 \times 1 \times \frac{1}{2} = \frac{3}{2}$

therefore, the absolute maximum is $\frac{15}{2}$ when $x = -3$.

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Continue problem 5 on page 13.

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Work for problem 5(c)

$$\frac{g(7)-g(-3)}{7-(-3)} = \frac{\frac{3}{2}-\frac{15}{2}}{10} = \frac{\frac{\cancel{-12}^{\,6}}{\cancel{2}}}{10} = -\frac{3}{5}$$

Work for problem 5(d)

$$\frac{g'(7)-g'(-3)}{7-(-3)} = \frac{1-(-4)}{10} = \frac{1}{2}$$

But Mean Value Theorem ~~[illegible]~~ doesn't guarantee a value of c such that $g''(c) = \frac{1}{2}$

because ~~function~~ function g' is not differentiable ~~for all values for~~ at some points.

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Form B
AB/BC 5
5B1

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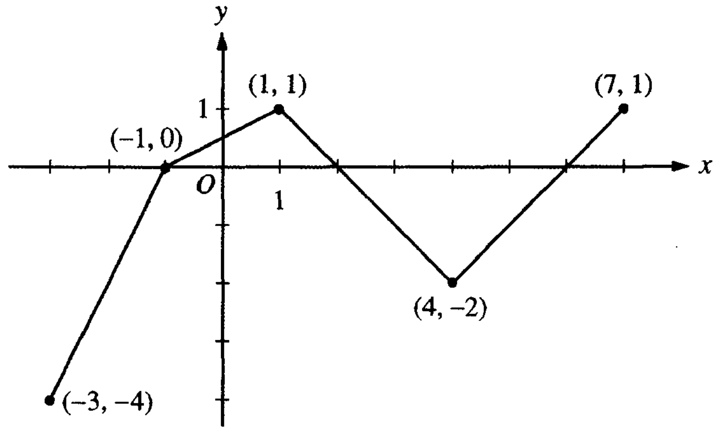

Graph of g'

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Work for problem 5(a)

$\boxed{x=1, \quad x=4}$

Since inflection of the graph (g(x)) is the point that g'(x) increasing become decrease or g'(x) decreasing become increase.

Work for problem 5(b)

$x=-3$

$g(-3)-4+\frac{3}{2}=5$

$\therefore g(-3)=\boxed{\frac{15}{2}}$

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Continue problem 5 on page 13.

Work for problem 5(c)

$$\frac{g(7)-g(-3)}{7-(-3)} = \frac{\frac{3}{2}-\frac{15}{2}}{10} = \frac{-6}{10} = \boxed{-0.6}$$

Meanwhile, $g(7) = g(2) - 4 + \frac{1}{2}$

$= 5 - 4 + \frac{1}{2}$

$= \frac{3}{2}$ "

$g(-3) = \frac{15}{2}$ "

Work for problem 5(d)

$$\frac{g'(7)-g'(-3)}{7-(-3)} = \frac{1-(-4)}{10} = \boxed{\frac{1}{2}}$$

No, it doesn't guarantee. Since $g'(x)$ is not ~~derivati~~ derivated at $x = -1, 1, 4.$,

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Form B
AB/BC 5
5C1

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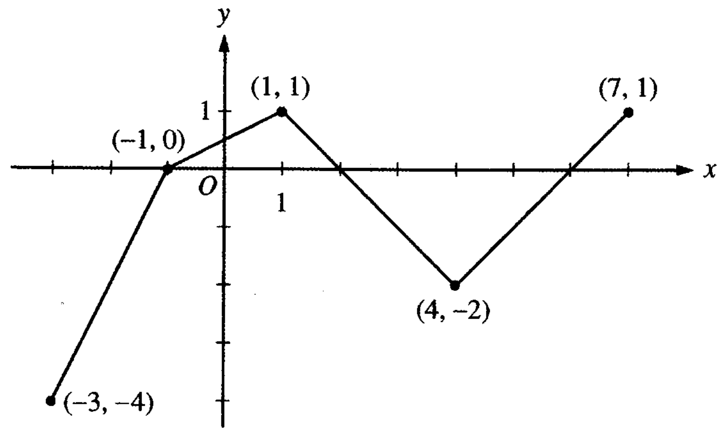

Graph of g'

Work for problem 5(a)

all points of inflection must have $f'(x)=0$ and change signs.
so $x=-1$, 2 and 6

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Work for problem 5(b)

$\int_{-3}^{-1} g'(x)dx = 4$.

$\int_{-1}^{2} g'(x)dx = 1.5$.

$\int_{2}^{6} g'(x)dx = 4$

$\int_{6}^{7} g'(x)dx = 0.5$

since $g(2)=5$

so $g(-1)=3.5$

$g(-3)=7.5$

$g(6)=1$

$g(7)=1.5$.

so the absolute maximum value of g on $-3\le x\le 7$ is 7.5.

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Continue problem 5 on page 13.

Work for problem 5(c)

average rate of change $= \dfrac{\int_{-3}^{7} g'(x)\,dx}{10}$

$= \dfrac{4+1.5+0.5+4}{10} = 1$

Work for problem 5(d)

average change of $g'(x) = \frac{2}{2} + \frac{1}{2} - \frac{1}{1} - \frac{1}{2} + \frac{1}{3} = \frac{1}{12}$

The Mean Value Theorem applied on the Interval $-3 \le x \le 7$ can not guarantee a value of c for $-3 < c < 7$ such that $g''(c)$ is equal to this average change. Since the rate of change is not continuous on the interval

Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). The student presents correct work in parts (a), (c), and (d). In part (b) the student does not identify $x = 2$ as a candidate, so the first point was not earned. The student finds the value of $g(-3)$ but does not find the value at the other endpoint, so the second point was not earned. The student did not earn the justification point since the work is not sufficient to state that $\frac{15}{2}$ is the maximum value.

Sample: 5C
Score: 4

The student earned 4 points: no points in part (a), 3 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not identify the correct values for the points of inflection. The student presents correct work in part (b). In part (c) the student uses the fact that the average rate of change is the average value of $g'(x)$ and presents a correct integral. The student makes an error in calculating the value of the integral so earned only 1 of the 2 points. In part (d) the student has an incorrect result for the average value of $g'(x)$, so the first point was not earned. Although the student declares that "[t]he Mean Value Theorem . . . can not guarantee a value of c" with the stated properties, the response includes the incorrect statement that "the rate of change is not continuous." Thus the student did not earn the second point.