Student Performance Q&A:

2015 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2015 free-response questions for AP Calculus AB and Calculus BC were written by the Chief Reader, Stephen Kokoska of Bloomsburg University. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1

What was the intent of this question?

In this problem students were given $R(t)$, the rate of flow of rainwater into a drainpipe, in cubic feet per hour, and $D(t)$, the rate of flow of water out of the pipe, in cubic feet per hour. Both $R(t)$ and $D(t)$ are defined on the time interval $0 \le t \le 8$. The amount of water in the pipe at time $t = 0$ is also given. In part (a) students needed to use the definite integral to compute the amount of rainwater that flows into the pipe during the interval $0 \le t \le 8$. Students had to set up the definite integral $\int_0^8 R(t)\,dt$ and evaluate the integral using the calculator. In part (b) students should have recognized that the rate of change of the amount of water in the pipe at time t is given by $R(t) - D(t)$. Students were expected to calculate $R(3) - D(3)$ using the calculator and find that the result is negative. Therefore, the amount of water in the pipe is decreasing at time $t = 3$. In part (c) students had to find the time t, $0 \le t \le 8$, at which the amount of water in the pipe is at a minimum. Students were expected to set up an integral expression such as $30 + \int_0^t [R(x) - D(x)]\,dx$ for the amount of water in the pipe at time t. Students should have realized that an absolute minimum exists since they are working with a continuous function on a closed interval, and this minimum must occur at either a critical point or at an endpoint of the interval. Students were expected to use the calculator to solve $R(t) - D(t) = 0$ and find the single critical point at $t = 3.272$ on the interval $0 < t < 8$. Students should have stored the full value for t in the calculator and used the calculator to evaluate the function at the critical point and the endpoints. In this case the amount of water is at a minimum at the single critical point. In part (d) students were asked to write an equation involving one or more integrals that gives the time w when the pipe will begin to overflow. Students were expected to set up an equation using the initial condition, an integral expression, and the holding capacity of the pipe such as

$$30 + \int_0^w [R(t) - D(t)]\,dt = 50.$$

How well did students perform on this question?

In general, students performed better on this problem than on the first problem of the 2014 exam, which is also in a context. The context of this problem, rainwater flowing into and out of a drainpipe, may have been more familiar to students. The mean score was 3.42 for AB students and 5.10 for BC students out of a possible 9 points. Approximately 16 percent of AB students did not earn any points on this problem.

Most students found parts (a) and (b) easier than parts (c) and (d). In part (b) many students knew to consider $R(3) - D(3)$ but did not properly communicate this difference. In part (c) many students simply announced $t = 3.272$ without a supporting mathematical equation. In addition, many students did not provide sufficient justification for the minimum value. Some students rounded intermediate values, and some students made decimal presentation errors (e.g., not rounding the final answer accurately to 3 decimal places).

Many students earned at least 1 point in part (d). Some students did not use proper mathematical notation, and others did not include the initial condition.

What were common student errors or omissions?

In part (a) some students included the initial condition, 30, either in the integrand or added to the integral. Some students used $R(t) - D(t)$ as the integrand.

In part (b) some students simply miscalculated either $R(3)$ or $D(3)$. Many students did not write accurate descriptions of water flowing into and out of the pipe. These students did not connect the functions to rates at $t = 3$, did not specify any rate, or presented a reason based on the total amount of water in the pipe over the interval $[0, 3]$. Some students compared $R'(3)$ and $D'(3)$.

In part (c) many students did not present the mathematical equation, $R(t) - D(t) = 0$, that led to the critical point. In addition, many students did not present sufficient justification for the minimum value. These students did not give a complete argument including the endpoints or presented only a local argument. Some students found the minimum of the function $R(t) - D(t)$.

In part (d) some students used an incorrect initial condition or no initial condition. Many students used constant limits or no limits of integration. Some students omitted the differential dt. In this case, if the initial condition was after the integral, the integrand was not well defined.

Some students solved this problem using the calculator in degree mode rather than radian mode. This was evident by the solutions presented. It also appeared that some students made errors in entering the functions R and D into the calculator.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should stress the importance of correct and consistent notation. In problems involving an integral, the differential must be included to indicate the integrand. In addition, mathematical expressions must be grouped properly, and parentheses must be used properly to convey the exact meaning of an equation or integral.

In a calculator-active question, students should be encouraged to store functions using calculator function names (e.g., Y1, Y2, f1(x), f2(x)) or by defining functions. These stored/defined functions can then be used when the calculator is needed for evaluating mathematical expressions and performing computations. In addition, teachers should remind students to use the names of given functions (such as R and D in this problem) in their work to avoid copy errors.

Students need more practice with rate-in/rate-out questions. Water flowing into and/or out of a tank or pipe are common applications, and students should be familiar with the concept of the integral as an accumulation function.

It is also important for students to remember that if they use the calculator for one of the four required capabilities, they must indicate the mathematical setup (e.g., the equation or integral) using proper mathematical notation.

Question AB2

What was the intent of this question?

In this problem students were given a graph of the boundary curves of two planar regions R and S in the first quadrant. One boundary curve is defined by $f(x) = 1 + x + e^{x^2-2x}$, and the other boundary is defined by $g(x) = x^4 - 6.5x^2 + 6x + 2.$ In part (a) students were asked to find the sum of the areas of regions R and $S.$ Two intersection points of the boundary curves, $(0, 2)$ and $(2, 4),$ are given, and students were expected to find the other point of intersection by using the calculator. The intersection point is $(A, B) = (1.032832, 2.401108).$ The sum of the areas of R and S is $\int_0^A (g(x) - f(x))\,dx + \int_A^2 (f(x) - g(x))\,dx.$ Students were expected to use the calculator to evaluate the integrals. In part (b) students were asked to find the volume of a solid with S as its base. Students had to interpret the area of the cross sections as $[f(x) - g(x)]^2$, and use the calculator to evaluate the volume as $\int_A^2 [f(x) - g(x)]^2\,dx.$ In part (c) students had to find the rate of change of the vertical distance, h, between the graphs of f and g at $x = 1.8.$ Students were expected to recognize and communicate $h'(x) = f'(x) - g'(x),$ then evaluate $h'(1.8)$ using the derivative at a point capability of the calculator.

How well did students perform on this question?

This was a typical area/volume problem with the additional condition that the upper and lower boundary curves changed for the two regions R and $S.$ The mean score was 4.68 out of a possible 9 points. Approximately 16 percent of students did not earn any points on this problem.

In part (a) many students were able to find the correct intersection point and calculate the area of each region. Students who consistently used the named functions, f and g, typically had very succinct, precise solutions. Some students guessed, by looking at the graph, that the point of intersection occurred at $x = 1.$ These students did not use the calculator. It also appeared that some students used the trace feature of the calculator to determine the point of intersection. Some students found the area using an integrand with absolute value.

In part (b) most students were able to write an expression for the cross-sectional area at an arbitrary point. Some students set up the correct integrand but included the constant π in the final expression for volume. Some students attempted to find volume using the method of washers.

In part (c) most students correctly associated the rate of change with the derivative. Some students did not communicate an expression for h or communicated an incorrect expression for h. Some students worked with an absolute value expression. There were several solutions presented without any supporting work, and some students tried to work with an average rate of change expression or an integral expression.

What were common student errors or omissions?

Some students did not store the functions f and g in the calculator in order to use those expressions efficiently. These students most often found the point of intersection correctly, but did not evaluate the integrals for either part (a) or (b) correctly. Many students did not use the function names when writing the expressions in all three parts of the question. Those students who wrote the algebraic expressions for f and g made arithmetic distribution mistakes or parentheses errors.

In part (a) some students either guessed the point of intersection occurred at $x = 1$ or used the calculator trace feature to report an inaccurate point of intersection. Many students used the y-coordinates of the points of intersection as limits of integration. Many students used 4 as the upper limit of integration on the second integral expression (the area of S). Some students correctly found the area of each region but did not add the areas, and some students wrote the final answer as 2, instead of 2.004. Some students presented indefinite integrals; some used poor or incorrect notation to express the area of the two regions. A common incorrect expression for the area was $\int_0^{1.033} R\,dx + \int_{1.033}^{2} S\,dx.$

In part (b) some students wrote the correct integrand but included the constant π in the expression for volume. Some students tried to use the method of washers, and some students presented an indefinite integral. In parts (a) and (b), some students were able to write correct integral expressions but were unable to evaluate the integrals correctly using the calculator.

In part (c) many students wrote $h'(x) = f(x) - g(x).$ It appeared many students did not have the use of the calculator when completing this part, or students did not know to use the numerical derivative capability. These students often tried to find the derivative of h analytically, and many made chain rule errors or could not evaluate the derivative at $x = 1.8.$ Some students tried to work with either an average rate of change or an integral expression.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students should use the calculator's solve or intersect capabilities to find a point of intersection rather than the trace capability. In addition, the numerical derivative at a point capability should be used to find the rate of change of a function at a point.

Students should use the names of functions that are presented in the statement of the question. Teachers should model this technique in their classes and on student assessments. Similarly, students should store given functions in the calculator and use these stored functions instead of entering the function expressions multiple times.

Students need additional practice on the method of finding the volume of solids with known cross sections, especially problems in which the cross sections are squares, rectangles, isosceles right triangles, equilateral triangles, or semicircles.

Communication and mathematical notation are important in presenting solutions. If a student uses a variable in the solution presentation, then the variable must be given a value. If a student uses new function names, for example, Y1 and Y2, then these functions must be defined. Complete expressions must be presented. For example, $x^4 - \cdots$ is not an acceptable integrand.

Question AB3/BC3

What was the intent of this question?

In this problem students were given a table of values of a differentiable function v, the velocity of a jogger, in meters per minute, along a straight path for selected values of t in the interval $0 \le t \le 40$. In part (a) students were expected to know that $v'(16)$ can be estimated by the difference quotient $\frac{v(20) - v(12)}{20 - 12}$. In part (b) students were expected to explain that the definite integral $\int_0^{40} |v(t)|\, dt$ gives the total distance jogged, in meters, by Johanna over the time interval $0 \le t \le 40$. Students had to approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the subintervals $[0, 12]$, $[12, 20]$, $[20, 24]$, $[24, 40]$, and values from the table. In part (c) students were given a cubic function B, the velocity of a bicyclist, in meters per minute, along the same straight path used by Johanna for $0 \le t \le 10$. Students should have known that $B'(t)$ gives Bob's acceleration at time t. Students were expected to find $B'(t)$ using derivatives of basic functions and then evaluate $B'(5)$. In part (d) students had to set up the definite integral $\frac{1}{10}\int_0^{10} B(t)\, dt$ that gives Bob's average velocity during the interval $0 \le t \le 10$. Students needed to evaluate this integral using basic antidifferentiation and the Fundamental Theorem of Calculus.

How well did students perform on this question?

In general, students performed well on this problem. Tabular problems like this have appeared on recent AP Calculus Exams, and teachers seem to be preparing students for this type of question. The mean score was 4.43 for AB students and 5.71 for BC students out of a possible 9 points.

Most students were able to answer part (a). They correctly presented a difference quotient, and although it is not required, simplified correctly.

In part (b) many students did not include all three required items ("distance," units, and time interval) in the explanation of the definite integral. Most students were able to write 7 out of 8 Riemann sum components correctly. However, some students made errors in simplification.

Most students did very well in part (c). In part (d) for those students who started with the correct integral expression, most earned all 3 points.

What were common student errors or omissions?

In part (a) the most common errors were simple subtraction errors. Some students omitted the difference quotient and reported only a final answer. Some students did not declare the value of $v'(16)$ after finding an equation for the secant line through $(12, 200)$ and $(20, 240)$.

In part (b) in the explanation, many students omitted either the time interval or the units of meters. In the expression for the Riemann sum, some students used -220 instead of $|-220|$. Some students found a left Riemann sum or trapezoidal sum.

In part (c) the most common error involved notation. However, many students were able to correctly find an expression for $B'(t)$ and use this expression to compute $B'(5)$.

In part (d) many students used an incorrect function as the integrand. For example, $B'(t)$ or $B''(t)$ were frequently presented as the integrand. Some students incorrectly used the constant $\frac{1}{10}$ in the evaluation of the definite integral. A common error was to use an equals sign to present a string of equalities, linking expressions that are not equal.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Since students made many arithmetic mistakes in this problem, teachers should remind students that unless otherwise indicated, numerical answers do not need to be simplified. This also suggests that students need more practice on problems in which a calculator is not permitted.

Teachers should continue to practice questions involving all types of Riemann sums. Students should communicate Riemann sums clearly before calculating the final answer.

Question AB4/BC4

What was the intent of this question?

In this problem students were to consider the first-order differential equation $\frac{dy}{dx} = 2x - y.$ In part (a) students were given an *xy*-plane with 6 labeled points and were expected to sketch a slope field by drawing a short line segment at each of the six points with slopes of $2x - y.$ In part (b) students needed to use implicit differentiation and the fact that $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ to obtain $\frac{d^2y}{dx^2} = 2 - 2x + y.$ Students were expected to explain that for points in Quadrant II, $x < 0$ and $y > 0$ so $\frac{d^2y}{dx^2} > 0.$ Thus, any solution curve for the differential equation that passes through a point (x, y) in Quadrant II must be concave up at $(x, y).$ In part (c) students were asked to consider the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 3.$ Students had to determine if $(2, 3)$ is the location of a relative minimum or relative maximum for f and justify the answer. Students were expected to show that $\frac{dy}{dx} \neq 0$ at $(2, 3)$ and conclude that $(2, 3)$ is neither the location of a relative maximum nor a relative minimum. In part (d) students were asked to find the values of the constants m and b so that the linear function $y = mx + b$ satisfies the differential equation $\frac{dy}{dx} = 2x - y.$ Students were expected to show that if $y = mx + b,$ then $\frac{dy}{dx} = m.$ Using a substitution in $\frac{dy}{dx} = 2x - y$ leads to $2x - y = m$ and thus $2x - (mx + b) = m.$ This equation enabled the student to find the values of m and $b.$

How well did students perform on this question?

In general, both Calculus AB and Calculus BC students found this problem to be challenging. Most students presented a lot of work in all parts of the problem. The mean score was 2.67 for AB students and 4.40 for BC students out of a possible 9 points. Approximately 15 percent of AB students did not earn any points on this problem.

In part (a) many students were able to sketch a correct slope field. Errors in this part were associated with miscalculating the slope values or poor sketches of line segments. There were some students who left part (a) blank and some who drew graphs that did not resemble slope fields.

Many students were unable to work with this non-separable differential equation. Some students simply guessed at answers in parts (b) and (c), and others left these parts completely blank. Some students even attempted to solve the differential equation.

In part (b) it appeared that some students did not understand the Leibniz notation for a second derivative and were, therefore, unable to answer the question. Some students were unable to provide a complete reason for the conclusion *concave up*.

Many students earned points in part (c). Most students understood how to use the first derivative to answer this question. However, many students tried to solve the differential equation in part (c), believing that this was necessary to answer the question.

Most students did not answer part (d) correctly. Some students attempted to solve the differential equation here also. Some students believed this part was an extension of part (c) and found an equation of a tangent line at the point $(2, 3)$. Some students presented the correct answer without any supporting work. Some BC students were able to solve the differential equation using an integrating factor, which is not included in the topic outline for Calculus BC. These students were able to find the linear solution.

What were common student errors or omissions?

In part (a) for those students who sketched incorrect segments, the most common errors were at the point $(0, -1)$ or $(1, -1)$. Some students sketched a vertical segment at the point $(1, 2)$ instead of a horizontal segment. Some students simply left part (a) blank.

In part (b) many students did not recognize Leibniz notation for the second derivative. Some students wrote inconsistent or misleading reasons involving concavity. For example, some students talked about *concave positive* and others tried to discuss concavity as a number or point. Many students could not generalize the second derivative for points in Quadrant II. Some students set the second derivative equal to zero and others argued only using an interval involving x. Specifically, many students presented a reason using the intervals $(-\infty, 2)$ or $(-\infty, 1)$ for x. Many students who did attempt to generalize in Quadrant II did not present sufficient evidence for the statement $\frac{d^2y}{dx^2} > 0$. Some students used a finite set of test points in Quadrant II and then generalized to the entire quadrant. Some students did not distribute the subtraction in simplifying their answer. Many students left the second derivative in terms of $\frac{dy}{dx}$.

In part (c) some students attempted to solve the differential equation. Some students attempted to justify their answer using a sign chart, declaring that $\frac{dy}{dx}$ did not change sign at $x = 2$ without any evidence for that claim.

In part (d) some students also attempted to solve the differential equation. Some tried to incorrectly use antidifferentiation. Some students also continued to use the point $(2, 3)$ from part (c) and found an equation of a tangent line. Some students set the two equations equal: $y = \frac{dy}{dx}$ or $2x - y = mx + b$.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Slope fields have been tested regularly on the AP Calculus Exams, and students should be able to create a portion of a slope field for a differential equation. Although students are not expected to solve a non-separable differential equation, they are expected to be able to obtain information from and work with any differential equation.

Teachers should emphasize to students that the behavior of a particular solution and properties of the family of solutions can be obtained from the differential equation without actually solving the differential equation. In addition, teachers should use various notation for derivatives, including Leibniz notation.

Question AB5

What was the intent of this question?

In this problem students were given the graph of f', the derivative of a twice-differentiable function f on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ were given. In part (a) students had to find all x-coordinates at which f has a relative maximum. Students should have known that a relative maximum can only occur at a critical point of f or at one of the endpoints of the interval. Students had to find the critical points $x = -2$, $x = 1$, and $x = 4$ from the graph of f' and apply the First Derivative Test to conclude that the relative maximum occurs at $x = -2$. In part (b) students were asked to determine the open intervals where the graph of f is both concave down and decreasing. Students needed to use the graph of f' to determine the open intervals where f' was both decreasing and negative in order to answer the question. In part (c) students were asked to find the x-coordinates of all points of inflection for the graph of f. Students needed to use the graph of f' to determine the x-coordinates of the points where f' changes from increasing to decreasing or from decreasing to increasing in order to answer the question. In part (d) students were asked to write an expression for $f(x)$ that involves an integral given that $f(1) = 3$. Students were expected to use the Fundamental Theorem of Calculus to produce $f(x) = 3 + \int_1^x f'(t)\,dt$. Students had to use properties of the definite integral, including the relationship of the definite integral to the area under the curve to find $f(4)$ and $f(-2)$.

How well did students perform on this question?

Many students performed well on this question, and most students presented significant work in all four parts. The mean score was 3.75 out of a possible 9 points. Approximately 17 percent of students earned no points on this problem.

In part (a) many students correctly identified $x = -2$ as the x-coordinate of a relative maximum and many were able to provide an acceptable reason. However, some students presented vague language. In part (b) many students were able to find the correct intervals. However, many struggled to express their reasons.

In part (c) most students were able to correctly identify the x-coordinates of the points of inflection. However, many could not present an acceptable reason. Many argued that horizontal tangents or that $f''(x) = 0$ justify points of inflection.

In part (d) many students were able to earn the conceptual point for having $f'(x)$ as the integrand in a definite integral. However, few students provided an expression for $f(x)$. Many students were unable to work with regions below the x-axis and limits of integration in which the lower limit is greater than the upper limit.

What were common student errors or omissions?

In part (a) the most common error was ineffective communication of the reason. Many students did not specifically reference the given graph of f'. Many of these students cited a general definition of a relative maximum involving f or used vague language about derivatives.

In part (b) many students simply presented the incorrect intervals, omitted an interval, or made vague statements about slopes and derivatives in their reasons. Some students did not specify which slope or derivative they were referencing. Many students did not earn the reason point because they indicated that the slope of f' is decreasing. Some students reported the interval $(-2, 1)$ instead of $(-2, -1)$.

In part (c) some students used horizontal tangents of the graph of f' or $f''(x) = 0$ as the reason for points of inflection. There were many vague references to *the slope* or *the derivative*.

In part (d) the most common errors were failure to provide an expression for $f(x)$, failure to incorporate the initial condition, mishandling the area of a region below the x-axis, and inability to evaluate $\int_1^{-2} f'(t)\, dt$. Some very common incorrect answers presented by students included $f(4) = 15$ and $f(-2) = -6$.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should emphasize the importance of clear, concise language when providing a reason for an answer. Standard notation and symbols should be used in these reasons. Teachers and students should avoid using non-standard notation or shorthand. For example, we do not read "$f \uparrow$" as f is increasing.

Students must be specific in references to a derivative, slope, or graph. Students should use standard notation for derivatives, $f'(x)$ and $f''(x)$, rather than the phrases the first derivative or the second derivative. It is important for students to refer to specific functions by their names.

Teachers should remind students of the importance of using given information, such as a given initial condition, in solving a problem and writing reasons and justifications. Correct mathematical notation is important, including the use of the differential in the notation in an integral.

Question AB6

What was the intent of this question?

In this problem students were given the equation of a curve, $y^3 - xy = 2$, with $\frac{dy}{dx} = \frac{y}{3y^2 - x}$. In part (a) students had to find an equation for the line tangent to the curve at the point $(-1, 1)$. Students were expected to use the given $\frac{dy}{dx}$ to find the slope of the curve at the point $(-1, 1)$. In part (b) students were asked to find the coordinates of all points on the curve at which there is a vertical tangent line. These are the points on the curve where $3y^2 - x = 0$, but $y \neq 0$. Students were expected to solve $y^3 - xy = 2$ with the condition that $3y^2 - x = 0$ and report only those pairs (x, y) where $y \neq 0$. In part (c) students were asked to evaluate $\frac{d^2y}{dx^2}$ at the point $(-1, 1)$ on the curve. Students had to use implicit differentiation with $\frac{dy}{dx}$ to find an expression for $\frac{d^2y}{dx^2}$, which required use of the chain rule and either the product rule or the quotient rule. The expression can be written in terms of x and y or can involve $\frac{dy}{dx}$. In either case, students needed to evaluate the expression for $\frac{d^2y}{dx^2}$ at $(-1, 1)$.

How well did students perform on this question?

Many students were able to earn some points in all parts of this problem. The mean score was 3.29 out of a possible 9 points. Approximately 20 percent of students earned no points on this problem.

In part (a) most students were able to compute the slope and find an equation of the tangent line. Some presented a tangent line equation with little supporting work.

In part (b) it appeared that many students did not understand the question. Some of these students set $3y^2 - x = 0$ but never connected this to the curve given by the equation $y^3 - xy = 2$.

In part (c) most students started this part by using the quotient rule. However, many students omitted terms or made evaluation errors.

What were common student errors or omissions?

In part (a) the most common errors were simplification and arithmetic errors. For example, some students incorrectly evaluated $\frac{dy}{dx}$ or incorrectly simplified their equation for the tangent line.

In part (b) some students did not know to set the denominator of $\frac{dy}{dx}$ equal to 0. Some set the numerator equal to 0. Some students were unable to write an equation in one variable. Of those who produced an equation in either x or y, some were unable to correctly solve the equation. Some students simply tested points and were unable to present a correct answer.

In part (c) most students used the quotient rule to find $\frac{d^2y}{dx^2}$. Some students made errors in applying the quotient rule. The most common errors included a reversal in the numerator, a missing factor of y in the second term of the numerator, and a missing square in the denominator. There were also many presentation errors such as missing parentheses. Most students were able to substitute for $\frac{dy}{dx}$. However, many students made simplification errors in arriving at a final answer.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should emphasize the connection between $\frac{dy}{dx}$ and the slope of a curve. Students need more practice in finding the second derivative when $\frac{dy}{dx}$ is given. Students must use proper notation and grouping symbols when presenting a mathematical expression.

Question BC2

What was the intent of this question?

In this problem students were given the velocity vector of a particle moving in the xy-plane with position $(x(t), y(t))$. The particle is at the point $(3, 5)$ at time $t = 1$. In part (a) students had to find the x-coordinate of the position of the particle at time $t = 2$. The x-coordinate of the position of the particle at $t = 1$ added to the net change from $t = 1$ to $t = 2$ produces the x-coordinate at $t = 2$, which is $x(1) + \int_1^2 \cos\left(t^2\right) dt$. Students were expected to evaluate this expression with the calculator. In part (b) students were given that there is a point on the curve at which the line tangent to the curve has a slope of 2. Students needed to find the time at which the particle was at that point. Students had to realize that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and then solve the equation $\frac{dy}{dx} = 2$ using the calculator. In part (c) students were asked to find the time at which the speed of the particle is 3. Students needed to solve the equation $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 3$ using the calculator. In part (d) students were asked to find the total distance traveled by the particle from time $t = 0$ to time $t = 1$. Students needed to set up the integral expression $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ and then evaluate this expression using the calculator.

How well did students perform on this question?

Most students were able to enter this question and present some correct work. The mean score was 4.87 out of a possible 9 points. Approximately 15 percent of students did not earn any points on this problem.

In part (a) most students presented a definite integral to represent the change in the x-coordinate of the particle. Some students did not use the correct lower limit and others attempted to find a closed form antiderivative. The most common lower limit was $t = 1$, and most students added the initial condition, $x(1) = 3$, to obtain the correct answer using the calculator. Some students used a lower limit of $t = 0$, and some were able to incorporate the definite integral from $t = 0$ to $t = 1$ to obtain the correct answer.

In part (b) most students presented a correct expression for the slope in terms of t and obtained the correct answer. However, many students introduced alternate notation without definitions, and some students incorrectly treated $v(t)$ as a position vector rather than a velocity vector.

In part (c) most students presented a correct expression for the speed of the particle. Some students continued to use alternate expressions without any explicit declaration. Some students made presentation errors in writing $\cos^2\left(t^2\right)$.

In part (d) most students were able to present the correct definite integral for the distance traveled by the particle from time $t = 0$ to $t = 1$. Students generally used their expression for speed from part (c) and many continued to use undeclared expressions.

What were common student errors or omissions?

In part (a) some students presented an indefinite integral, and others attempted to find an elementary antiderivative for $\cos\left(t^2\right)$. There were several linkage issues in this problem; that is, some students connected unequal expressions with equal signs. Some students presented ambiguous integrals by excluding the differential dt (e.g., $\int_1^2 \cos\left(t^2\right) + 3$). Some students did not present the mathematical expression that led to the calculator answer.

In part (b) some students presented an expression for the slope using one of the pairs $x'(t),\ y'(t)$, or $\frac{dx}{dt},\ \frac{dy}{dt}$, or $v_x(t),\ v_y(t)$, to identify the *x*-component and the *y*-component of the velocity vector, respectively, without explicitly declaring this identification. Some students incorrectly treated $v(t)$ as a position vector rather than a velocity vector.

In part (c) some students made presentation errors, omitting a parenthesis. Some students substituted 3 into the expression for speed rather than finding when the speed is 3. A few students represented the speed of the particle as $\left|\frac{dy}{dx}\right|$.

In part (d) there were also some parentheses presentation errors. Some students presented $\int_0^1 \sqrt{\cos^2\left(t^2\right)}\ dt + \int_0^1 \sqrt{e^t}\ dt$ as the distance traveled by the particle. Some students attempted to use the arc length formula to compute distance.

In general, some students presented confusing expressions for slope, speed, and distance in this problem. Several students obviously did not use the calculator, and there were some decimal presentation errors. Some students did not present the mathematical expression that led to their numerical answer. Most students used the calculator to correctly evaluate a definite integral. However, some students were unable to use the calculator to solve an equation in parts (b) and (c).

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students must remember to present the limits on a definite integral and to include the differential to ensure that the integrand is well defined. Teachers should emphasize to students that presentation and mathematical notation is very important, including the use of balanced parentheses, proper placement of exponents, and appropriate use of equal signs.

Students still need practice with the four required calculator capabilities, especially solving equations numerically. Students seem to be able to navigate the graph screen well and to use the intersect capability to explore and estimate solutions. However, students need to better understand how to solve equations numerically with the calculator in order to obtain more accurate solutions.

Teachers should remind students to use the calculator to store/define functions that are given and named in the statement of a question. This will help to ensure that the correct expressions are used in subsequent work with the functions in the calculator. Students must explicitly define all other symbolic representations used in their expressions, equations, and integrals, particularly when the presentation may appear to be a formula.

Question BC5

What was the intent of this question?

In this problem students were given $f(x) = \dfrac{1}{x^2 - kx}$ and $f'(x) = \dfrac{k - 2x}{\left(x^2 - kx\right)^2}$, where the parameter, k, is a nonzero constant. In part (a) for $k = 3,$ students were asked to write an equation for the line tangent to the graph of f at the point with $x = 4.$ Students needed to compute $f(4)$ and $f'(4)$ and then use those values and the functions to produce an equation. In part (b) for $k = 4,$ students needed to determine whether f had a relative minimum, a relative maximum, or neither at $x = 2.$ Students were expected to confirm that $f'(2) = 0$ and apply the First Derivative Test. Since f' changes sign from positive to negative at $x = 2,$ students should have concluded that f has a relative maximum at $x = 2.$ In part (c) students had to find the value of k for which f has a critical point at $x = -5.$ Students were expected to solve $f'(-5) = 0$ to determine that $k = -10.$ In part (d) students were expected to use partial fraction decomposition to rewrite $f(x)$ as a sum of rational expressions. The result is used to find $\int f(x)\,dx.$ The partial fraction decomposition yields

$\dfrac{1}{x(x-6)} = \dfrac{A}{x} + \dfrac{B}{x-6} = \dfrac{-1/6}{x} + \dfrac{1/6}{x-6},$ and the general antiderivative is $-\frac{1}{6}\ln|x| + \frac{1}{6}\ln|x-6| + C.$

How well did students perform on this question?

The overall performance on this question was very good. The mean score was 6.21 out of a possible 9 points. Only 4 percent of students did not earn any points on this problem.

In part (a) most students did very well. Some students made numerical or algebraic errors. Most students understood the concepts of slope and tangent line.

In part (b) some students attempted to use a sign chart alone as justification. Some students had difficulty expressing the justification and confused the behavior of the function f with its derivative. There were some students who used the Second Derivative Test for justification. Many of these students did not compute the second derivative correctly.

In part (c) there were many students who did not know how to start the problem. Many students did not treat k as an unknown parameter. Some students included an additional solution, $-5,$ by considering where the first derivative does not exist.

In part (d) although most students performed well, many did not explicitly state the partial fraction decomposition. Many students did not include the absolute value symbols in their final solution, and some forgot the $+C.$

What were common student errors or omissions?

In part (a) some students wrote a correct point-slope equation for the tangent line and then went on to obtain an incorrect slope-intercept form equation.

In part (b) some students did not tie their answer and justification to the sign of $f'.$ These students often presented a justification that was formulaic and not specifically associated with the problem.

In part (c) the most common error was an additional solution, -5. Students who reported both -5 and -10 were not penalized.

In part (d) some students had difficulty factoring the denominator, and some reported an incorrect partial fraction decomposition. Some students used pattern recognition to arrive at the partial fraction decomposition. Some students did not include the absolute value symbols, and some omitted the $+C$. There were some errors in simplification.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should emphasize that it is important to read questions carefully. Some students did not earn points in this problem by not explicitly producing the partial fraction decomposition as required. Students should be reminded that unless otherwise indicated, algebraic and numerical simplification are not required on the AP Calculus Exams. For example, in part (d) some students who gave the correct answer as $\frac{-\ln|x|}{6} + \frac{\ln|x-6|}{6} + C$ simplified this expression incorrectly.

Teachers should remind students that it is necessary to include proper, explicit mathematical steps to reach the solution. Proper mathematical notation, including parentheses, is also important and necessary.

Students need to remember that when using the First Derivative Test to characterize, with justification, the behavior of a function at a critical point, they must tie their justification to the sign of the first derivative of the function, and not just the sign at selected points near the critical point.

Question BC6

What was the intent of this question?

In this problem students were presented with the Maclaurin series $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n} x^n + \cdots$ for a function f. The Maclaurin series converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series. In part (a) students were asked to use the ratio test to find R. Students were expected to evaluate $\lim_{n \to 0} \left| \frac{a_{n+1}}{a_n} \right|$ and use this limit to find R. Evaluating the limit results in a conclusion of $|x| < \frac{1}{3}$, and thus the radius of convergence is $R = \frac{1}{3}$. In part (b) students were asked to write the first four nonzero terms of the Maclaurin series for f', then express f' as a rational function for $|x| < R$. By using term-by-term differentiation, the first four nonzero terms are $1 - 3x + 9x^2 - 27x^3$. Because this series is geometric with a common ratio of $-3x$, the rational function is $f'(x) = \frac{1}{1+3x}$. In part (c) students needed to write the first four nonzero terms of the Maclaurin series for e^x and use this series to write a third-degree Taylor polynomial for $g(x) = e^x f(x)$ about $x = 0$. After showing that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$, students were expected to multiply to determine that the third-degree Taylor polynomial desired is $T_3(x) = x - \frac{1}{2}x^2 + 2x^3$.

How well did students perform on this question?

The mean score was 3.96 out of a possible 9 points. This was good for a series question that was the last problem on the exam. There were multiple entry points in this problem that contributed to positive student performance. Approximately 9 percent of students did not earn any points on this problem.

In part (a) it was often difficult to follow student solutions. There were many presentation and notation errors.

In part (b) most students were not able to find the sum of the geometric series.

In part (c) although many students earned the first point, some students did not know the Maclaurin series expansion for e^x. Some students simply did not know to multiply the series expressions for e^x and $f(x)$, and some only multiplied terms with the same power.

What were common student errors or omissions?

In part (a) many students set up a ratio without using consecutive terms of the series. It appeared that many of these errors were due to misuse of parentheses. Some students did not use limit notation correctly or consistently. Some students had difficulty evaluating $(-3)^{-1}$, and others confused the concepts of interval of convergence and radius of convergence.

In part (b) most students did not attempt to write a rational function using the sum of a geometric series. It was not clear whether these students could not identify the common ratio or did not know the formula for the sum of a geometric series. Many students presented a formula using sigma notation.

In part (c) some students attempted to multiply the two series approximations by corresponding terms rather than distributing the multiplication. Some students did not collect terms and, therefore, did not present a third-degree Taylor polynomial.

Based on your experience of student responses at the AP® Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Teachers should emphasize the use of proper mathematical notation and interpretation (e.g., the difference between -3^n and $(-3)^n$). Absolute value symbols and limit notation must be used properly in the application of the ratio test.

In problems similar to part (a), students must use consecutive terms when applying the ratio test and must be cautious if they omit the expression involving the variable x. In this problem, the ratio of the consecutive terms simplifies to $K|x|$. Interpreting or using K to find the radius of convergence is a routine task. In a series where the ratio of consecutive terms simplifies to $K\left|x^p\right|$, where p is a power other than 1 or 0, finding the radius of convergence is more difficult. Teachers should introduce their students to these type of problems.

Teachers should emphasize the importance and multiple uses of the formula for the sum of a geometric series. Students should have more practice with performing operations on series and manipulating series.