AP® CALCULUS BC
2012 SCORING GUIDELINES

Question 4

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

(a) $f(1) = 15, \quad f'(1) = 8$

An equation for the tangent line is
$y = 15 + 8(x - 1)$.

$f(1.4) \approx 15 + 8(1.4 - 1) = 18.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(b) $\int_1^{1.4} f'(x)\,dx \approx (0.2)(10) + (0.2)(13) = 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)\,dx$

$f(1.4) \approx 15 + 4.6 = 19.6$

$3 : \begin{cases} 1 : \text{midpoint Riemann sum} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(c) $f(1.2) \approx f(1) + (0.2)(8) = 16.6$

$f(1.4) \approx 16.6 + (0.2)(12) = 19.0$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$

(d) $$T_2(x) = 15 + 8(x - 1) + \frac{20}{2!}(x - 1)^2$$
$$= 15 + 8(x - 1) + 10(x - 1)^2$$

$f(1.4) \approx 15 + 8(1.4 - 1) + 10(1.4 - 1)^2 = 19.8$

$2 : \begin{cases} 1 : \text{Taylor polynomial} \\ 1 : \text{approximation} \end{cases}$

x	1	1.1	1.2	1.3	1.4
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4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y - 15 = 8(x-1)$

$\boxed{y = 8x + 7}$

$y(1.4) \approx 8(1.4) + 7$

$= 11.2 + 7$

$= \boxed{18.2}$

1.4
$\times 8$
11.2

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$\int_1^{1.4} f'(x)\,dx \approx (1.2-1)(10) + (1.4-1.2)(13)$

$= (0.2)(10) + (0.2)(13)$

$= 2 + 2.6 = \boxed{4.6}$

$f(1.4) \approx f(1) + \int_1^{1.4} f'(x)\,dx$

$= 15 + 4.6$

$= \boxed{19.6}$

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4A 2

16.6
2.4
19.0

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(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$f(1.2) \approx f(1) + f'(1)(0.2) = 15 + (8)(0.2) = 15 + 1.6 = 16.6$

$f(1.4) \approx f(1.2) + f'(1.2)(0.2) = 16.6 + (12)(0.2) = 16.6 + 2.4$
$= 19$

$\therefore \boxed{f(1.4) \approx 19}$

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(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$$P_2(x) = \frac{f(1)(x-1)^0}{0!} + \frac{f'(1)(x-1)^1}{1!} + \frac{f''(1)(x-1)^2}{2!}$$

$$P_2(x) = 15 + \frac{8(x-1)}{1!} + \frac{20(x-1)^2}{2!}$$

$$\boxed{P_2(x) = 15 + 8(x-1) + 10(x-1)^2}$$

$$f(1.4) \approx 15 + 8(1.4-1) + 10(1.4-1)^2$$
$$= 15 + 8(0.4) + 10(0.4)^2$$
$$= 15 + 3.2 + 1.6$$
$$= \boxed{19.8}$$

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NO CALCULATOR ALLOWED

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y = mx + b$

$m = 8$

$15 = 8(1) + b$

$7 = b$

$f(x) = 8x + 7$

$f(x) = 8(1.4) + 7$

$= 14.2$

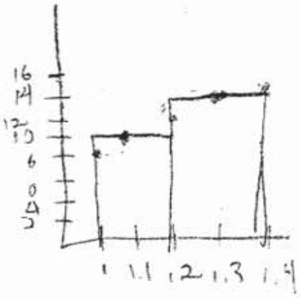
$$\begin{array}{r} 1.4 \\ \times\ 8 \\ \hline 11.2 \end{array}$$

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$[(1.2-1)(10) + (1.4-1.2)(13)]$

$[0.2(10) + 0.2(13)]$

$[2 + 2.6] \approx 4.6$



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4B 2

NO CALCULATOR ALLOWED

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$\Delta x = 0.2$

$f(1) = 15 \quad (1, 15)$

$f^2 = 15 + 8(0.2) = 15 + 1.6 = 16.6 \quad (1.2, 16.6)$

$f(1.4) = 16.6 + 12(0.2) = 16.6 + 2.4 = 17.4 \quad (1.4, 17.4)$

$f(1.4) \approx 17.4$

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$\frac{15(x-1)^0}{0!} + \frac{8}{1!}(x-1)^1 + \frac{20}{2!}(x-1)^2$

$P_2(x) = 15 + 8(x-1) + 10(x-1)^2$

$= 15 + 8(1.4 - 1) + 10(1.4 - 1)^2$

$= 15 + 8(0.4) + 10(0.4)^2$

$= 15 + 3.2 + 10(.16)$

$= 18.2 + 1.60 \approx 19.80$

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NO CALCULATOR ALLOWED

x	1	1.1	1.2	1.3	1.4
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4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y - y_1 = m(x - x_1)$ $\quad m = dy/dx = f'(x)$

$y - 15 = 8(x-1)$

$y = 8x + 14$

$y = (8)(1.4) + 14$

$y = 11.2 + 14$

$y = 25.2$

$1.4 \times 8 = 11.2$

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$\frac{1}{2}\int_1^{1.4} 10(2) + 13(2)$

$\frac{1}{2}\int 20 + 26$

$\frac{1}{2}(46)$

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4C 2

NO CALCULATOR ALLOWED

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$\Delta y = m \cdot \Delta x$

$\Delta y = 8 \cdot .2$

$\Delta y = 1.6$

$y = 15 + 1.6 = 16.6$

$(1.2, 16.6)$

$\Delta y = 12 \cdot .2$

$\Delta y = 2.4$

$y = 16.6 + 2.4$

$(1.4, 19.0)$

$8 \times .2 = 1.6$

$16.6 + 2.4 = 19.0$

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$$8 + 12(x-1)^1 + \frac{14.5(x-1)^2}{2!}$$

$$8 + 12(1.4-1) + \frac{14.5(1.4-1)^2}{2!}$$

$8 + 4.8 + 1.45 =$

$f(1.4) = 14.25$

$.4^2$ $\quad .4 \times .4 = .16$ $\quad 12 \times .16$: 72, 120

$6(.2)$

$12 \times .4 = 4.8$

$7.25 \times .2 = 1.450$

$8 + 4.8 + 1.45 = 14.25$

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AP® CALCULUS BC
2012 SCORING COMMENTARY

Question 4

Overview

Students were presented with a table of values for f' at selected values of x given that f is a twice-differentiable function. The values for $f(1)$ and $f''(1)$ are also given. Part (a) asked students to write an equation for the line tangent to the graph of f at $x = 1$ and then use this line to approximate $f(1.4)$. Students should have used the given values for $f(1)$ and $f'(1)$ to construct an equation equivalent to $y = f(1) + f'(1)(x - 1)$. Students could then substitute $x = 1.4$ to obtain the desired approximation. Part (b) asked students to use a midpoint Riemann sum with two subintervals of equal length, based on values in the table, to approximate $\int_1^{1.4} f'(x)\, dx$. They were then asked to use this approximation to estimate $f(1.4)$. This estimate is obtained by using the midpoint Riemann sum in the expression $f(1) + \int_1^{1.4} f'(x)\, dx$. Part (c) asked students to use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Part (d) asked for the second-degree Taylor polynomial for f about $x = 1$, which was then used to obtain yet another approximation for $f(1.4)$. Students should have used the given values for $f(1)$, $f'(1)$, and $f''(1)$ to write the Taylor polynomial.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In parts (a) and (d) the student's work is correct. In part (b) the student presents a correct midpoint Riemann sum, so the first point was earned. In part (c) the student correctly presents Euler's method with two steps, so the first point was earned. The student's arithmetic error in the last sum leads to an incorrect value for the approximation.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student presents the correct tangent line and earned the first point. The student makes an algebra error in subsequent work and did not earn the second point. In part (b) the student's work is incorrect. In part (c) the student's work is correct. In part (d) the student uses incorrect values for $f(1)$, $f'(1)$, and $f''(1)$.