# AP® CALCULUS BC
# 2012 SCORING GUIDELINES

## Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time $t$ is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the $x$-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

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(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Because $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\left.dy/dt\right|_{t=2}}{\left.dx/dt\right|_{t=2}} = 3.055$ (or 3.054)

$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$
$= 0.651$ (or 0.650)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2. For $t \geq 0$, a particle is moving along a curve so that its position at time $t$ is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$

@ $t=2$,

$\frac{dx}{dt} = \frac{\sqrt{2+2}}{e^2}$

$\frac{dx}{dt} = \frac{\sqrt{4}}{e^2}$

$\frac{dx}{dt} = \frac{2}{e^2} > 0$

Therefore, the horizontal movement of the particle is to the right at $t=2$ because $\frac{dx}{dt}$ is positive at $t=2$.

$\frac{dy}{dx}$ = slope

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$\frac{dy}{dx} = \frac{\sin^2 t}{\frac{\sqrt{t+2}}{e^t}}$

@ $t=2$,

$\frac{dy}{dx} = \frac{\sin^2(2)}{\left(\frac{\sqrt{2+2}}{e^2}\right)}$

$\frac{dy}{dx} = \frac{e^2 \sin^2(2)}{\sqrt{4}}$

$\frac{dy}{dx} = \frac{e^2 \sin^2(2)}{2}$

$\frac{dy}{dx} = 3.055$

∴ The slope of the path of the particle is 3.055 at $t=2$.

(b) Find the $x$-coordinate of the particle's position at time $t = 4$.

$X(t) = 1 + \int_2^t \frac{dx}{dT}\, dT$

$X(t) = 1 + \int_2^t \frac{\sqrt{T+2}}{e^T}\, dT$

$X(4) = 1 + \int_2^4 \frac{\sqrt{T+2}}{e^T}\, dT$

$X(4) = 1.253$

∴ The x-coordinate of the particle's position at time $t=4$ is 1.253

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + (\sin^2 t)^2}$

Speed $= \sqrt{\frac{(t+2)}{e^{2t}} + \sin^4(t)}$

@ $t = 4$,

speed $= \sqrt{\left(\frac{4+2}{e^{2(4)}}\right) + \sin^4(4)}$

speed = .575

∴ The speed of the particle at time $t=4$ was .575

$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$

$v(t) = \left\langle \frac{\sqrt{t+2}}{e^t}, \sin^2 t \right\rangle$

$a(t) = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$

$a(t) = \left\langle \frac{(e^{2t})\frac{1}{2\sqrt{t+2}} - e^{2t}\sqrt{t+2}}{e^{2t}}, 2\sin(t)\cos(t) \right\rangle$

$a(t) = \left\langle \frac{\frac{1}{2\sqrt{t+2}} - \sqrt{t+2}}{e^t}, 2\sin(t)\cos(t) \right\rangle$

$a(4) = \left\langle \frac{\frac{1}{2\sqrt{4+2}} - \sqrt{4+2}}{e^4}, 2\sin(4)\cos(4) \right\rangle$

$a(4) = \langle -.041, .9897 \rangle$

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(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

distance $= \int_2^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + (\sin^2 t)^2}\, dt$

distance = .651

∴ The distance traveled by the particle from time $t=2$ to $t=4$ was .651

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2. For $t \geq 0$, a particle is moving along a curve so that its position at time $t$ is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t} \Rightarrow x'(2) = \frac{\sqrt{2+2}}{e^2} = \frac{2}{e^2} \approx 0.271$

To the right b/c $\frac{dx}{dt}$ [which represents (horizontal) velocity in the $\pm x$ direction] is positive.

slope $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin^2 t}{\left(\frac{\sqrt{t+2}}{e^t}\right)} \Rightarrow @\ t=2 = \frac{\sin^2(2)}{\left(\frac{\sqrt{2+2}}{e^2}\right)} \approx 3.0547 \approx \boxed{3.055}$

(b) Find the $x$-coordinate of the particle's position at time $t = 4$.

$x(t) = x(2) + \int_2^4 \frac{dx}{dt} = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\, dt \approx 1 + 0.252954408$

$x(4) \approx \boxed{1.253}$

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(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(\frac{\sqrt{4+2}}{e^4}\right)^2 + [\sin^2(4)]^2} = 0.574504453$

$= \boxed{0.575}$

$a(4) = \frac{d^2y/dt^2}{d^2x/dt^2} = \frac{2\sin 4 \cos^2 4}{\left(\frac{\frac{1}{2}(4+2)^{-1/2}(e^4)(e^4)(\sqrt{4+2})}{e^{2(4)}}\right)} =$

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

dist $= \int_2^4 \left|\frac{dy}{dx}\right| = \int_2^4 \left|\frac{\sin^2 t}{\left(\frac{\sqrt{t+2}}{e^t}\right)}\right| = 5.233420614 = \boxed{5.233}$

2. For $t \geq 0$, a particle is moving along a curve so that its position at time $t$ is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$. 1

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin^2 t}{\frac{\sqrt{t+2}}{e^t}} \quad t=2 \quad = \frac{\sin^2(2)}{\frac{\sqrt{2+2}}{e^2}} = 3.055$$

slope of the particle @ $t=2$.

The horizontal movement of the particle at $t=2$ is moving to the right b/c the slope is positive and $t \geq 0$.

(b) Find the $x$-coordinate of the particle's position at time $t = 4$.

$$\int \frac{dx}{dt}$$

$$\int \frac{\sqrt{t+2}}{e^t}\, dt$$

$$\int \frac{(t+2)^{1/2}}{e^t}\, d$$



Continue problem 2 on page 7.



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2C2

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + (\sin^2 t)^2}$$

$$= \sqrt{\left(\frac{2}{e^2}\right)^2 + (\sin^2(2))^2}$$

$$= \boxed{.7569}$$

$$\text{acceler} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$$

$$= \sqrt{\quad}$$

$= \sin(t)^2$

$2(\sin(t)) \cdot \cos t$

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(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

$$TDT = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$TDT = \int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + (\sin^2 t)^2}\, dt$$

$$\boxed{TDT = .9004}$$

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# AP® CALCULUS BC
# 2012 SCORING COMMENTARY

## Question 2

### Overview

This problem described the path of a particle whose position at time $t$ is given by $(x(t), y(t))$, where $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$. Part (a) asked whether the particle's horizontal direction of motion is toward the left or toward the right at time $t = 2$. Students should have determined the sign of $\frac{dx}{dt}$ at this time to establish the direction of motion. Students were asked to find the slope of the particle's path at that time. The slope can be found by evaluating $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ at $t = 2$. Part (b) asked students to find the $x$-coordinate of the particle's position at time $t = 4$. This is calculated using the expression $x(4) = x(0) + \int_0^4 x'(t)\,dt$. Part (c) asked for the speed of the particle at time $t = 4$ seconds. This value is found by evaluating $\sqrt{(x'(t))^2 + (y'(t))^2}$ at time $t = 4$. Students were then asked for the acceleration vector at this time, which is given by $\langle x''(4), y''(4) \rangle$. Part (d) asked for the distance traveled by the particle over the interval $2 \le t \le 4$ seconds. This is found by integrating $\sqrt{(x'(t))^2 + (y'(t))^2}$ over the interval $2 \le t \le 4$.

**Sample: 2A**
**Score: 9**

The student earned all 9 points.

**Sample: 2B**
**Score: 6**

The student earned 6 points: 3 points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In parts (a) and (b) the student's work is correct. In part (c) the student correctly evaluates the speed. The expression for acceleration is incorrect. In part (d) the student presents an incorrect integral for distance.

**Sample: 2C**
**Score: 3**

The student earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student considers $\frac{dy/dt}{dx/dt}$ and calculates the slope correctly. The student's reason for the horizontal movement of the particle is incorrect. In parts (b) and (c) the student's work is not sufficient to earn any points. In part (d) the student's integral is correct, so 1 point was earned.