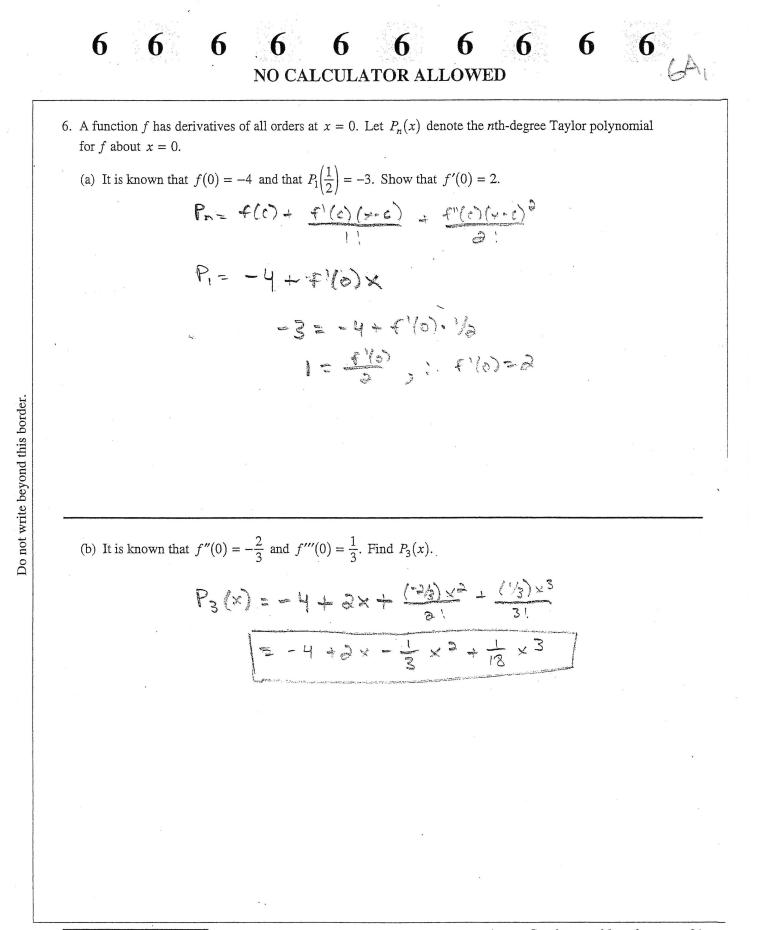
AP[®] CALCULUS BC 2013 SCORING GUIDELINES

Question 6

A function f has derivatives of all orders at x = 0. Let $P_n(x)$ denote the *n*th-degree Taylor polynomial for f about x = 0.

- (a) It is known that f(0) = -4 and that $P_1\left(\frac{1}{2}\right) = -3$. Show that f'(0) = 2.
- (b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.
- (c) The function h has first derivative given by h'(x) = f(2x). It is known that h(0) = 7. Find the third-degree Taylor polynomial for h about x = 0.

| (a) | $P_{1}(x) = f(0) + f'(0)x = -4 + f'(0)x$ | $2: \begin{cases} 1 : \text{uses } P_1(x) \\ 1 : \text{verifies } f'(0) = 2 \end{cases}$ |
|-----|--|---|
| | $P_{1}\left(\frac{1}{2}\right) = -4 + f'(0) \cdot \frac{1}{2} = -3$ $f'(0) \cdot \frac{1}{2} = 1$ f'(0) = 2 | |
| (b) | $P_3(x) = -4 + 2x + \left(-\frac{2}{3}\right) \cdot \frac{x^2}{2!} + \frac{1}{3} \cdot \frac{x^3}{3!}$ $= -4 + 2x - \frac{1}{3}x^2 + \frac{1}{18}x^3$ | $3: \begin{cases} 1: \text{ first two terms} \\ 1: \text{ third term} \\ 1: \text{ fourth term} \end{cases}$ |
| (c) | Let $Q_n(x)$ denote the Taylor polynomial of degree <i>n</i> for <i>h</i> about $x = 0$. | 4 : $\begin{cases} 2 : \text{applies } h'(x) = f(2x) \\ 1 : \text{constant term} \\ 1 : \text{remaining terms} \end{cases}$ |
| | $h'(x) = f(2x) \Rightarrow Q_3'(x) = -4 + 2(2x) - \frac{1}{3}(2x)^2$ $Q_3(x) = -4x + 4 \cdot \frac{x^2}{2} - \frac{4}{3} \cdot \frac{x^3}{3} + C; \ C = Q_3(0) = h(0) = 7$ $Q_3(x) = 7 - 4x + 2x^2 - \frac{4}{9}x^3$ | |
| | OR 9 ¹ | |
| | $h'(x) = f(2x), \ h''(x) = 2f'(2x), \ h'''(x) = 4f''(2x)$ $h'(0) = f(0) = -4, \ h''(0) = 2f'(0) = 4, \ h'''(0) = 4f''(0) = -\frac{8}{3}$ | |
| | $Q_3(x) = 7 - 4x + 4 \cdot \frac{x^2}{2!} - \frac{8}{3} \cdot \frac{x^3}{3!} = 7 - 4x + 2x^2 - \frac{4}{9}x^3$ | |



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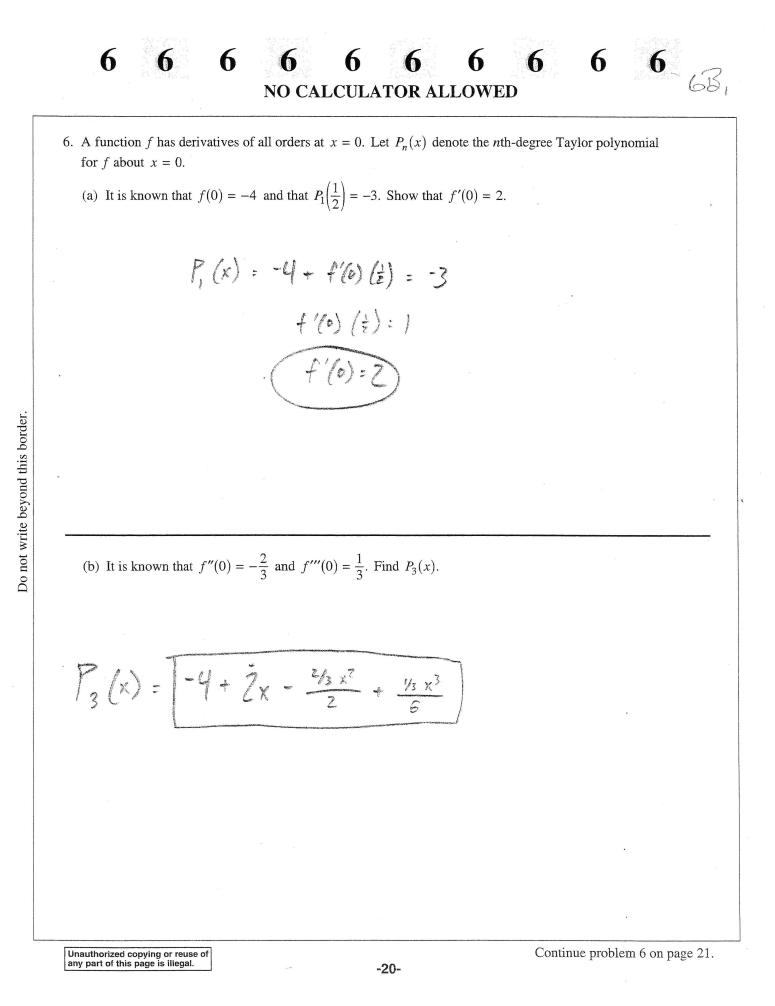
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Continue problem 6 on page 21.

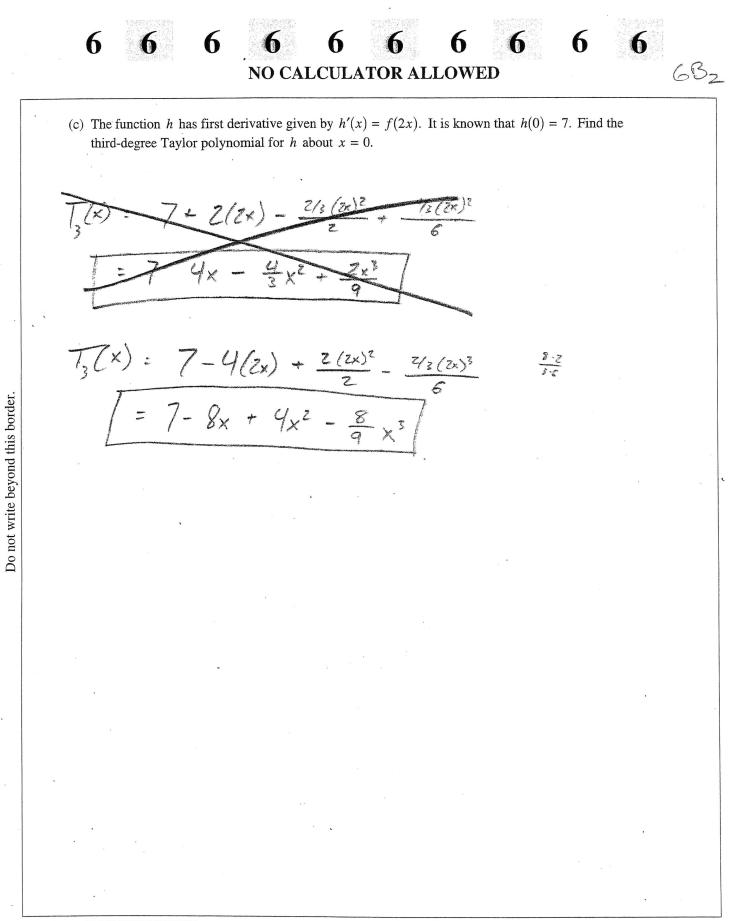
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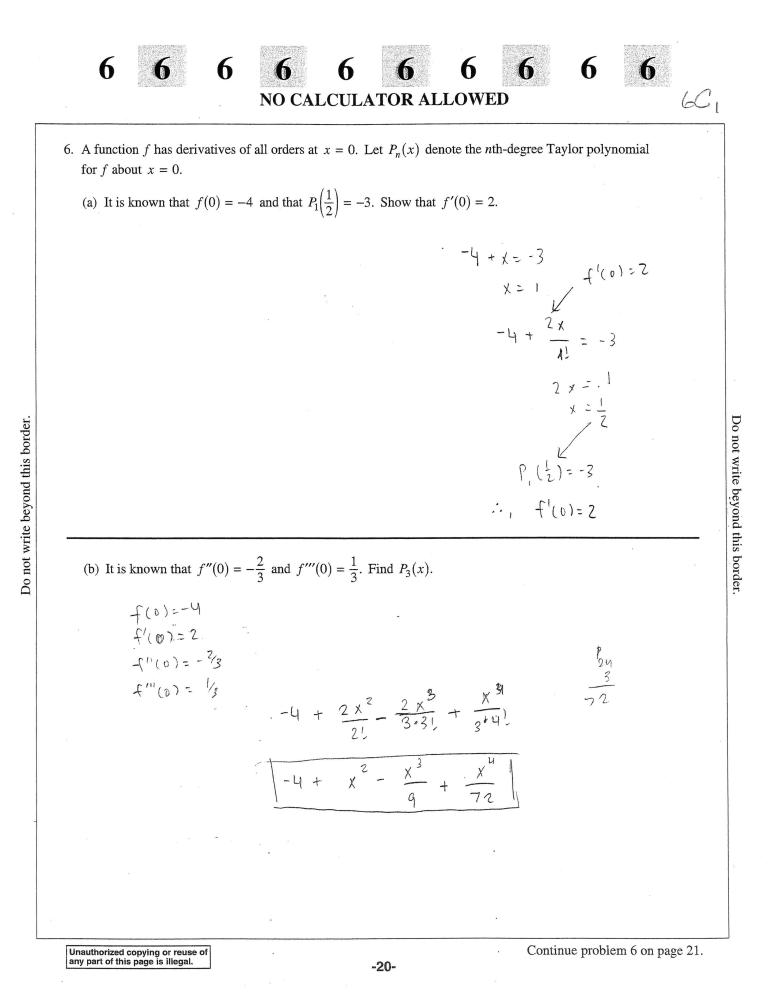


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(c) The function h has first derivative given by h'(x) = f(2x). It is known that h(0) = 7. Find the third-degree Taylor polynomial for h about x = 0.

h(o) = 7 h'(o) = f(o)h'(v) = f(2v) h''(o) = f'(2v)(2)7 + Чx

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AP[®] CALCULUS BC 2013 SCORING COMMENTARY

Question 6

Overview

This problem described a function f known to have derivatives of all orders at x = 0. In part (a) information was provided about the value of the function at x = 0, as well as about the value of its first-degree Taylor polynomial

about x = 0 at the point $x = \frac{1}{2}$. Students needed to use this information to verify that f'(0) = 2.

In part (b) students were given information about the second and third derivatives of f at x = 0. Students needed to use this additional information to find the third-degree Taylor polynomial for f about x = 0. In part (c) a new function h was defined in terms of f. Students needed to use information provided about h(0), as well as information already provided about f, to find the third-degree Taylor polynomial for h about x = 0.

Sample: 6A Score: 9

The student earned all 9 points.

Sample: 6B Score: 6

The student earned 6 points: 2 points in part (a), 3 points in part (b), and 1 point in part (c). In parts (a) and (b), the student's work is correct. No supporting work was required. In part (c) the student appears to have antidifferentiated $P_2(x)$ and replaced x by 2x in the result. This is not a legitimate method, and the student did not earn the fourth point. The student earned the third point because the answer has a constant term of 7 in a polynomial of degree three or higher.

Sample: 6C Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (c). In part (a) the student's actual solution begins on the third line and earned both points in part (a). The student shows that f'(0) = 2 together with the given values for $P_1\left(\frac{1}{2}\right)$ and f(0) implies that $x = \frac{1}{2}$, thereby verifying that f'(0) = 2. In part (b) the student's work is incorrect. In part (c) the student earned 1 of 2 points. The student would have earned the second point with a correct expression for h'''(0). Because the student attempts to use a legitimate method, the student was eligible to earn the fourth point. No additional points were earned since the answer is not a polynomial of degree three or higher, and the student does not have a value for h'''(0).