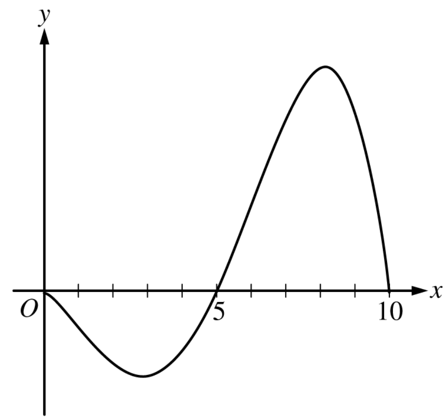
AP® CALCULUS BC
2011 SCORING GUIDELINES (Form B)

Question 4

The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure above. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.



Graph of f

(a) Find the average value of f on the interval $0 \le x \le 5$.

(b) Evaluate $\int_0^{10} (3f(x) + 2)\, dx$. Show the computations that lead to your answer.

(c) Let $g(x) = \int_5^x f(t)\, dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.

(d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

(a) Average value $= \frac{1}{5}\int_0^5 f(x)\, dx = \frac{-10}{5} = -2$	1 : answer
(b) $\int_0^{10} (3f(x) + 2)\, dx = 3\left(\int_0^5 f(x)\, dx + \int_5^{10} f(x)\, dx\right) + 20$ $= 3(-10 + 27) + 20 = 71$	2 : answer
(c) $g'(x) = f(x)$ $g'(x) < 0$ on $0 < x < 5$ $g'(x)$ is increasing on $3 < x < 8$. The graph of g is concave up and decreasing on $3 < x < 5$.	3 : $\begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{analysis} \\ 1 : \text{answer and reason} \end{cases}$
(d) Arc length $= \int_0^{20} \sqrt{1 + (h'(x))^2}\, dx = \int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\, dx$ Let $u = \frac{x}{2}$. Then $du = \frac{1}{2}dx$ and $\int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\, dx = 2\int_0^{10} \sqrt{1 + (f'(u))^2}\, du = 2(11 + 18) = 58$	3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{substitution} \\ 1 : \text{answer} \end{cases}$

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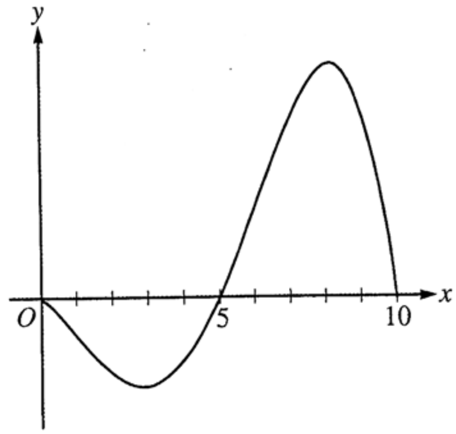

Graph of f

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Work for problem 4(a)

$$\frac{1}{5-0}\int_0^5 f(x)\,dx = \frac{1}{5-0}(-10) = \frac{-10}{5} = -2$$

-2

Work for problem 4(b)

$$\int_0^{10}(3f(x)+2)\,dx = 3\int_0^{10} f(x) + \int_0^{10} 2\,dx$$

$$= 3(-10+27) + [2x]_0^{10}$$

$$= 3(17)+20 = 51+20$$
$$= 71$$

71

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Continue problem 4 on page 11.

Work for problem 4(c)

$g(x) = \int_5^x f(t)\,dt \quad \therefore g'(x) = f(x)$

graph of g is decreasing on $0 < x < 5$ because $g'(x) < 0$

graph of g is concave up on $3 < x < 8$ because $g''(x) > 0$

graph of g is both concave up and decreasing on $3 < x < 5$ (because $g'(x) < 0$ and $g''(x) > 0$)

$3 < x < 5$

Work for problem 4(d)

$\int_0^{20} \sqrt{1 + \left(\frac{dh}{dx}\right)^2} \cdot dx = \int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\,dx = A$

$\frac{x}{2} = t \quad \frac{dt}{dx} = \frac{1}{2} \quad dt = \frac{1}{2}dx \qquad A = 2\int_0^{10} \sqrt{1 + f'(t)^2}\,dt$

$\int_0^{10} \sqrt{1 + (f'(x))^2}\,dx = 11 + 18 = 29$

$A = 2 \times 29 = 58$

58

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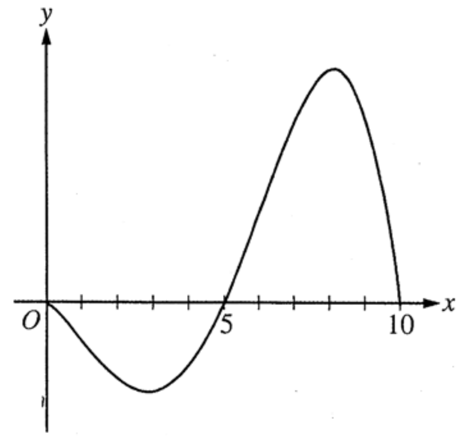

Graph of f

Work for problem 4(a)

$$\frac{\int_0^5 f(x)\,dx}{5-0} = 2$$

Work for problem 4(b)

$$\int_0^{10} (3f(x)+2)\,dx = 3\int_0^{10} f(x)\,dx + \int_0^{10} 2\,dx = 3\times 37+20 = 131$$

Continue problem 4 on page 11.

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Work for problem 4(c)

To be concave up and decreasing, $g'(x)<0$, $g''(x)>0$

$g'(x) = f(x) < 0$

~~$\therefore 0<x<5$~~ $\therefore$ ~~$0<x<3,\ 8<x<10$~~ $\therefore 0<x<5$

$g''(x) = f'(x) > 0$

~~$\therefore 3<x<8$~~ ~~$\therefore 3<$~~ $\therefore 3<x<8$

$\therefore 3<x<5$

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Work for problem 4(d)

$\int_0^{20} \sqrt{1+\{f'(x)\}^2}\,dx$

$\because = \int_0^{10} \sqrt{1+\{f'(x)\}^2}\,dx$

$= \int_0^5 \sqrt{1+\{f'(x)\}^2}\,dx + \int_5^{10} \sqrt{1+\{f'(x)\}^2}\,dx = 11+18 = 29$

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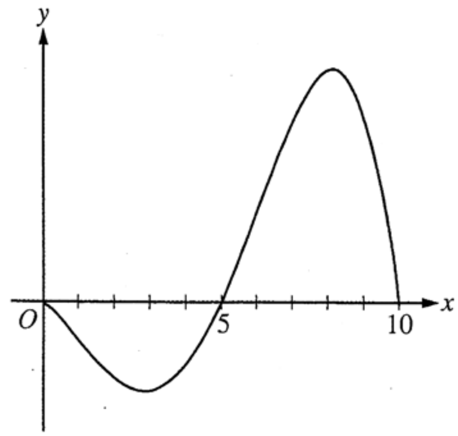

Graph of f

Work for problem 4(a)

$\frac{1}{5-0}\int_0^5 f(x)\,dx$

$= \frac{1}{5} \times 10 = 2$

Work for problem 4(b)

$\int_0^{10} (3f(x)+2)\,dx = 3\int_0^{10} f(x)\,dx + \int_0^{10} 2\,dx$

$= 17 \times 3 + 20$

$= 51 + 20$

$= 71$

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Continue problem 4 on page 11.

Work for problem 4(c)

$g(x) = \int_5^x f(t)\,dt$

$g(x) = F(x) - F(5)$

$g'(x) = f(x) - f(5) \ni g'(x) > 0 \rightarrow$ g(x) is increasing for any x at x > 5

$g''(x) = f'(x) - f'(5)$

Thus, there is no interval that the graph of g both concave up and decreasing.

Work for problem 4(d)

since $h(x) = 2f\left(\frac{x}{2}\right)$

arc length of $h(20)$ is $2f(10)$.

thus,

arc length of $h_1(20)$ is $= 2(11+18)$

$= 58$

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: no point in part (a), 1 point in part (b), 3 points in part (c), and 2 points in part (d). In parts (a) and (b) the student does not work correctly with the region below the x-axis. The student's work earned 1 of the 2 points in part (b). In part (c) the student's work is correct. In part (d) the student may be attempting a substitution because there are new correct limits of integration, but the factor of 2 is missing, so only 2 of the 3 possible points were earned.

Sample: 4C
Score: 3

The student earned 3 points: no point in part (a), 2 points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student does not work correctly with the region below the x-axis. In part (b) the student's work is correct. In part (c) the student's work is incorrect. In part (d) the student has the correct answer, which is achieved by doubling the arc length of the graph of $y = f(x)$ from $x = 0$ to $x = 10$ but gives no indication as to why that method works. The student earned 1 of the 3 possible points.