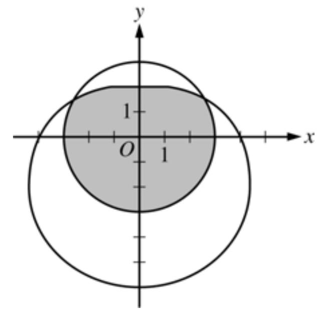
AP® CALCULUS BC
2013 SCORING GUIDELINES

Question 2



The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

(a) $\text{Area} = 6\pi + \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4 - 2\sin\theta)^2 \, d\theta = 24.709$ (or 24.708)

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$$

(b) $x = r\cos\theta \Rightarrow x(\theta) = (4 - 2\sin\theta)\cos\theta$

$x(t) = \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right)$

$x(t) = -1$ when $t = 1.428$ (or 1.427)

$$3 : \begin{cases} 1 : x(\theta) \text{ or } x(t) \\ 1 : x(\theta) = -1 \text{ or } x(t) = -1 \\ 1 : \text{answer} \end{cases}$$

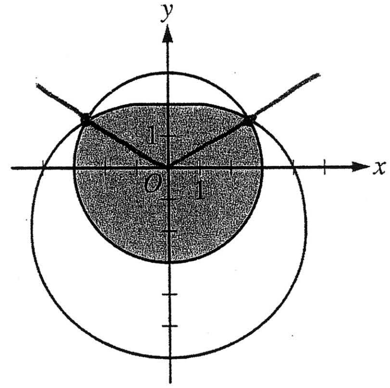
(c) $y = r\sin\theta \Rightarrow y(\theta) = (4 - 2\sin\theta)\sin\theta$

$y(t) = \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right)$

$$\begin{aligned} \text{Position vector} &= \langle x(t), y(t) \rangle \\ &= \left\langle \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right), \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right) \right\rangle \end{aligned}$$

$$\begin{aligned} v(1.5) &= \langle x'(1.5), y'(1.5) \rangle \\ &= \langle -8.072, -1.673 \rangle \text{ (or } \langle -8.072, -1.672 \rangle \text{)} \end{aligned}$$

$$3 : \begin{cases} 2 : \text{position vector} \\ 1 : \text{velocity vector} \end{cases}$$



2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

$S = \frac{2}{3}\pi(3)^2 + \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4-2\sin\theta)^2 = 24.709$

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(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$r = 4 - 2\sin(t^2)$

$x = r\cos\theta$

$x = (4 - 2\sin(t^2))(\cos(t^2))$

$-1 = 4\cos t^2 - 2\sin t^2 \cos t^2$

$\boxed{t = 1.428}$ ← only t in interval $[1,2]$

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

$\langle x(t), y(t) \rangle$

$x(t) = (4\cos t^2 - 2\sin t^2 \cos t^2)$

$y = r\sin\theta$

$y = (4 - 2\sin t^2)(\sin t^2)$

$y = 4\sin t^2 - 2\sin^2 t^2$

Position vector:

$\boxed{\langle 4\cos t^2 - 2\sin t^2 \cos t^2, 4\sin t^2 - 2\sin^2 t^2 \rangle}$

$\left.\frac{dx}{dt}\right|_{t=1.5} = -8.0721$

Velocity vector at $t = 1.5$

$\langle -8.072, -1.673 \rangle$

$\left.\frac{dy}{dt}\right|_{t=1.5} = -1.6729$

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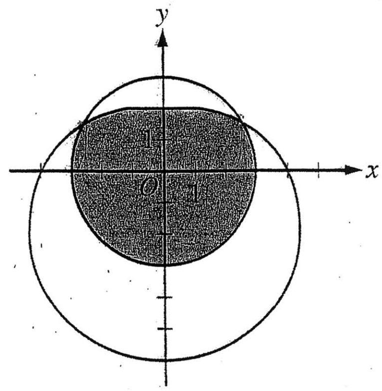

2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

$S = \pi(3)^2 - \int_{\pi/6}^{5\pi/6} \frac{1}{2}(4-2\sin\theta)^2 =$

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(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$x = r\cos\theta$

$x = (4-2\sin\theta)(\cos\theta)$

$-1 = (4-2\sin(t^2))(\cos(t^2))$

$t = 1.380$

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

position $\langle [(4-2\sin(t^2))(\cos(t^2))], [(4-2\sin(t^2))(\sin(t^2))] \rangle$

velocity at $t=1.5$:

$\langle 1.265, 5.865 \rangle$

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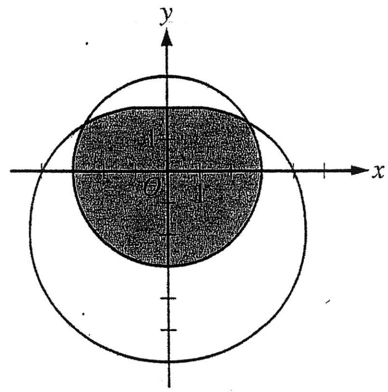

2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

$$\int_{\pi/6}^{5\pi/6} 3^2 - (4-2\sin\theta)^2 \, d\theta = 7.131$$

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(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$\cancel{r = 4 - 2\sin t^2}$

$\cancel{-1 = 4 - 2\sin t^2}$

$\cancel{-3 = -2\sin t^2}$

$\cancel{\frac{3}{2} = \sin t^2}$

$x = r\cos\theta$

$x = 4 - 2\sin\theta\,(\cos\theta)$

$-1 = 4 - 2\sin(t^2)(\cos t^2)$

$-3 = -2\sin t^2 \cos t^2$

$\frac{3}{2} = \sin t^2 \cos t^2$

$-1 = r\cos\theta$

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

$x = r\cos\theta$

$y = r\sin\theta$

$(r\cos t^2, r\sin t^2)$

$((4 - 2\sin t^2)\cos t^2, (4 - 2\sin t^2)\sin t^2)$

$((4 - 2\sin(1.5)^2)\cos(1.5)^2, (4 - 2\sin(1.5)^2\sin(1.5)^2)$

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Question 2

Overview

This problem provided the graphs of two curves defined by polar equations, along with values of θ at which the curves intersect. Part (a) asked students to find the area of the region common to the interiors of both graphs. This required students to divide the region into two subregions, bounded by arcs determined by the given values of θ, and then to apply the formula for polar area on each subregion to find the total area of the region. Part (b) described a particle moving along one of the curves and asked students to find the time when the x-coordinate of the particle is -1. Students needed to express the x-coordinate of the particle in terms of the angle θ, express θ in terms of time t, and, setting the resulting expression for x in terms of t equal to -1, solve for t in the desired time interval. Part (c) asked students to find the position vector of the particle in terms of time t, which required also determining an expression for y in terms of t, and then to find the velocity vector at a given time. This final step required finding the numerical derivative of each expression in the position vector at the given time.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student earned 2 points for providing the integrand and the correct limits and constant. In part (b) the student earned 2 points for writing $x(\theta)$ and setting $x(t)$ equal to -1. In part (c) the student earned 2 points for the x- and y-components of the position vector.

Sample: 2C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). In part (a) the student earned 1 point for the integrand. In part (b) the student does not include parentheses in the expression $4 - 2\sin\theta$, and continues to attempt a solution with an incorrect equation. In part (c) the student earned 2 points for the x- and y-components of the position vector.