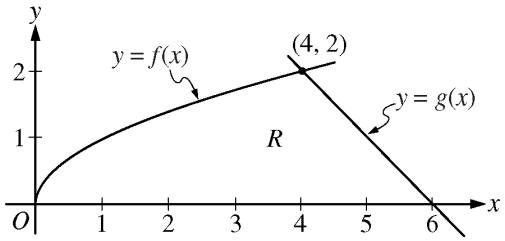
AP® CALCULUS BC
2011 SCORING GUIDELINES (Form B)

Question 3

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) $y = \sqrt{x} \Rightarrow x = y^2$

$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

3 : { 2 : integrand; 1 : answer }

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

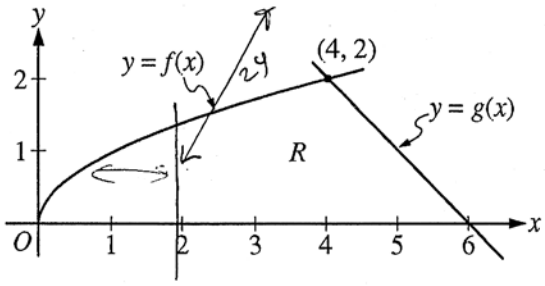
The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

3 : { 1 : $f'(x)$; 1 : equation; 1 : answer }

NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

$R = \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx =$

$= \frac{2}{3}x^{3/2}\Big|_0^4 + \left(6x - \frac{x^2}{2}\right)\Big|_4^6 = \frac{16}{3} + 36 - 18 - 24 + 8 =$

$= 2 + \frac{16}{3} = \frac{22}{3}$

Do not write beyond this border.

Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b)

for $0 \le y \le 2$: ~~R is between~~ ~~y = f(x) & the vertical axis~~;

$y = \sqrt{x} \Leftrightarrow x = y^2$; $\begin{cases} x=0 \\ y=0 \end{cases}$; $\begin{cases} x=4 \\ y=2 \end{cases}$;

$y = 6 - x \Leftrightarrow x = 6 - y \Rightarrow$

$$V = \int_0^2 \left((6 - y - y^2) \times 2y\right) dy$$

Work for problem 3(c)

tangent line to f;

$y_t = f(x_0) + f'(x_0)(x - x_0)$

$y_t = \sqrt{x_0} + \frac{1}{2\sqrt{x_0}}(x - x_0)$, y_t is $\perp$ to $g(x) \Rightarrow$

$\frac{1}{2\sqrt{x_0}} = -\frac{1}{g'(x_0)}$, $g'(x_0) = (6 - x_0)' = -1 \Rightarrow$

$\frac{1}{2\sqrt{x_0}} = 1 \Leftrightarrow 2\sqrt{x_0} = 1$

$\sqrt{x_0} = \frac{1}{2}$

$x_0 = \frac{1}{4}$, $y_0 = \sqrt{x_0} = \frac{1}{2} \Rightarrow$

$P(0.25, 0.5)$

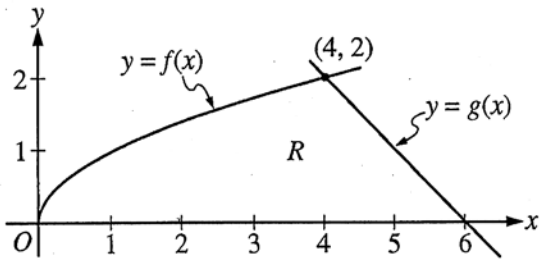
Do not write beyond this border.

Do not write beyond this border.

NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

a) $A = \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= \frac{2}{3}x^{3/2}\Big]_0^4 + \left(6x - \frac{x^2}{2}\right)\Big]_4^6 = \frac{2}{3}(4)^{3/2} + \left[\left((6\times 6) - \frac{36}{2}\right) - (24-8)\right]$

$= \frac{2}{3}(2)(4) + (18-16) = \frac{16}{3} + 2 = \frac{16}{3} + \frac{6}{3} = \frac{22}{3}\,(\text{unit})^2.$

Do not write beyond this border.

Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b)

$h = 2y$

$y = 6 - x \Rightarrow x = 6 - y$

$y = \sqrt{x} \Rightarrow x = y^2$

$$V = \int_a^b A(x)\,dx = \int_0^2 \left([(6-y) - y^2] \times 2[(6-y) - y^2] \right) dy$$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad g'(x) = -1$

$\frac{1}{2\sqrt{x}} = 1$ $\quad 1 = 2\sqrt{x}$

$\sqrt{x} = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

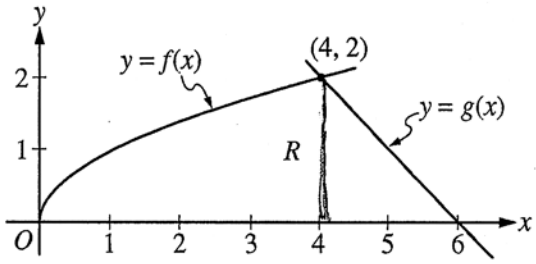
$f(\frac{1}{4}) = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow P\left(\frac{1}{4}, \frac{1}{2}\right)$

Do not write beyond this border.

NO CALCULATOR ALLOWED

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

(a). $R = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= 2x^{\frac{3}{2}}\Big|_0^4 + \left(6x - \frac{x^2}{2}\right)\Big|_4^6$

$= 2\times 4^{\frac{3}{2}} - 0 + 6\times 6 - \frac{36}{2} - 6\times 4 + \frac{16}{2}$

$= 16 + 36 - 18 - 24 + 8$

$= 8$

Do not write beyond this border.

Continue problem 3 on page 9.

Work for problem 3(b)

$V = \int_0^6 |6 - x - \sqrt{x}| \, dy \qquad (0 \le y \le 2)$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}} \qquad g'(x) = -1$

Since the line is perpendicular to $g(x)$, when $g'(x) = -1$.
the slope of the line is 1

Therefore $\frac{1}{2\sqrt{x}} = 1 \qquad x = \frac{1}{4}$

So $f(\frac{1}{4}) = \frac{1}{2}$ the coordinates of point P is $(\frac{1}{4}, \frac{1}{2})$

AP® CALCULUS BC
2011 SCORING COMMENTARY (Form B)

Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 3 points in part (a), no points in part (b), and 3 points in part (c). In parts (a) and (c) the student's work is correct. In part (b) the student's integrand was not eligible for any points.

Sample: 3C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), and 3 points in part (c). In part (a) the student earned the integral point. The student's antidifferentiation is incorrect. The student was eligible for the answer point, but the work contains an arithmetic error. In part (b) the student's integrand is incorrect and so was not eligible for any points. In part (c) the student's work is correct.