AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 2

The velocity vector of a particle moving in the plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0$, the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.

(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

(c) Find the speed of the particle at $t = 1$.

(d) Find the acceleration vector of the particle at $t = 1$.

(a) The tangent line is vertical when $x'(t) = 0$ and $y'(t) \ne 0$.

On $0 < t < 1.5$, this happens at $t = 1.253$ and $t = 1.144$ or 1.145.

$2: \begin{cases} 1: \text{sets } \dfrac{dx}{dy} = 0 \\ 1: \text{answer} \end{cases}$

(b) $\left.\dfrac{dy}{dx}\right|_{t=1} = \dfrac{y'(1)}{x'(1)} = 0.863447$

$x(1) = -2 + \int_0^1 x'(t)\,dt = 9.314695$

$y(1) = 3 + \int_0^1 y'(t)\,dt = 4.620537$

The line tangent to the path of the particle at $t = 1$ has equation $y = 4.621 + 0.863(x - 9.315)$.

$4: \begin{cases} 1: \left.\dfrac{dy}{dx}\right|_{t=1} \\ 1: x(1) \\ 1: y(1) \\ 1: \text{equation} \end{cases}$

(c) Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$

1 : answer

(d) Acceleration vector: $\langle x''(1), y''(1)\rangle = \langle -28.425, 2.161\rangle$

$2: \begin{cases} 1: x''(1) \\ 1: y''(1) \end{cases}$

Work for problem 2(a)

Because the line tangent to the path of the partical is vertical

$\frac{dx}{dt} = 0$

$1 + 4\cos(t^2)\sin(e^t) = 0$

for $0 < t < 1.5$ $t = 1.145$ or $t = 1.253$

Work for problem 2(b)

the slope of the particle's path at $t=1$

is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1+2\sin(t^2)}{1+4\cos(t^2)\sin(e^t)} = \frac{1+2\sin(1)}{1+4\cos(1)\sin(e)}$

$= 0.863$

~~the path vector of the particle is (∫1+4cos(t²)sin(e^t), ∫1+2sin(t²)) with a position (-2,3) at t=0~~

The position of the particle at $t=1$ is

$(-2 + \int_0^1 1+4\cos(t^2)\sin(e^t)\,dt,\ 3 + \int_0^1 1+2\sin(t^2)\,dt)$

$(9.315, 4.621)$

Therefore, the line tangent to the path of the particle at $t=1$ is $y - 4.621 = 0.863(x - 9.315)$

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2A

Work for problem 2(c)

The speed of the particle at t=1 is

$\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2} = \sqrt{[14\cos(t^2)\sin(e^t)]^2+[1+2\sin(t^2)]^2}$

$= \sqrt{(14\cos 1\sin e)^2+(1+2\sin 1)^2}$

$= 4.105$

Work for problem 2(d)

The acceleration vector of the particle at t=1 is

$(\frac{d}{dt}(\frac{dx}{dt}), \frac{d}{dt}(\frac{dy}{dt}))$

$(14(-\sin(t^2)\cdot 2t\sin(e^t)+\sin(e^t)e^t\cos t^2, 2\cos(t^2)\cdot 2t)$

$\langle -28.425, 2.161 \rangle$

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 2B1

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Work for problem 2(a)

slope of the tangent $= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^t)}$.

When $\frac{dx}{dt} = 0$ the slope $\to \infty$ the line will be vertical

Let $14\cos(t^2)\sin(e^t) = 0 \quad 0 < t < 1.5$

$t_1 = 1.1447$.
$t_2 = 1.2533$.

Work for problem 2(b)

When $t=1$
Slope $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{1+2\sin 1}{14\cos 1 \sin e} = a$

$a = 0.8634$.

$x = \int \frac{dx}{dt} = \int 14\cos(t^2)\sin(e^t)\, dt$.
$y = \int \frac{dy}{dt} = \int 1+2\sin(t^2)\, dt = t - \frac{2\cos t^2}{t} + C$.

$y(t=0) = 0 - 0 + C = 3 \quad C = 3$.

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2B₂

Work for problem 2(c)

when $t=1$

$\frac{dx}{dt} = v(x) = 14\cos 1 \cdot \sin e = a \qquad a = 3.1072$

$\frac{dy}{dt} = v(y) = 1 + 2\sin 1 = b \qquad b = 2.6829$

speed $= \sqrt{v(x)^2 + v(y)^2} = \sqrt{a^2+b^2} = c$

$c = \text{speed} = 4.1052$

Work for problem 2(d)

acceleration $(x) = \frac{d^2x}{dt^2} = (14\cos(t^2)\sin e^t)'$

$= 14\cos(t^2)\cos(e^t)\, e^t - 14\sin(t^2)\, 2t \cdot \sin e^t$

acceleration $(x, t=1) = 14\cos 1 \cos(e) \cdot e - 14\sin(1)\, 2 \cdot \sin e = d$

$d = -28.4253$

acceleration $(y) = \frac{dy^2}{dt^2} = (1+2\sin(t^2))'$

$= 0 + 2\cos(t^2)\, 2t$

acceleration $(y, t=1) = 0 + 2\cos(1) \cdot 2 = f$

$f = 2.1612$

Thus the acceleration vector at $t=1$ is $(-28.4253, 2.1612)$

Work for problem 2(a)

solve $\frac{dx}{dt}=0$, yields

$\therefore \cos t^2=0$ or $\sin(e^x)=0$.

$\therefore t=\frac{\sqrt{2}}{2}\sqrt{\pi}$ or $t=\ln\pi$

$\therefore$ when ~~t equal~~ $t=1.253$ or $t=1.145$,
the line tangent to the path of the particle is vertical.

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Work for problem 2(b)

$$\frac{dy}{dx}=\frac{1+2\sin(t^2)}{14\cos(t^2)\sin(e^t)}\Big|_{t=1}=\frac{1+2\sin 1}{14\cos 1\cdot\sin e}\doteq 0.209$$

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Work for problem 2(c)

$V(1) = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} = \sqrt{[14\cos(t^2)\sin(e^t)]^2 + [1+2\sin(t^2)]^2} \Big|_{t=1}$

$\doteq 4.105$

Work for problem 2(d)

$a(1) = \sqrt{(\frac{d^2x}{dt^2})^2 + (\frac{d^2y}{dt^2})^2} = 28.5073$

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points. In part (a) an ideal solution would include that $\frac{dy}{dt} \neq 0$ at the two points. In part (d) the student's intermediate symbolic work contains an error. Since this question was on the calculator portion of the exam, it was presumed that the student corrected the error when producing a correct numerical result with the calculator.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student correctly evaluates $\frac{dy}{dx}$ at $t = 1.$ In parts (c) and (d), the student's work is correct.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student's numerical slope value is incorrect. In part (c) the student's work is correct. In part (d) the student's work is incorrect.