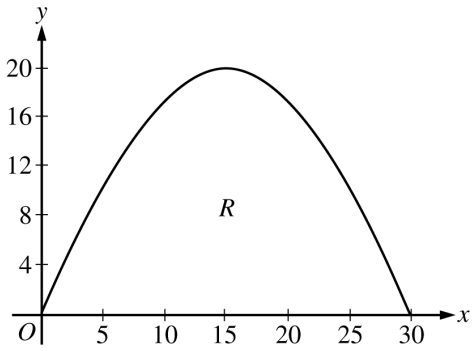
AP® CALCULUS BC
2009 SCORING GUIDELINES (Form B)

Question 1

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

(c) Find the perimeter of the base of the cake.

(a) Area $= 30 \cdot 20 - \int_0^{30} f(x)\,dx = 218.028$ cm^2

3 : { 2 : integral; 1 : answer }

(b) Volume $= \int_0^{30} \frac{\pi}{2}\left(\frac{f(x)}{2}\right)^2 dx = 2356.194$ cm^3

Therefore, the baker needs $2356.194 \times 0.05 = 117.809$ or 117.810 grams of chocolate.

3 : { 2 : integral; 1 : answer }

(c) Perimeter $= 30 + \int_0^{30} \sqrt{1 + (f'(x))^2}\,dx = 81.803$ or 81.804 cm

3 : { 2 : integral; 1 : answer }

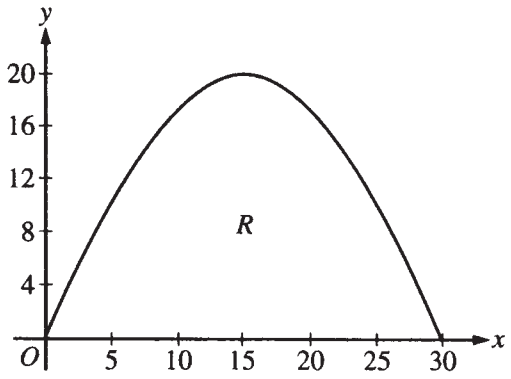
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

Area of $R = \int_0^{30} 20\sin\left(\frac{\pi x}{30}\right)dx \approx 381.972$

remaining cardboard $= 30 \times 20 - R$

$\approx 218.028 \text{ cm}^2$

Work for problem 1(b)

area of semicircle $= \frac{1}{2}r^2\pi$

$r = \frac{1}{2}y$ $\quad \therefore$ area of semicircle $= \frac{1}{8}y^2\pi$

$\therefore$ Volume $= \frac{1}{8}\pi \int_0^{30} \left(20\sin\left(\frac{\pi x}{30}\right)\right)^2 dx$

$\approx 2356.19449 \text{ cm}^3$

amount of Chocolate $= 0.05 \times 2356.19449 = 117.8097 \text{ g}$

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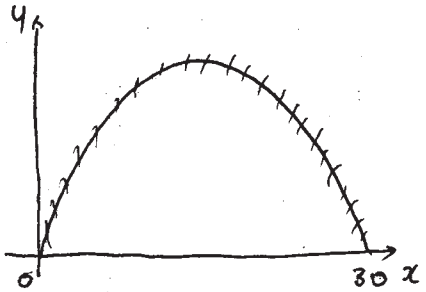
Continue problem 1 on page 5.

Work for problem 1(c)



perimeter = shaded line + a portion of x-axis

→ a portion of x-axis = 30 cm

shaded line = $\int_0^{30} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= \int_0^{30} \sqrt{1+\left(\frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)\right)^2}\,dx$

≈ 51.80370374 cm

shaded line + a portion of x-axis

$= 81.80370374$ cm

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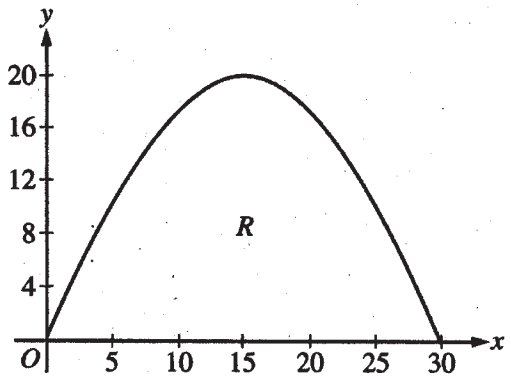
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

Area of original cardboard: $30 \times 20 = 600\ cm^2$ — ①

Area of $R = \int_0^{30} f(x)\,dx = \left[-\frac{600}{\pi}\cos\left(\frac{\pi x}{30}\right)\right]_0^{30} = 381.972\ cm^2$ — ②

Area of discarded cardboard: ① − ② $= 600 - 381.972 = 218.028\ cm^2$

Note that R does not go beyond the height (y-value) of 20, making the above calculation valid.

Work for problem 1(b)

Area of a semi-circle with radius $r = \frac{1}{2}\pi r^2$, thus area of a cross-section of R: $\frac{1}{2}\left(\frac{1}{2}\cdot 20\sin\left(\frac{\pi x}{30}\right)\right)^2\pi$. Integrating this from x=0 to 30, we get $\frac{\pi}{8}\int_0^{30}\left(20\sin\left(\frac{\pi x}{30}\right)\right)^2 dx$, we get $2356.194\ cm^3$ of cake.

Each cubic centimeter of the cake has 0.05 gram of chocolate; so $2356.194\ cm^3 \cdot 0.05\ g/cm^3 = 117.810\ g$. of chocolate will be in the cake.

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Continue problem 1 on page 5.

Work for problem 1(c)

The perimeter of the base of the cake can be divided into two parts.
The straight component on the x-axis is one, with length $30-0=30$ cm.
The curve of $f(x)$ from $x=0$ to 30 is the other.
The length of a curve is given by $\int_a^b \left(1+\left(\frac{dx}{dy}\right)^2\right)dx$.
$f'(x)$ is given by the question, thus the curve has length
$\int_0^{30}\left(1+\left(\frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)\right)^2\right)dx = 95.797$ cm.
Adding both components, the perimeter of $R = 30+95.797 = 125.797$ cm.

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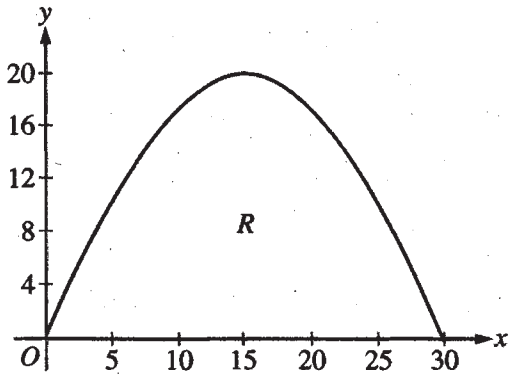
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$A_T = 30 \cdot 20 = 600 \text{ cm}^2$

$A_R = \int_0^{30} 20\sin\left(\frac{\pi x}{30}\right) dx$

$A_R = 381.972 \text{ cm}^2$

The area of the discarded cardboard will be 218.028 cm^2

$A_D = A_T - A_R$

$= 600 - 381.972$

$A_D = 218.028 \text{ cm}^2$

Work for problem 1(b)

$A = \frac{1}{2}\pi r^2 \qquad r = 20\sin\left(\frac{\pi x}{30}\right)$

$A = \frac{\pi}{2}\left(20\sin\left(\frac{\pi x}{30}\right)\right)^2$

$V = \int_0^{30} A = \frac{\pi}{2}\int_0^{30} \left[20\sin\left(\frac{\pi x}{30}\right)\right]^2 dx$

$V = 6000 \text{ cm}^3$

$6000 \cdot 0.05 = 300$

The cake will contain 300 grams of unsweetened chocolate.

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Continue problem 1 on page 5.

Work for problem 1(c)

P = arc length + 30

$L = \int_0^{30} \sqrt{\left(\frac{dy}{dx}\right)^2}\, dx$

$L = \int_0^{30} \sqrt{\left(\frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)\right)^2}\, dx$

$L = 65.747$

$P = 65.747 + 30$

$P = 95.797$ cm

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AP® CALCULUS BC
2009 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 3 points in part (b), and no points in part (c). In part (c) the student does not have an arclength integral and was not eligible for the answer point.

Sample: 1C
Score: 4

The student earned 4 points: 3 points in part (a), 1 point in part (b), and no points in part (c). In part (a) the student's work is correct. In part (b) the student has an error in the constant factor and earned only 1 of the integral points. The student was eligible for the last point, but the answer is not consistent with the work shown. In part (c) the student does not have an arclength integral and was not eligible for the answer point.