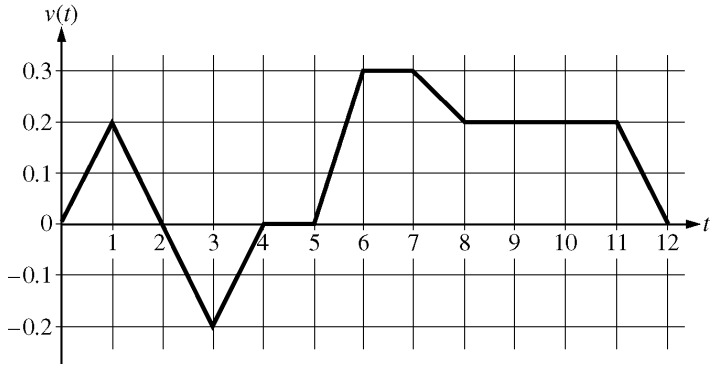
AP® CALCULUS BC
2009 SCORING GUIDELINES

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

(a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2

2 : { 1 : answer; 1 : units }

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

$$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$$
$$= 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$

2 : { 1 : meaning of integral; 1 : value of integral }

(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.

2 : { 1 : answer; 1 : reason }

(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

3 : { 2 : Larry's distance from school (1 : integral; 1 : value); 1 : Caren's distance from school and conclusion }

CALCULUS BC

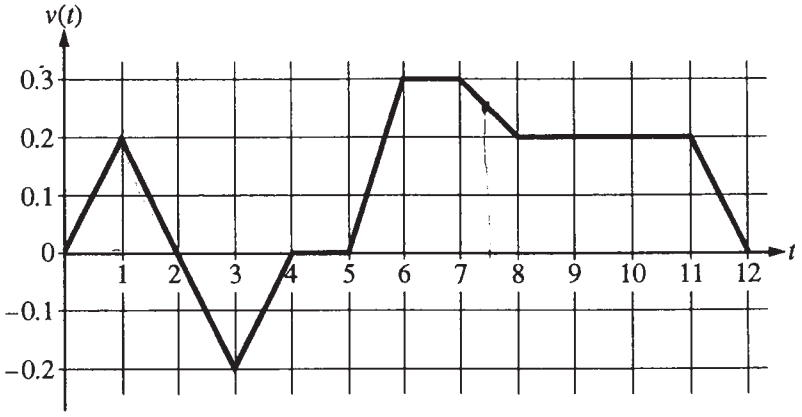
SECTION II, Part A

Time—45 minutes

Number of problems—3

1A1

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(a)

$a(7.5) = v'(7.5) = \dfrac{v(8)-v(7)}{8-7} = 0.2-0.3 = -0.1$

$\therefore a(7.5) = -0.1 \text{ mi}/\text{min}^2$

Work for problem 1(b)

$\int_0^{12} |v(t)|\,dt = 0.2+0.2+0.15+0.3+0.2+0.05+6(0.1)+0.1$

$= 1.8 \text{ miles}$

$\therefore \int_0^{12} |v(t)|\,dt$ is the total distance that Caron traveled from time $t=0$ min to $t=12$ min to arrive to the school, which is 1.8 mi.

Continue problem 1 on page 5

Work for problem 1(c)

She turns around at $t = 2$ minutes because that is when her velocity changes from positive to negative.

Work for problem 1(d)

$\int_0^{12} w(t)\,dt = \int_0^{12} \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)dt = 1.6$ mi

The distance from Lamy's house to school : 1.6mi

$\int_0^{12} v(t)\,dt = 0.15+0.3+0.2+0.05+0.6+0.1 = 1.4$ mi

The distance from Caren's house to school : 1.4mi

∴ Caren lives closer to school because the distance from school to her house is smaller than that to Lamy's house.

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1B1

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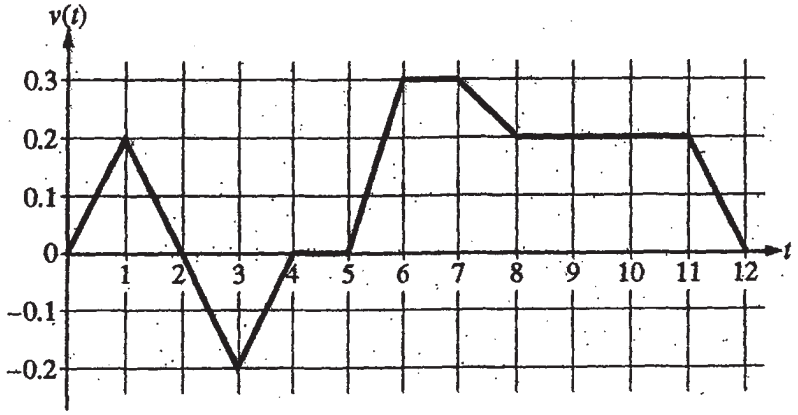
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(a)

$a(t) \quad \frac{dv}{dt} \quad \frac{v_2 - v_1}{t_2 - t_1} \quad \frac{-.2 - .3}{8 - 7} \quad \frac{-.1}{1} \quad -.1$

$-.1$ miles per minute

Work for problem 1(b)

$\int_0^{12} |v(t)|\,dt$ would show the total distance in miles that Caren traveled in 12 minutes

$\int_0^2 v(t)\,dt - \int_2^4 v(t)\,dt + \int_4^{12} v(t)\,dt$ $\quad .3/2 + .11 + .1 + .05$

$.2 \quad -.2 + .41 \quad .81$

Continue problem 1 on pag

1 1 1 1 1 1 1 1 1 1

Work for problem 1(c)

$t = 2$ because her velocity changes from + to − and the $\int_0^2 |V(t)|\,dt = \int_2^4 |V(t)|\,dt$

Work for problem 1(d)

$W(t) = \frac{11}{15}\sin\left(\frac{11}{12}t\right)$

Caren $\int_0^{12} |V(t)|\,dt = .81$ miles

Larry $\int_0^{12} W(t)\,dt = 1.6$ miles

Caren lives closer to school because she traveled less distance to get there

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CALCULUS BC

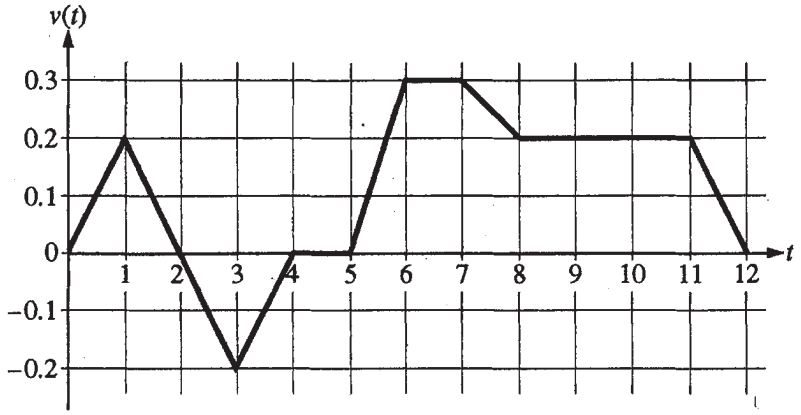
SECTION II, Part A

Time—45 minutes

Number of problems—3

1C, 1

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(a)

acceleration = $v'(t)$.

$v'(7.5)$

Work for problem 1(b)

∫ of velocity is the position.

so, $\int_0^{12} |v(t)|\,dt$ means how much or how far of distance Caven rode bicycle in 12 minutes.

$\int_0^{12} |v(t)|\,dt$ = Signed area. $\left[\left[\frac{(1\times0.2)}{2}\times 4\right] + \left(\frac{1\times0.3}{2}\right) + (1\times0.3) + (1\times0.2) + \left(\frac{1\times0.1}{2}\right) + ((1\times0.2)\times3) + \left(\frac{1\times0.2}{2}\right)\right] = 1.8$

Continue problem 1 on page 5

Work for problem 1(c)

She turns around to go back home at $t = 2$. the graph in the interval $1 \le x \le 3$, changes the direction.

~~She was~~

Work for problem 1(d)

~~At t=6, $W(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{2}\right) = \frac{\pi}{15} = 0.2093$. Larry's position from home~~

Larry's ~~is~~ distances to school : $\int_0^{12} \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)dt = 1.6$.

Caren's " $= \int_0^{12} f(t)dt = 1.8$ · (from answer (b))

So. Larry lives closer to school.

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AP® CALCULUS BC
2009 SCORING COMMENTARY

Question 1

Overview

This problem opened with a piecewise-linear graph. The graph models the velocity function $v(t)$ for bicycle rider Caren during a 12-minute period in which she travels along a straight road, starting at home at time $t = 0$ and arriving at school at time $t = 12$. Part (a) asked for Caren's acceleration at a particular time during her trip, which required students to recognize that acceleration is the derivative of velocity and to acquire the value of this derivative from the slope of the appropriate line segment on the given velocity graph. Part (b) asked for an interpretation of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip, as well as for the value of this integral. Part (c) provided the additional information that Caren needed to return home to retrieve her homework shortly after starting her journey. Students needed to associate Caren's direction of motion with the sign of her velocity to determine at what time she turned around. (Students were not required to observe that the distances traveled in each direction match.) In part (d) the velocity function for another bicycle rider, Larry, was modeled by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$ for the same 12-minute period, $0 \le t \le 12$. This part asked who lives closer to school, Caren or Larry. To respond, students needed to compute the two home-to-school distances, $\int_0^{12} v(t)\, dt$ (which equals $\int_5^{12} v(t)\, dt$) and $\int_0^{12} w(t)\, dt$.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student earned the first point for evaluating a correct difference quotient. The student's units of miles per minute are incorrect. In part (b) the student earned the first point for a correct interpretation of the meaning of the integral using correct units. The student's evaluation of the integral is incorrect. In part (c) the student's work is correct. The statement regarding the integrals of $|v(t)|$ on the two different intervals is correct but was not required to earn the point. In part (d) the student earned 2 points for Larry's distance from school by stating and evaluating the correct definite integral. The student's value for Caren's distance from school is incorrect, so the last point was not earned.

Sample: 1C
Score: 4

The student earned 4 points: no points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In part (b) the student does not indicate the correct units of miles, so the first point was not earned. The student earned the second point for a correct evaluation of the integral. In part (c) the student earned the first point for a correct answer. The student's reason is not valid. In part (d) the student earned 2 points for Larry's distance from school by stating and evaluating the correct definite integral. The student's value for Caren's distance from school is incorrect, so the last point was not earned.