AP® CALCULUS AB/CALCULUS BC
2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8,$ where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8.$ Selected values of $R(t)$ are shown in the table above. At time $t = 0,$ there are 50,000 liters of water in the tank.

(a) Estimate $R'(2).$ Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8,$ is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \dfrac{R(3) - R(1)}{3 - 1} = \dfrac{950 - 1190}{3 - 1} = -120$ liters/hr^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t)\,dt.$

$$\begin{aligned}\int_0^8 R(t)\,dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters}\end{aligned}$$

This is an overestimate since R is a decreasing function.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $$\begin{aligned}\text{Total} &\approx 50000 + \int_0^8 W(t)\,dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters}\end{aligned}$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0,$ $W(8) - R(8) < 0,$ and $W(t) - R(t)$ is continuous.

Therefore, the Intermediate Value Theorem guarantees at least one time $t,$ $0 < t < 8,$ for which $W(t) - R(t) = 0,$ or $W(t) = R(t).$

For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

$$R'(2) \approx \frac{R(3) - R(1)}{3-1} = \frac{950-1190}{2}$$

$$R'(2) \approx -120 \text{ L/hr}^2$$

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

$$\int_0^8 R(t)\,dt \approx L_4 = (1)(1340) + (2)(1190) + (3)(950) + (2)(740)$$

$$\int_0^4 R(t)\,dt \approx 8{,}050 \text{ L}$$

This is an overestimate because we are taking the Left Riemann sum of a decreasing function. Therefore the left endpoints, are greater than the right endpoints in each subinterval.

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(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

$t=0 \rightarrow 50{,}000$ L

Total Left $\approx 50{,}000 + \left(\int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt\right)$

$\int_0^8 W(t)\,dt = 7836.19532455$

Total Left $\approx 50{,}000 + (7836.19532455 - 8050)$

$\boxed{\text{Total Left} \approx 49{,}786 \text{ L}}$

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

$T(t) = W(t) - R(t)$

$T(0) = 2000 - 1340 = 600$

$T(8) = 81.524 - 700 = -618.47$

$\boxed{-618.47 < 0 < 600}$

When $T(t) = 0$, the rate of water being pumped in to the tank is equal to the rate of water being pumped out. Because the function $T(t)$ is continuous and differentiable, by IVT there exists some t on the interval $[0, 8]$ where $T(t) = 0$.

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t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

$$R'(2) \approx \frac{950 - 1190}{3 - 1} = \boxed{-120 \text{ liters per hour}^2}$$

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

$1340 + (2)(1190) + (3)(950) + (2)(740) = 8050$

8050 liters is the total amount of water removed during the 8 hours. The left Riemann sum is an overestimate since R(t) is decreasing and the graph is below the Riemann sum.

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Continue problem 1 on page 5.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

$$\int_0^8 (W(t) - R(t))\,dt$$

$$\cancel{\int_0^8 (W(t)) = 8}$$

$$50{,}000 + \left(\int_0^8 (W(t))\,dt - 8050\right)$$

$$50{,}000 + (7836.1953 - 8050)$$

$$49786.7953 \text{ liter}$$

$\boxed{\text{49,787 liters of water is in the tank.}}$

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

No there is no rate that is the same as the rate at which water is being removed because R'(t) is decreasing at a slower rate than W'(t) so the rate at which W'(t) is decreasing is much greater than R'(t).

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$\frac{y_2 - y_1}{x_2 - x_1}$

x — t (hours)	0	1	3	6	8
y — $R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1190 - 1340}{1 - 0} = -150 \ \frac{L^2}{hr}$

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

$1(1340) + 2(1190) + 3(950) + 2(740) = 21370 \ \frac{\text{liters}}{\text{hour}}$

over estimate b/c left Riemann sum.

Continue problem 1 on page 5.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

$50{,}000 + \int_0^8 w(t)\,dt - 21370 =$ 36466 Liters in the tank after 8 hours

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

yes b/c the ammount of water pumped out is increasing.

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Question 1

Overview

In this problem students were given a function W that models the rate, in liters per hour, at which water is pumped into a tank at time t hours. They were also given a function R that models the rate, in liters per hour, at which water is removed from the tank. W is defined as an exponential function on the interval $0 \le t \le 8,$ and R is differentiable and decreasing on $0 \le t \le 8.$ Selected values of $R(t)$ are given in a table. The amount of water in the tank, in liters, at time $t = 0$ is given. In part (a) students needed to estimate $R'(2)$ by calculating the value of an appropriate difference quotient based on the values in the table. Units of liters/hr^2 are required. In part (b) students needed to use a left Riemann sum approximation for $\int_0^8 R(t)\,dt$ to estimate the total amount of water removed from the tank during the interval $0 \le t \le 8.$ Students needed to use the appropriate function values from the table with the four subintervals $[0, 1],$ $[1, 3],$ $[3, 6],$ and $[6, 8].$ By applying the given information that R is decreasing, students needed to conclude that the left Riemann sum approximation is an overestimate. In part (c) students needed to estimate the total amount of water in the tank at time $t = 8.$ This required adding the amount of water in the tank at time $t = 0$ to the amount of water pumped into the tank during the interval $0 \le t \le 8,$ and then subtracting the overestimate found in part (b). The definite integral $\int_0^8 W(t)\,dt$ gives the amount of water pumped into the tank during the interval $0 \le t \le 8$ and is evaluated using the calculator. In part (d) students needed to apply the Intermediate Value Theorem to $W(t) - R(t).$ This theorem guarantees at least one time t on the interval $0 < t < 8$ for which $W(t) - R(t) = 0$ or $W(t) = R(t).$ For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

Sample: 1A
Score: 9

The response earned all 9 points.

Sample: 1B
Score: 6

The response earned 6 points: 2 points in part (a), 3 points in part (b), 1 point in part (c), and no points in part (d). In parts (a) and (b) the student's work is correct. In part (c) the student earned the first point for the correct definite integral. The student did not earn the second point for the estimate because of an arithmetic error. In part (d) the student earned no points.

Sample: 1C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student calculates an estimate using an incorrect interval, and the units are incorrect. In part (b) the student has a correct left Riemann sum and earned the first point. The student has an incorrect estimate and does not support the answer of an overestimate with a valid reason. In part (c) the student earned the first point for the definite integral. The estimate is consistent with the student's estimate from part (b), so the second point was earned. In part (d) the student earned no points.