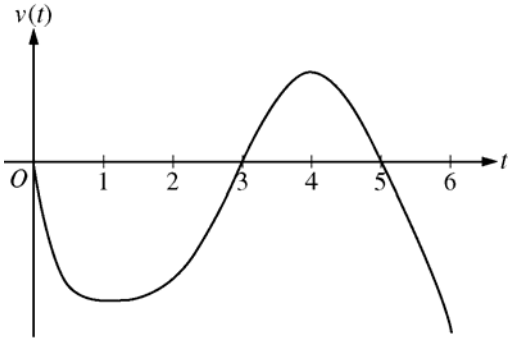
AP® CALCULUS BC
2008 SCORING GUIDELINES

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.

$$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.

3 : { 1 : identifies $t = 3$ as a candidate; 1 : considers $\int_0^6 v(t)\,dt$; 1 : conclusion }

(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.

By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.

3 : { 1 : positions at $t = 3$, $t = 5$, and $t = 6$; 1 : description of motion; 1 : conclusion }

(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.

1 : answer with reason

(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.

2 : { 1 : answer; 1 : justification }

4A,

NO CALCULATOR ALLOWED

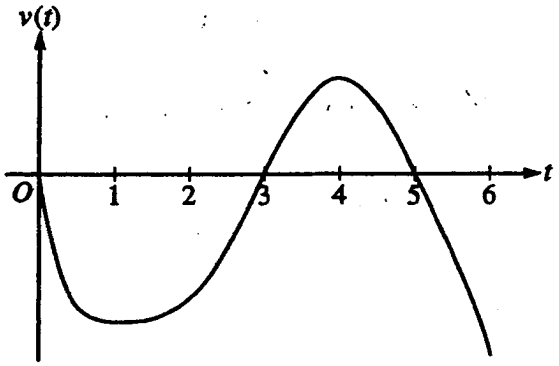
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

$x(t) = \int v(t)\,dt$

$x(0) = -2$

farthest to the left = greatest negative $x(t)$

from $[0,3]$ constantly moving to left

position: $x(0) - 8 = x(3)$

$-2 - 8 = x(3)$

$-10 = x(3)$

during $[3,5]$ moving to right

position: $x(0) - 8 + 3 = x(5)$ $\quad -2 - 8 + 3 = -7 = x(5)$

→ $[5,6]$ moving left

total displacement:

$x(0) - 8 + 3 - 2 =$

$-2 - 8 + 3 - 2 = -9$

farthest to the left @ $t = 3$, where $x = -10$

Work for problem 4(b)

$[0,3]$ particle moves from -2 to -10 (passes $x = -8$ once here)

$[3,5]$ moves from -10 to -7 (passes $x = -8$ once here)

$[5,6]$ from -7 to -9 passes -8

3 times particle is at -8 ble displacement calculations crossed $x = -8$ three times.

Continue problem 4 on page 11.

Work for problem 4(c)

the speed of the particle is decreasing because acceleration ($v'(t)$) and the direction of movement are in opposite directions. (acceleration is + but $v(t)$ is negative)

Work for problem 4(d)

Acceleration is negative from $[0,1) \cup (4,6]$ because the slope of velocity ($v'(t)$) is negative there.

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4B1

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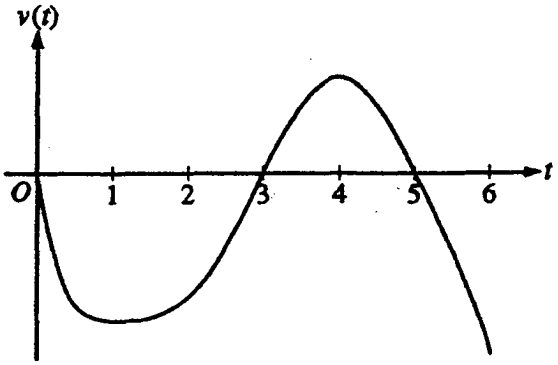
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

The point at which the particle is farthest to the left is $t=3$ here the particle would be at $x=-10$

This is so because the particle starts at position $x=-2$ then decreases on the graph for a total of 8 from $(0,3)$. The point farthest left could not be at any other point because after $t=3$ the velocity is positive so the particle is moving right. This positive value from $(3,5)$ has a greater area than $(5,6)$ so the particle ends farther to the right than at 3

Work for problem 4(b)

On the interval $0 \le t \le 6$ $x=-8$ three times

on the interval $(0,3)$ it adds -6 to the -2 so it passes -8 on some point and continues to -10 on $(3,5)$ it adds 3 bringing it to -7 passing -8 again and finally on $(5,6)$ it adds -2 bringing it to $x=-9$ passing -8 for the third time.

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Continue problem 4 on page 11.

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Work for problem 4(c)

on the interval $2 \leq t \leq 3$ the speed of the particle is increasing this is because from (2,3) the graph has a positive slope and the graph of v is velocity so if it is a positive slope it has positive acceleration so the particle is increasing in speed.

Work for problem 4(d)

The acceleration of the particle is negative on the intervals $0 \leq t \leq .5$ and $4 \leq t \leq 6$ this is because the slope of the graph of velocity is negative at these points.

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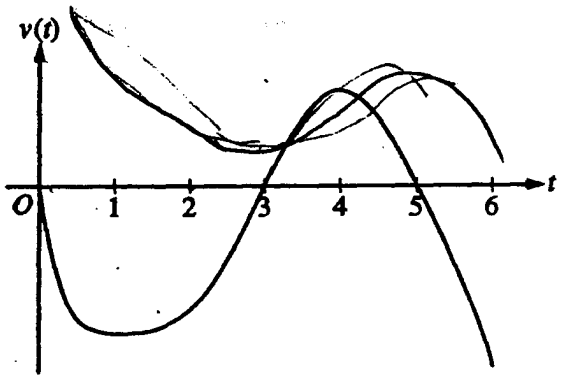
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

$\frac{dx}{dt} = v(t)$ (-2,0)

$\frac{dx}{dt} = 0$

at $t = 3$, it is the farthest left b/c $\frac{dx}{dt}$ goes from negative to positive

position = 8 − 2

The position is at $x = 6$

Work for problem 4(b)

the particle is at $x = -8$ at t-value, $t = 3$ because $\int_0^x v(t)\,dt = -8$ only when $x = 3$

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Continue problem 4 on page 11.

Work for problem 4(c)

$s = |v(t)|$, the speed is increasing because the graph of $v(t)$ is increasing on the interval $(2,3)$ and speed is the absolute value of the velocity ($a(t) = v'(t)$ is pos. on this interval.)

Work for problem 4(d)

the acceleration is negative on $(0,1)$ and $(4,6)$ because the velocity is decreasing on these intervals

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AP® CALCULUS BC
2008 SCORING COMMENTARY

Question 4

Overview

This problem presented students with the graph of a velocity function for a particle in motion along the *x*-axis for $0 \le t \le 6$. Areas of regions between the velocity curve and the *t*-axis were also given. Part (a) asked for the time and position of the particle when it is farthest left, so students needed to know that velocity is the derivative of position, and they had to be able to determine positions at critical times from the particle's initial position and areas of regions bounded by the velocity curve and the *t*-axis. Part (b) tested knowledge of the Intermediate Value Theorem applied to information about the particle's position function derived from its initial position and the supplied graph of its derivative. Part (c) asked students to interpret information about the speed of the particle from the velocity graph: namely, that if velocity is negative but increasing, then its absolute value, speed, is decreasing. Part (d) asked for the time intervals over which acceleration is negative, so students had to recognize that acceleration is the derivative of velocity. The sign of acceleration can be read from the intervals of increase/decrease of the velocity function.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 3 points in part (a), 3 points in part (b), no points in part (c), and no points in part (d). In part (a) the student clearly uses $t = 3$, describes the motion of the particle over each interval, and draws the correct conclusion. The student earned all 3 points. In part (b) the student finds the positions of the particle at the appropriate times, describes how the particle passes $x = -8$ on each interval, and draws the correct conclusion. The student earned all 3 points. In part (c) the student concludes that the speed of the particle is increasing so did not earn the point. In part (d) the student provides intervals that are not correct. Since the intervals do not have correct endpoints, the student did not earn any points.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student identifies $t = 3$ as a candidate but uses a local minimum justification. As a result, the student did not earn the last 2 points. In part (b) the student has a correct conclusion based on the work presented in part (a) but provides a reason that is not correct. The student did not earn the first point because the positions at $t = 5$ and $t = 6$ are not considered. The student does not describe the motion of the particle from position to position so did not earn the second or third points. In part (c) the student concludes that the speed of the particle is increasing so did not earn the point. In part (d) the student provides correct intervals and justification and earned both points.