AP® CALCULUS BC
2015 SCORING GUIDELINES

Question 2

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

(a) $x(2) = 3 + \int_1^2 \cos\left(t^2\right)\, dt = 2.557$ (or 2.556)

3 : { 1 : integral; 1 : uses initial condition; 1 : answer }

(b) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{e^{0.5t}}{\cos\left(t^2\right)}$

$\dfrac{e^{0.5t}}{\cos\left(t^2\right)} = 2$

$t = 0.840$

2 : { 1 : slope in terms of t; 1 : answer }

(c) Speed $= \sqrt{\cos^2\left(t^2\right) + e^t}$

$\sqrt{\cos^2\left(t^2\right) + e^t} = 3$

$t = 2.196$ (or 2.195)

2 : { 1 : speed in terms of t; 1 : answer }

(d) Distance $= \int_0^1 \sqrt{\cos^2\left(t^2\right) + e^t}\, dt = 1.595$ (or 1.594)

2 : { 1 : integral; 1 : answer }

2. At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

($\frac{dx}{dt}$, $\frac{dy}{dt}$)

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

$$x(2) = 3 + \int_1^2 \cos(t^2)\,dt = 2.557$$

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

$$\frac{dy}{dt} = e^{0.5t}$$

$$\frac{dx}{dt} = \cos(t^2)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{0.5t}}{\cos(t^2)} = 2$$

$t = .840$ ~~-2.780~~ not in interval $0 < t < 1$

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(c) Find the time at which the speed of the particle is 3.

$\|v(t)\| = \sqrt{[\cos(t^2)]^2 + [e^{0.5t}]^2} = 3$

$t = 2.196$

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

Arc Length $= \int_0^1 \sqrt{[\cos(t^2)]^2 + [e^{0.5t}]^2}\, dt$

$= 1.595$

2. At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

$$3 + \int_1^2 \cos(t^2)\,dt = 4$$

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

$$\frac{y'(t)}{x'(t)} = 2 = \frac{e^{0.5t}}{\cos(t^2)}$$

$$2\cos t^2 = e^{0.5t}$$

$$t = 0.84$$

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Continue problem 2 on page 7.

(c) Find the time at which the speed of the particle is 3

$\sqrt{(\cos(t^2))^2 + (e^{0.5t})^2} = 3$ $\quad \cos^2(t^2) + e^t = 9$

$t = 2.19$

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

$\int_0^1 \sqrt{x'(t)^2 + y'(t)^2}\,dt$ $\int_0^1 \sqrt{\cos^2(t^2) + e^t}\,dt$

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2. At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

position functions $\left(\frac{1}{2t}\sin(t^2), 2e^{.5t}\right)$ $x = \frac{1}{2t}\sin(t^2) + C$

$\left(\frac{1}{2(2)}\sin(2^2), 2e^{.5(2)}\right)$ $3 = \frac{1}{2}\sin(1) + C$

$\left(\frac{1}{4}\sin(4) + 5.158\right)$ $6 - .8415 = C$

$C = 5.158$

4.969

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

$\frac{dy}{dx} = \frac{e^{.5t}}{\cos(t^2)} = \frac{2}{1}$ $t = 4.159$

$\frac{1}{2}e^{.5t} = 2\cos(t^2)$

$\cos^{-1}\left(\frac{1}{2}e^{.5t}\right) = t^2$

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Continue problem 2 on page 7.

(c) Find the time at which the speed of the particle is 3.

$$3 = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$3 = \sqrt{(\cos(t^2))^2 + (e^{.5t})^2}$$

$$t = 2.196$$

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

$$\int_0^1 \left| \frac{e^{.5t}}{\cos(t^2)} \right| dt = 1.501$$

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AP® CALCULUS BC
2015 SCORING COMMENTARY

Question 2

Overview

In this problem students were given the velocity vector of a particle moving in the *xy*-plane with position $(x(t), y(t))$. The particle is at the point $(3, 5)$ at time $t = 1$. In part (a) students had to find the *x*-coordinate of the position of the particle at time $t = 2$. The *x*-coordinate of the position of the particle at $t = 1$ added to the net change from $t = 1$ to $t = 2$ produces the *x*-coordinate at $t = 2$, which is $x(1) + \int_1^2 \cos\left(t^2\right) dt$. Students were expected to evaluate this expression with the calculator. In part (b) students were given that there is a point on the curve at which the line tangent to the curve has a slope of 2. Students needed to find the time at which the particle was at that point. Students had to realize that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and then solve the equation $\frac{dy}{dx} = 2$ using the calculator. In part (c) students were asked to find the time at which the speed of the particle is 3. Students needed to solve the equation $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 3$ using the calculator. In part (d) students were asked to find the total distance traveled by the particle from time $t = 0$ to time $t = 1$. Students needed to set up the integral expression $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ and then evaluate this expression using the calculator.

Sample: 2A
Score: 9

The response earned all 9 points.

Sample: 2B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student presents a definite integral equal to the change in the *x*-coordinate of the particle from time $t = 1$ to $t = 2$, so the first point was earned. The student presents the correct use of the initial condition, so the second point was earned. The student does not state the correct answer. In part (b) the student's work is correct. In part (c) the student presents an expression for the speed in terms of t set equal to the given speed of 3, so the first point was earned. The student does not state the correct answer to three decimal places, so the second point was not earned. In part (d) the student presents a definite integral equal to the distance the particle traveled from time $t = 0$ to $t = 1$, so the first point was earned. The student does not state an answer.

Sample: 2C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student does not present an integral, so the first point was not earned. The student attempts to anti-differentiate $\cos\left(t^2\right)$. Neither the second nor third points were earned. In part (b) the student presents a correct expression for slope in terms of *t*, so the first point was earned. The student does not state the correct answer. In part (c) the student's work is correct. In part (d) the student presents a definite integral whose integrand does not represent the speed of the particle.