Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \dfrac{C(4) - C(3)}{4 - 3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) C is differentiable $\Rightarrow C$ is continuous (on the closed interval)

$\dfrac{C(4) - C(2)}{4 - 2} = \dfrac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.

$2 : \begin{cases} 1 : \dfrac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\dfrac{1}{6}\int_0^6 C(t)\,dt \approx \dfrac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$

$\qquad = \dfrac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$

$\qquad = \dfrac{1}{6}(60.6) = 10.1$ ounces

$\dfrac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min

$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

3A₁   3A₁

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(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.6 \frac{oz}{min}$$

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

since $C(t)$ is differentiable for all values in $[2,4]$,

we can say that there must be some value of t in $(2,4)$, such

as $t = a$, such that $C'(a) = \frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$

by the mean value theorem

So, yes!

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$$\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6} \cdot 2 \cdot (5.3 + 11.2 + 13.8)$$

$$\approx 10.1 \text{ oz}$$

this is the average value, in oz, of the amount of coffee in the cup over the interval $0 \leq t \leq 6$

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$$B'(t) = \frac{32}{5} e^{-0.4t}$$

$$B'(5) = \frac{32}{5} e^{-2}$$

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3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$\frac{f(4) + f(3)}{2} = \frac{12.8 + 11.2}{2} = \boxed{12 \; oz}$$

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

Yes, according to the Mean Value Theorem.

$$C'(t) = \frac{f(b) - f(a)}{b - a}$$

$$\boxed{c'(t) = \frac{12.8 - 8.8}{4 - 2} = \frac{4}{2} = 2}$$

-14-

$3B_2$ $3B_2$

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\, dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\, dt$ in the context of the problem.

stepsize of 2

$(5.3)2 + (11.2)2 + (13.8)2$

$10.6 + 22.4 + 27.6 = 60.6$

$\frac{1}{6}\int_0^6 C(t)\, dt \approx \frac{1}{6}(60.6) = 10.1\, oz$

Over the course of six hours, the coffeemaker's average production is 10.1 oz per hour.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$\frac{16}{4} \cdot \frac{4}{9}$

$B'(t) = 6.4e^{-0.4t}$

$B'(5) = 6.4/e^2$

GO ON TO THE NEXT PAGE.

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(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$C'(3.5) = \frac{12.8 - 11.2}{4-3} = \frac{1.6}{1} = \boxed{1.6 = C'(3.5)}$$

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

$$C'(t) = \frac{f(b) - f(a)}{b-a}$$

$$= \frac{14.5 - 8.8}{6-2} = \frac{5.7}{4} < 2$$

$\frac{14.7}{-9.0}$

NO _ because $\frac{5.7}{4}$ is

less than 2

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$$\Delta x \cdot h + \Delta x \cdot h + \Delta x \cdot h$$

$$2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8$$

the meaning of $\frac{1}{6}\int_0^6 C(t)dt$ is the amount of ounces of water that was poured into the cup over the course of 0 seconds to 6 seconds.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$\frac{2}{5} = .4$

$$B(t) = 16 - 16e^{-.4t}$$

$$B'(t) = -16 \cdot .4e^{-.4t}$$

$$B'(t) = \frac{+32}{5}e^{-.4(t)} \qquad -\frac{2}{5} \cdot 5$$

$$B'(5) = \frac{32}{5}e^{-.4(5)}$$

$$\boxed{B'(5) = \frac{32}{5}e^{-1}}$$

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Do not write beyond this border.

Question 3

Overview

In this problem, a table was provided giving values of a differentiable function C at selected times between $t = 0$ and $t = 6$ minutes, where $C(t)$ represented the amount of coffee, in ounces, in a cup at time t. Part (a) asked students to approximate the derivative of the function C at $t = 3.5$ and to indicate units of measure. Because $t = 3.5$ fell between values of t given in the table, students should have constructed a difference quotient using the temperature values across the smallest time interval containing $t = 3.5$ that is supported by the table. Students should have recognized this derivative as the rate at which the amount of coffee in the cup is increasing, in ounces per minute, at time $t = 3.5$. Part (b) asked students whether there is a time t, $2 \le t \le 4$, at which $C'(t) = 2$. Students should have recognized that the hypotheses for the Mean Value Theorem hold because C is differentiable and then applied the theorem to the function on the interval $[2, 4]$ to conclude that there must be such a time t. Part (c) asked for an interpretation of $\frac{1}{6} \int_0^6 C(t)\, dt$ and a numeric approximation to this expression using a midpoint sum with three subintervals of equal length as indicated by the data in the table. Students should have recognized this expression as providing the average amount, in ounces, of coffee in the cup over the 6-minute time period. Students needed to use the values in the table at times $t = 1$, $t = 3$, and $t = 5$, with interval length 2, to compute this value. In part (d) students were given a symbolic expression for a function B that modeled the amount of coffee in the cup on the interval $0 \le t \le 6$. Students were asked to use this model to determine the rate at which the amount of coffee in the cup is changing when time $t = 5$. This was answered by computing the value $B'(5)$.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student does not construct a numeric difference quotient, so no points were earned. The use of f instead of C is ignored. In part (b) the student's work is correct. The use of f instead of C is ignored because the student correctly uses C-values from the table and invokes the Mean Value Theorem. In part (c) the student earned the first 2 points. The interpretation point was not earned because the student does not commit to ounces as the units. The student also uses "ounces per hour" in the interpretation. In part (d) the student's work is correct.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (c), and 1 point in part (d). In part (a) the student earned the approximation point, but not the units point. In part (b) the student's work is incorrect. In part (c) the student earned the midpoint sum point, but not the approximation point. The interpretation does not include "average," so the third point was not earned. In part (d) the student has the correct derivative but incorrectly multiplies $-0.4(5)$, so the answer point was not earned.