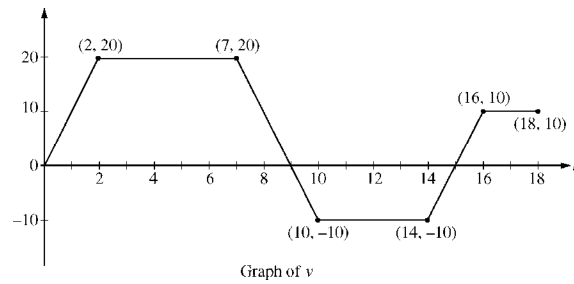
Question 4

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.



Graph of v

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

$2 : \begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

$2 : \begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

(c) The total distance traveled is $\int_0^{18} |v(t)|\, dt = 140 + 50 + 25 = 215.$

1 : answer

(d) For $7 < t < 10$, $a(t) = \dfrac{20 - (-10)}{7 - 10} = -10$

$v(t) = 20 - 10(t - 7) = -10t + 90$

$x(7) = \dfrac{7+5}{2} \cdot 20 = 120$

$$x(t) = x(7) + \int_7^t (-10u + 90)\, du$$

$$= 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t}$$

$$= -5t^2 + 90t - 265$$

$4 : \begin{cases} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{cases}$

 4A

NO CALCULATOR ALLOWED

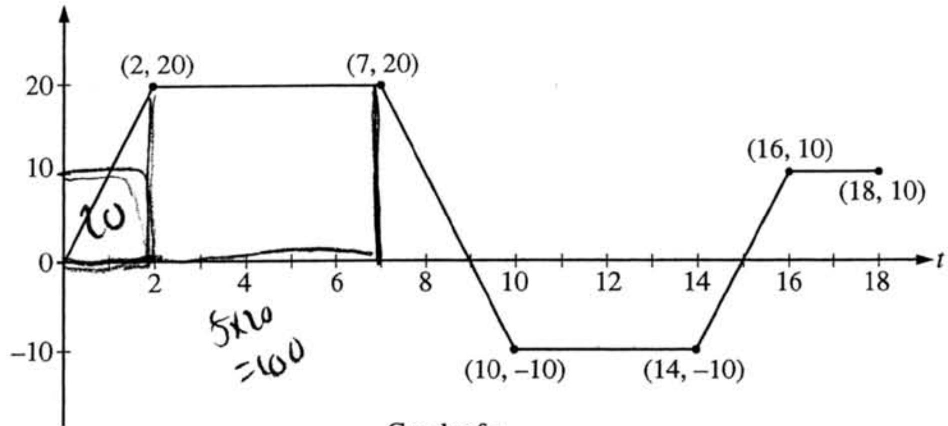
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

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Work for problem 4(a)

The squirrel changes direction for $t=15$ and $t=9$ because velocity changes from negative to positive and vice versa on those points.

Work for problem 4(b)

distance of squirrel from A, at t : $S(t)$

$S(9) = \int_0^9 v(t)\,dt = 140$

$S(15) = \int_0^{15} v(t)\,dt = 140 - 50 = 90$

$S(18) = \int_0^{18} v(t)\,dt = 90 + 25 = 115$

∴ The squirrel is farthest from the building when $t=9$. The squirrel is 140 away from the building A

Continue problem 4 on page 1

Work for problem 4(c)

$\int_0^{18} |v(t)| = 140 + 50 + 25 = \underline{215}$ //

Work for problem 4(d)

In $(7,10)$

$a(t) = v'(t) = \frac{-10-20}{10-7} = \frac{-30}{3} = \underline{-10}$ //

$v(9) = 0$

velocity: $y - 0 = -10(x-9)$

$y = -10x + 90$

$\therefore \underline{v(t) = -10x + 90}$ //

$x(t) = x(7) + \int_7^t v(t)\,dt$

$= 120 + [-5x^2 + 90x]_7^t$

$= 120 + (-5t^2 + 90t) - (385)$

$= \underline{-5t^2 + 90t - 265}$ //

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NO CALCULATOR ALLOWED

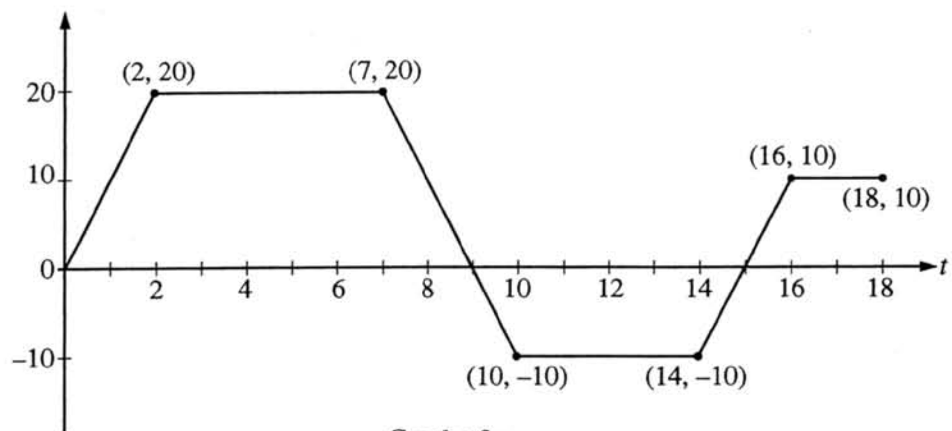
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

The squirrel changes direction at $t = 9$ and $t = 15$. His velocity changes from positive to negative.

Work for problem 4(b)

At $t = 9$ the squirrel is farthest from the building A. At $t = 9$, the squirrel is 140 units away from building A.

$\frac{1}{2} \cdot 20 \cdot (9 + 5) = 140$

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Continue problem 4 on page 1

NO CALCULATOR ALLOWED

Work for problem 4(c)

$\frac{1}{2} \cdot 20 \cdot (14) + \frac{1}{2} \cdot 10 \cdot (2+3) + \frac{1}{2} \cdot 10 \cdot (6+4)$

$140 + 25 + 50 = 215$

Total distance traveled $= 215$ units.

Work for problem 4(d)

$\boxed{a(t) = -10}$

$\frac{-10-20}{10-7} = \frac{-30}{3} = -10$

$\boxed{v(t) = -10x + 90}$

$x(t) = -5x^2 + 90x + 120$

$C = \frac{1}{2} \cdot 20 \cdot (5+7)$

$C = 120$

$\int -10x + 90 \, dx$

$-5x^2 + 90x + C$

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 4C1

NO CALCULATOR ALLOWED

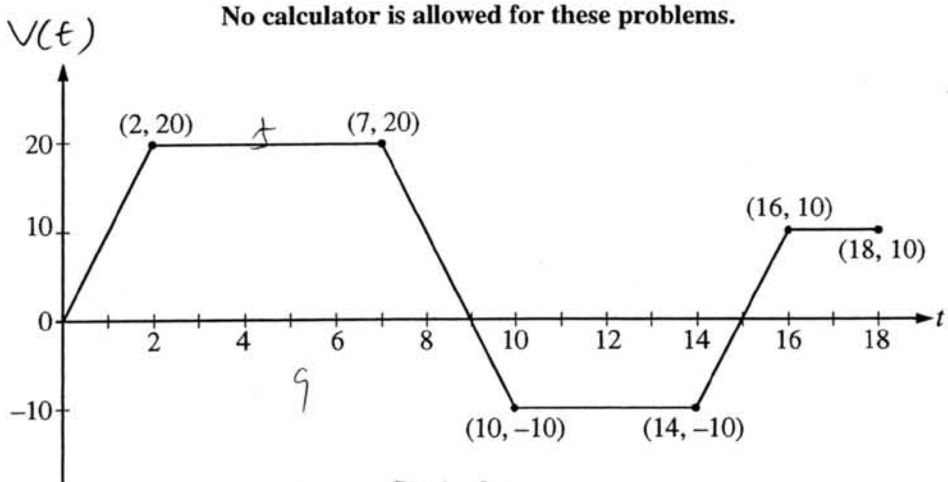
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

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Work for problem 4(a)

1. at $9 < t < 15$, the squirrel changes its direction since its velocity changes from positive to negative.

Work for problem 4(b)

(1) at $t = 9$. because that's when the area between the graph of $v(t)$ and the x-axis is the largest.

(2) $S = \frac{(5+9)\times 20}{2} = 140$

Continue problem 4 on page

 4C₂

NO CALCULATOR ALLOWED

Work for problem 4(c)

Total Distance :

$= \left| \int_0^9 v(t)\,dt \right| - \left| \int_9^{15} v(t)\,dt \right| + \left| \int_{15}^{18} v(t)\,dt \right|$

$= 140 - 50 + 25$

$= 115$

Work for problem 4(d)

$v(t)$ @, $7 < t < 10$ is a straight line.

passing $(7, 20), (10, -10)$

$\therefore \; v(t) = \underline{-10t + 90.}$

According to motion Theorems,

$a(t) = v'(t) = \underline{-10}$

$x(t) = \int v(t)\,dt = \underline{-5t^2 + 90t}$ ..

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student identifies the two points at which the graph of v crosses the t-axis but does not correctly explain why the squirrel changes direction at those two points. The given explanation applies to only one of the two points. In part (b) the student does not identify all candidates but does evaluate the distance at $t = 9.$ The second point was earned. In part (c) the student's work is correct. In part (d) the student has correct expressions for $a(t)$ and $v(t),$ but the expression for $x(t)$ does not incorporate the initial condition. One of the points for $x(t)$ was earned.

Sample: 4C
Score: 3

The student earned 3 points: no points in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d). In part (a) the student presents an interval instead of points. In part (b) the student does not identify all candidates but does evaluate the distance at $t = 9.$ The second point was earned. In part (c) the student finds displacement rather than total distance traveled. In part (d) the student has correct expressions for $a(t)$ and $v(t)$ but not for $x(t).$