CollegeBoard AP

Student Performance Q&A:

2010 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2010 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Michael Boardman of Pacific University in Forest Grove, Ore. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1

What was the intent of this question?

This problem supplied two rate functions related to the amount of snow on Janet's driveway during a nine-hour period. One function, f, given by $f(t) = 7te^{\cos t}$, measured in cubic feet per hour, models the rate of accumulation on the driveway for t between 0 and 9 hours after midnight. A second function, g, is a step function that gives the rate at which Janet removes snow from the driveway during this period. For part (a) students needed to use the definite integral $\int_0^6 f(t)\,dt$ to calculate the accumulation of snow on the driveway by 6 A.M. — integrating the rate of accumulation of snow over a time interval gives the net accumulation of snow during that time period. Part (b) asked for the rate of change of the volume of snow on the driveway at 8 A.M.; students needed to recognize this as the difference $f(8) - g(8)$ between the rate of accumulation and the rate of removal at time $t = 8$. Part (c) asked the students to recover a function h measuring the total amount of snow removed from the driveway for t between 0 and 9 hours after midnight. Students needed to integrate to obtain a piecewise-linear expression for h from the step function g. Part (d) asked for the amount of snow on the driveway at 9 A.M., which required students to compute the difference of two integrals, $\int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$.

How well did students perform on this question?

The mean score was 3.67 for AB students and 5.39 for BC students out of a possible 9 points. About 2.7 percent of AB students and 10.6 percent of BC students earned all 9 points. About 17.3 percent of AB students and 5.3 percent of BC students did not earn any points.

Students did particularly well on part (a). In general they earned both points, for setting up and evaluating the integral. Most students included correct units in their answers, even though units were not required.

Students generally performed well on part (b). As in part (a), students tended to include correct units even though they were not required.

Students experienced difficulty with part (c). Although the vast majority stated that $h(t) = 0$ on the interval $0 \le t \le 6$, most had incorrect expressions on the remaining two subintervals.

Most students performed better in part (d) than they did in part (c). Most never explicitly referred to $h(9)$ but wrote and computed $\int_0^9 f(t)\,dt$ and then later subtracted 125 and $2 \cdot 108$.

What were common student errors or omissions?

In part (b) some students mistakenly used $f'(8)$ rather than $f(8)$ in their calculations.

In part (c) the most common error was to write that $h(t) = 125t$ rather than $h(t) = 125(t-6)$ on $6 < t \le 7$, and similarly that $h(t) = 108t$ rather than $h(t) = 125 + 108(t-7)$ on $7 < t \le 9$. Some students added together the three rules for $h(t)$ on the three separate intervals. Students had difficulty expressing an integral with a variable limit of integration. Most students who wrote an integral used, for example, $\int_6^t 125\,dt$ rather than $\int_6^t 125\,ds$. No points were deducted for this.

In part (d) the most common error was to use $h(9) = 125 + 108$ rather than the correct $h(9) = 125 + 2 \cdot 108$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Algebra skills are important, even on calculator-active questions. In particular it is important for students to be adept with piecewise-defined functions.
- Free-response questions have multiple entry points. Students should attempt each part of a free-response question.

Question AB2/BC2

What was the intent of this question?

This problem involved a zoo's contest to name a baby elephant. Students were presented with a table of values indicating the number of entries $E(t)$, measured in hundreds, received in a special box and recorded at various times t during an eight-hour period. Part (a) asked for an estimate of the rate of deposit of entries into the box at time $t = 6$. Students needed to recognize this rate to be the derivative value $E'(6)$. Because $t = 6$ falls between the time values specified in the table, students needed to calculate the average rate of change of E across the smallest time subinterval from the table that brackets $t = 6$. Part (b) asked for an approximation to $\frac{1}{8}\int_0^8 E(t)\,dt$ using a trapezoidal sum and the subintervals of $[0, 8]$ indicated by the data in the table. Students were further asked to interpret this expression in context, with the expectation that they would recognize that it gives the average number of hundreds of entries in the box during the eight-hour period. In part (c) a function P is supplied that models the rate at which entries from the box were processed, by the hundred, during a four-hour period ($8 \le t \le 12$) that began after all entries had been received. This part asked for the number of entries that remained to be processed after the four hours. Students needed to recognize that the number of entries processed is given by $\int_8^{12} P(t)\, dt,$ so that the number remaining to be processed, in hundreds of entries, is given by the difference between the total number of entries in the box, $E(8),$ as given by the table, and the value of this integral. Part (d) cited the model $P(t)$ introduced in the previous part and asked for the time at which the entries were being processed most quickly. Students should have recognized this as asking for the time corresponding to the maximum value of $P(t)$ on the interval $8 \le t \le 12$ and applied a standard process for optimization on a closed interval.

How well did students perform on this question?

Student performance was average. The mean score was 2.60 for AB students and 3.92 for BC students out of a possible 9 points. About 0.5 percent of AB students and 1.8 percent of BC students earned all 9 points. About 19.2 percent of AB students and 5.9 percent of BC students did not earn any points. In general students had difficulty with the context and with function values presented in tabular form.

In part (a) most students were able to successfully approximate the rate of change.

In part (b) many students had difficulty giving an appropriate trapezoidal sum based on the tabular data. Some did not recognize $\frac{1}{8}\int_0^8 E(t)\, dt$ as the average value of $E(t)$.

In part (c) most students correctly set up and computed the integral $\int_8^{12} P(t)\, dt.$ Many went on to give the correct answer.

In part (d) most students entered the problem correctly, setting $P'(t) = 0.$ Many gave the correct answer as well, but many of those students did not show sufficient justification to earn the justification point.

What were common student errors or omissions?

Some students seemed confused over whether $E(t)$ represented the number of entries or the rate at which entries were being deposited in the box. This confusion affected their responses in the first three parts of the question.

In part (b) a number of students inappropriately applied the Trapezoidal Rule, failing to observe the variable widths in the intervals given in the table. For the third point in part (b), many students had difficulty addressing the units and the time interval correctly. Still more students failed to recognize the expression as an average value, not an average rate.

In part (d), although many students earned the first point for considering where the derivative was equal to zero, they often neglected to consider the endpoints in their justification of the absolute maximum.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students should be encouraged to show intermediate steps in their solutions. AP Exam Readers do not fill in the gaps in student explanations, and credit may not be given when the steps taken to arrive at an answer are not clear.

- It is important for students to understand a variety of methods for determining when a local maximum is a global maximum, and to understand the difference between a local argument and a global argument.
- Students need practice communicating mathematics using clear, mathematically correct and mathematically precise language.

Question AB3

What was the intent of this question?

The context for this problem was the line for an amusement-park ride. It was given that 700 people were in line when the ride began operation in the morning, and the rate $r(t)$, in people per hour, at which people join the ride was supplied via a piecewise-linear graph for $0 \le t \le 8.$ It was also given that during the eight hours the ride is in operation, people move onto it at the rate of 800 people per hour, provided there are people waiting. Part (a) asked for the number of people arriving at the ride between times $t = 0$ and $t = 3$ hours. Students needed to obtain this value by computing $\int_0^3 r(t)\, dt$ geometrically from the supplied graph. Part (b) asked whether the length of the line was increasing or decreasing between times $t = 2$ and $t = 3$ hours. Students could determine this by comparing the rate $r(t)$ at which the line grows to 800 people per hour, the rate at which people move from the line onto the ride. Part (c) asked for the time t when the line was longest and the length of the line at that time. Students needed to recognize that the line is growing when $r(t) > 800$ and shrinking when $r(t) < 800,$ so that the line is at its longest during the one time ($t = 3$) when the graph of r decreases through the value $r = 800.$ The number of people waiting in line at that time is computed by subtracting the $3 \cdot 800 = 2400$ people that move from the line onto the ride during the 3 hours from the sum of the 700 people in line at time $t = 0$ and $\int_0^3 r(t)\, dt,$ the number of people joining the line between times $t = 0$ and $t = 3$ hours. Part (d) asked for an equation whose solution gives the earliest time t at which there were no longer people in line. This occurs when the number of people that have joined the line by time $t,$ $700 + \int_0^t r(s)\, ds,$ matches the number that have moved from the line to the ride, $800t.$

How well did students perform on this question?

Students performed poorly, particularly in organizing their work and in giving complete and correct justifications and explanations. The mean score was 2.19 out of a possible 9 points. About 0.3 percent of students earned all 9 points. About 25 percent of students did not earn any points.

In part (a) most students attempted to compute the area between the t-axis and the graph of r, but many, even those with the correct answer, did not write the definite integral.

In part (b) most students stated correctly that the number of people waiting in line was increasing, but many did not provide valid arguments.

In part (c) some students correctly answered the problem but did not supply a complete justification.

In part (d) most students were unable to correctly construct the requested equation.

What were common student errors or omissions?

Many students did well in part (a). However, most students did not clearly show the method they used. Many students did not show that they were computing a definite integral, and work was difficult to follow in many cases.

In part (b) many students used an incorrect justification. The most common error was to misinterpret $r(t)$ as an amount rather than a rate.

In part (c) some students were able to identify $t = 3$ as the time that the line was longest, and they correctly stated that there were 1500 people in line at that time. However, they were not able to adequately justify this result. There were also students who claimed that the line was longest at $t = 2,$ the time at which the given graph achieved a maximum.

In part (d) it was very common for students to use the same variable for the upper limit of integration and the variable of integration, writing $\int_0^t r(t)\,dt$ rather than something like the form $\int_0^t r(s)\,ds,$ where the variables are different. Another notational difficulty was the absence of the differential from the integral. In this particular question, a response without a differential on the integral was ambiguous and may have been correct or incorrect depending on the placement of the implied differential. Some students wrote an indefinite integral, and others neglected to include the initial condition.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students must be careful in their use of mathematical notation. Missing or misplaced symbols and poor presentation of otherwise good ideas can lead to ambiguity or incorrect results. Students are expected to show intermediate work and give final answers in complete, correct form.
- Students need practice in providing clear, complete and unambiguous mathematical justifications.
- Students also need practice and facility with functions defined by an integral, such as $\int_0^t r(s)\,ds.$

Question AB4/BC4

What was the intent of this question?

In this problem students were given the graph of a region R bounded on the left by the y-axis, below by the curve $y = 2\sqrt{x},$ and above by the line $y = 6.$ In part (a) students were asked to find the area of R, requiring an appropriate integral (or difference of integrals), antiderivative, and evaluation. Part (b) asked for an integral expression that gives the volume of the solid obtained by revolving R about the line $y = 7.$ This is found by integrating cross-sectional areas that correspond to washers with outer radius $7 - 2\sqrt{x}$ and inner radius 1, where $0 \le x \le 9.$ Part (c) asked for an integral expression for the volume of a solid whose base is the region R and whose cross sections perpendicular to the y-axis are rectangles of height three times the lengths of their bases in R. Students needed to find the cross-sectional area function in terms of y and use this as the integrand in an integral with lower limit $y = 0$ and upper limit $y = 6.$

How well did students perform on this question?

Student performance was average. The mean score was 3.67 for AB students and 5.28 for BC students out of a possible 9 points. About 7.2 percent of AB students and 17.4 percent of BC students earned all 9 points. About 16.2 percent of AB students and 5 percent of BC students did not earn any points.

In part (a) most students correctly set up the integral representing the area of the region R. Most also correctly evaluated the integral.

Many students did not know how to approach part (b). Those who attempted this part generally were successful in writing a correct, or nearly correct, expression for the volume.

Part (c) proved quite difficult for most students. Many did not correctly solve for x in terms of y. Others had difficulty determining the area of the specified cross sections.

What were common student errors or omissions?

In part (a) some students had incorrect limits, writing $\int_0^6 \left(6 - 2\sqrt{x}\right) dx$ rather than $\int_0^9 \left(6 - 2\sqrt{x}\right) dx$. A number of students made algebra and arithmetic errors when simplifying their answers. Others did not correctly antidifferentiate $2\sqrt{x}$.

In part (b) some students had the terms in the integrand reversed, writing $\pi\int_0^9 \left((7-6)^2 - \left(7 - 2\sqrt{x}\right)^2\right) dx$ rather than the correct $\pi\int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2\right) dx$. Many had difficulty with the placement of parentheses. Some students attempted to simplify their integrands and made algebraic errors in the process. Many students did not write the differential, and, as in responses to Question AB3, this led to ambiguous or even incorrect results.

In part (c) some students did not solve for x in terms of y. Those students and others assumed the cross sections given were perpendicular to the x-axis rather than the y-axis.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students are expected to use correct mathematical notation at all times. Incorrect notation can communicate something other than what was intended. This includes placement of parentheses, use of the differential to terminate an integrand, and correct placement of limits of integration.
- Algebra skills are critical to success in calculus.
- Numerical answers can be left unsimplified when working on free-response questions. Emphasizing this and practicing with students will help them save time and reduce their chances of making an error.

Question AB5

What was the intent of this question?

This problem described a function g that is defined and differentiable on $[-7, 5]$ and with $g(0) = 5$. The graph of $y = g'(x)$ on $[-7, 5]$ was given, consisting of three line segments and a semicircle. Part (a) asked for values of $g(3)$ and $g(-2)$. These values are given by $5 + \int_0^3 g'(x)\,dx$ and $5 + \int_0^{-2} g'(x)\,dx$, respectively, with the definite integrals computed using geometry and properties of definite integrals. Part (b) asked for the x-coordinates of points of inflection for the graph of $y = g(x)$ on the interval $-7 < x < 5$. Students needed to reason graphically that these occur where the graph of g' changes from increasing to decreasing or vice versa. Part (c) introduced a new function, $h(x) = g(x) - \frac{1}{2}x^2$, and asked for x-coordinates of critical points of h and for the classification of each critical point as the location of a relative minimum, relative maximum or neither. Students needed to find that $h'(x) = g'(x) - x$ in order to determine the x-coordinates of critical points and apply a sign analysis of h' to classify these critical points.

How well did students perform on this question?

Students performed very poorly. The mean score was 1.75 out of a possible 9 points. About 0.3 percent of students earned all 9 points. About 38.9 percent of students did not earn any points.

In part (a) students generally recognized that answering the question involved computing areas associated with the given graph of g', but many did not perform the calculation correctly.

In part (b) most students listed either too many or too few points.

In part (c) many students were able to find $h'(x)$. Few students were able to identify $x = 3$ as an x-coordinate of a critical point, and fewer still were able to identify $x = \sqrt{2}$. Students also had difficulty in using $h'(x)$ or $h''(x)$ to classify each critical point as a relative minimum or relative maximum.

What were common student errors or omissions?

In part (a) many students did not use the initial condition $g(0) = 5$. Many miscalculated the area of the quarter circle typically as 2π, 4π, or $\frac{\pi}{2}$. Some students computed $g(-2)$ as $5 + \pi$ rather than the correct $5 - \pi$. Some made arithmetic errors and some used incorrect limits, writing, for example, $g(3) = 5 + \int_{-7}^3 g'(x)\,dx$ rather than the correct $g(3) = 5 + \int_0^3 g'(x)\,dx$. Some students attempted to determine a piecewise-defined expression for $g'(x)$ and subsequently antidifferentiate this expression. They were generally unsuccessful with this approach.

In part (b) most students listed either a superset or a subset of the correct set of x-coordinates of the points of inflection. Many of those students did not include $x = 2$ or $x = 3$ in their answers. Some students included $x = -2$. Many students with the correct answer had difficulty correctly explaining it.

In part (c) most students did not determine that $x = \sqrt{2}$ was the x-coordinate of one of the critical points of h. Some students incorrectly stated that $x = 3$ was the location of a relative maximum of h. Some included $x = 2$ and $x = 4.5$ as x-coordinates of critical points of h.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students must be able to draw conclusions about the behavior of the function f given the graph of its derivative, f'.
- Given a single value, $f(a)$, of the function f and either a symbolic or graphical presentation of f', students should be able to use the Fundamental Theorem of Calculus, $f(x) = f(a) + \int_a^x f'(t)\,dt$, to recover values of the function f.
- Students need practice in writing unambiguous and complete mathematical justifications for their results.

Question AB6

What was the intent of this question?

This problem identified f as a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$. It was also given that solutions to this differential equation satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Part (a) asked for an equation of the line tangent to the graph of f at $x = 1$. Students needed to evaluate the given expression for $\frac{dy}{dx}$ at the point $(1, 2)$ to find the slope of this line. Part (b) asked for an approximation to $f(1.1)$ using the tangent line equation from part (a). Given that $f(x) > 0$ for $1 < x < 1.1$, students were asked to determine whether this approximation is greater than or less than $f(1.1)$. To make the determination, students needed to use the given second derivative together with the fact that f is positive on the interval to ascertain the relative position of the tangent line and the graph of $y = f(x)$ for $1 < x < 1.1$. Part (c) asked for the particular solution $y = f(x)$ with initial condition $f(1) = 2$. Students should have used the method of separation of variables.

How well did students perform on this question?

Student performance was average. The mean score was 3.14 out of a possible 9 points. About 1.3 percent of students earned all 9 points. About 32.8 percent of students did not earn any points.

In part (a) most students earned both points. Some students incorrectly computed the slope.

In part (b) most students correctly computed the approximation. However, few attempted the explanation, and of those who did, few provided a correct explanation.

In part (c) most students began the process of solving the differential equation correctly, but most of them did not arrive at a correct solution.

What were common student errors or omissions?

In part (a) some students used the expression for the second derivative to compute the slope of the line.

In part (b) some students made arithmetic errors in computing their approximations. Most students did not attempt an explanation of the inequality. Of those who did, most either did not offer a valid approach or gave information only on the sign of $\frac{d^2y}{dx^2}$ at the point $(1, 2)$ rather than on the entire interval $1 \le x < 1.1.$

In part (c) some students incorrectly separated variables, writing, for example, $\int y^3\, dy = \int x\, dx.$ Many students incorrectly wrote that $\ln y^3$ was an antiderivative of $\frac{1}{y^3}.$ Many who had done correct work up to and including determining the constant of integration had difficulty using algebra to arrive at a correct expression for $y.$

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students should understand that the second derivative of a function f determines the concavity of the graph of $y = f(x),$ and that the concavity of this graph can be used to give information about tangent line estimates.
- It is important for students to know that the only standard power function with a logarithm as an antiderivative is $f(x) = x^{-1}.$
- Algebra skills are critical to success in calculus.

Question BC3

What was the intent of this question?

This problem described the path of a particle whose motion is described by $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ satisfies $\frac{dy}{dt} = te^{t-3} - 1.$ Part (a) asked for the speed of the particle at time $t = 3$ seconds. Part (b) asked for the total distance traveled by the particle for $0 \le t \le 4$ seconds. This is found by integrating $\sqrt{(x'(t))^2 + (y'(t))^2}$ over the interval $0 \le t \le 4.$ Part (c) asked for the time $t,$ $0 \le t \le 4,$ at which the line tangent to the particle's path is horizontal and whether the particle's direction of motion is toward the left or toward the right at that time. Students needed to solve $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ for t and determine the sign of $\frac{dx}{dt}$ at this time to establish the left-to-right direction of motion. In part (d) it was given that there is a point with x-coordinate 5 through which the particle passes twice. Students were asked for (i) the two values of t when that occurs, (ii) the slopes of the lines tangent to the particle's path at that point, and (iii) the y-coordinate of that point, given that $y(2) = 3 + \frac{1}{e}.$ After solving $x(t) = 5$ for $t = 1$ and $t = 3,$ the slopes can be found by evaluating $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ at each value of t and the y-coordinate by evaluating $y(3) = y(2) + \int_2^3 \frac{dy}{dt}\, dt$ (or the corresponding expression for $t = 1$).

How well did students perform on this question?

Student performance was average. The mean score was 3.89 out of a possible 9 points. About 4.1 percent of students earned all 9 points. About 14.3 percent of students did not earn any points.

In part (a) most students found the correct answer. Those who did not typically used the incorrect formula for speed.

In part (b) most students used the appropriate arc length formula and produced the correct answer.

In part (c) many students correctly found the *t*-value for which the line tangent to the path of the particle is horizontal. Some students had general difficulty dealing with the parameterized curve and were unable to determine this *t*-value.

In part (d) many students found the *t*-values that give $x(t) = 5,$ but many students had difficulty with parts (ii) and (iii).

What were common student errors or omissions?

In part (a) some students incorrectly used $\left|\frac{dy}{dx}\right|$ for the speed.

In part (b) some students used $\left|\frac{dy}{dx}\right|$ as the integrand.

In part (c) most students provided incorrect reasoning that used either $\frac{dy}{dx}$ or the value of $x(t)$ evaluated at the *t*-value presented.

In part (d) some students made algebraic or numerical errors in solving $x(t) = 5.$ Many students did not use the chain rule to determine the slopes of the tangent lines. Some did not correctly use the Fundamental Theorem of Calculus to find the *y*-coordinate.

Many students did not show the setups for calculations they performed on their graphing calculators.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need to be reminded that they are expected to show their work on the free-response section of the exam. Answers without supporting work usually will not receive credit.
- Students need practice with parameterized motion, computing speed, acceleration and direction of motion.

Question BC5

What was the intent of this question?

This problem presented the differential equation $\frac{dy}{dx} = 1 - y$ with a particular solution $y = f(x)$ satisfying $f(1) = 0.$ It was also given that $f(x) < 1$ for all values of x. Part (a) asked students to use Euler's method with two steps of equal size to approximate $f(0).$ Part (b) asked for the evaluation of

$\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$, anticipating that students would recognize an invitation to apply L'Hospital's Rule. Part (c) asked for the particular solution $y = f(x)$ with initial condition $f(1) = 0$. Students should have used the method of separation of variables.

How well did students perform on this question?

Student performance was good. The mean score was 4.54 out of a possible 9 points. About 7.2 percent of students earned all 9 points. About 12.4 percent of students did not earn any points.

In part (a) most students were able to apply Euler's method, though many had difficulty organizing their information.

In part (b) many students used L'Hospital's Rule, but many did not include the justification needed for its use.

In part (c) most students correctly separated variables. Many students were able to correctly antidifferentiate, with some making minor errors, and many reported the correct answer.

What were common student errors or omissions?

In part (a) many students used a step-size of $\frac{1}{2}$ in Euler's method rather than the correct $-\frac{1}{2}$. Some students presented their work for Euler's method in a table but did not label the quantities. When there was an error, it was difficult to ascertain what these quantities represented and whether or not the student was correctly applying Euler's method. Many students made arithmetic errors that led to incorrect answers.

In part (b) many students did not include a statement or work justifying the use of L'Hospital's Rule. In particular it was important for students to state that $\lim_{x \to 1} f(x) = 0 = \lim_{x \to 1}\left(x^3 - 1\right)$. In using L'Hospital's Rule, some students made an error computing the derivative of $x^3 - 1$.

In part (c) many students incorrectly reported $\ln|1 - y|$ as an antiderivative of $\frac{1}{1 - y}$. Some students made arithmetic or algebraic errors leading to incorrect answers.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Although the tabular approach can provide a useful framework for a student to work through the steps of Euler's method, students who use this approach must also be able to show how they obtained the values they entered into the table. Students should be encouraged to show their work in calculating values in the table, regardless of the relative difficulty of the calculations.
- When applying a theorem, students should be explicit that the hypotheses of the theorem are satisfied.

Question BC6

What was the intent of this question?

This problem provided the function f defined by $f(x) = \frac{\cos x - 1}{x^2}$ for $x \neq 0$ and $f(0) = -\frac{1}{2}$. It was given that f has derivatives of all orders, and the function g is defined by $g(x) = 1 + \int_0^x f(t)\,dt$. Part (a) asked for the first three nonzero terms and the general term of the Taylor series for $\cos x$ about $x = 0$. Students were to use this with algebraic manipulation to find the first three nonzero terms and the general term of the Taylor series for f about $x = 0$. In part (b) students were asked to use the Taylor series for f about $x = 0$ to determine whether f has a relative maximum, relative minimum or neither at $x = 0$. From the series for f students can establish that $f'(0) = 0$ and $f''(0) = \frac{1}{4 \cdot 3}$ and resolve this issue using the Second Derivative Test. Part (c) asked for the fifth-degree Taylor polynomial for g about $x = 0$. In part (d) it was given that the Taylor series for g about $x = 0$ is an alternating series whose terms decrease in absolute value to 0. Students were asked to use the third-degree Taylor polynomial for g about $x = 0$ to estimate $g(1)$ and to explain why this estimate is within $\frac{1}{6!}$ of the actual value. The properties of the series for $g(1)$ allow us to bound the error in this approximation by the absolute value of the next term in the series.

How well did students perform on this question?

Most students performed poorly. The mean score was 1.96 out of a possible 9 points. About 2.2 percent of students earned all 9 points. About 43.6 percent of students did not earn any points.

In part (a) many students were able to write at least the first two terms of the series for cosine. Because the rest of the question relied on having a reasonable series in part (a), some students did not earn points in subsequent parts.

In part (b) students who had a Taylor series in part (a) were able to use to use the Second Derivative Test and did well.

In part (c) students who imported a Taylor series from part (a) and considered term-by-term integration did well.

Many students earned no points in part (d) because they had made insufficient progress in parts (a) and (c) to be able to complete part (d).

What were common student errors or omissions?

In part (a) many students either did not write a general term or wrote an incorrect general term for the series for $\cos x$ or for $f(x)$. Many had difficulty manipulating the series for $\cos x$ to obtain the series for f. Some students subtracted one from *each* term of the series, rather than just subtracting one. Many students had misplaced or missing parentheses in the general term; for example, many students wrote $2n!$ rather than the correct $(2n)!$ Some students attempted to compute the Taylor series for f from the definition of Taylor series rather than manipulating the series for $\cos x$. Those students were not successful, given the complexity of the derivatives of f.

Some students attempted to use the First Derivative Test for part (b). This was quite difficult, and most of them were unsuccessful.

In part (c) some students did not correctly antidifferentiate the terms in the series for f.

In part (d) many students did not apply the error bound for this type of alternating series.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students are expected to know the power series expansions for e^x, $\sin x$, $\cos x$ and $\frac{1}{1-x}$, as specified in the AP Calculus Course Description.
- Students need practice with creating new series from a given series using techniques such as substitution and algebraic manipulation.
- Students also need practice with writing complete and correct mathematical justifications.
- The topics listed in the AP Calculus Course Description for polynomial approximations and series are an important part of the Calculus BC course. Teachers should devote sufficient time to series topics because they provide the most challenging content of the course.