2017

AP CollegeBoard

AP Calculus BC

Sample Student Responses and Scoring Commentary

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Question 1

	1 : units in parts (a), (c), and (d)
(a) Volume $= \int_0^{10} A(h)\,dh$ $\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$ $= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$ $= 176.3$ cubic feet	$2 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$
(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.	1 : overestimate with reason
(c) $\int_0^{10} f(h)\,dh = 101.325338$ The volume is 101.325 cubic feet.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) Using the model, $V(h) = \int_0^h f(x)\,dx$. $\left.\frac{dV}{dt}\right\|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$ $= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$ $= f(5)\cdot 0.26 = 1.694419$ When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.	$3 : \begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$V \approx [2(50.3)+3(14.4)+5(6.5)]$

$\approx 176.3 \text{ ft}^3$

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

The approximation in part a is an overestimate because A is a decreasing function.

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Continue question 1 on page 5.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

$$V = \int_0^{10} \left(\frac{50.3}{e^{0.2h}+h}\right) dh$$

$$\approx 101.325 \text{ ft}^3$$

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

$h = 5$ ft

$\frac{dh}{dt} = 0.26$ ft/min

$\frac{dV}{dt} = ?$

$$V = \int_0^h \left(\frac{50.3}{e^{0.2h}+h}\right) dh$$

$$\frac{dV}{dt} = \frac{50.3}{e^{0.2h}+h} \cdot \frac{dh}{dt}$$

$$= \frac{50.3}{e^{0.2(5)}+(5)} \cdot 0.26$$

$$\approx 1.694 \text{ ft}^3/\text{min}$$

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h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$$V = [A(0)](2) + [A(2)](3) + [A(5)](5)$$
$$= (50.3)(2) + (14.4)(3) + (6.5)(5) = 176.3 \text{ ft}^3$$

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

The volume is overestimated, because the A(h) is continuosly decreasing, so after the value of A(h) at the left end of each subinterval, A(h) is lower for the rest of the subinterval (compared to the left end). which means the actual volume should be lower than the approximation.

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Continue question 1 on page 5.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \frac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

Volume equals to $\int_0^{10} f(h)\,dh \doteq 101 \text{ ft}^3$

(use of graphing calculator)

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

$V = \int_0^5 f(h)\,dh$, $\frac{dV}{dt} = [f(5) - f(0)]\frac{dh}{dt}$

$\therefore \frac{dV}{dt} = (6.52 - 50.3)(0.26) = 11.3828 \text{ ft}^3/\text{minute}$

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h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$2(14.4) + 3(6.5) + 5(2.9)$

$28.8 + 19.5 + 14.5 = 62.8 \text{ ft}^3$

The volume of the tank is the $\int_0^{10} A(h)$, therefore using a left Riemann sum, the volume of the tank is $\boxed{62.8 \text{ ft}^3}$.

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

Because the function A is decreasing over the interval between $0 \le h \le 10$, the LRAM is an under-estimate of the volume in the tank.

Continue question 1 on page 5.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

The Volume of the tank is the $\int f(h)\,dh$.

$$\int_0^{10} \frac{50.3}{e^{.2h}+h}\,dh = 101.325 \text{ ft}^3$$

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

$h=5$

$\frac{dh}{dt} = .26$ ft/min

$\frac{dv}{dt} = f(h) = \frac{50.3}{e^{.2h}+h}$ when $h=5$ $= 6.517$ ft/min²

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Question 1

Overview

In this problem students were presented with a tank that has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by a continuous and decreasing function A, where $A(h)$ is measured in square feet. Values of $A(h)$ for heights $h = 0,\ 2,\ 5,$ and 10 are supplied in a table. In part (a) students were asked to approximate the volume of the tank using a left Riemann sum and indicate the units of measure. Students needed to respond by incorporating data from the table in a left Riemann sum expression approximating $\int_0^{10} A(h)\,dh$ using the subintervals $[0, 2],\ [2, 5],$ and $[5, 10].$ [LO 3.2B/EK 3.2B2] In part (b) students needed to explain that a left Riemann sum approximation for the definite integral of a continuous, decreasing function overestimates the value of the integral. [LO 3.2B/EK 3.2B2] In part (c) the function f given by $f(h) = \frac{50.3}{e^{0.2h} + h}$ is presented as a model for the area, in square feet, of the horizontal cross section at height h feet. Students were asked to find the volume of the tank using this model, again indicating units of measure. Using the model f for cross-sectional areas of the tank, students needed to express the volume of the tank as $\int_0^{10} f(h)\,dh$ and use the graphing calculator to produce a numeric value for this integral. [LO 3.4D/EK 3.4D2] In part (d) water is pumped into the tank so that the water's height is increasing at the rate of 0.26 foot per minute at the instant when the height of the water is 5 feet. Students were asked to use the model from part (c) to find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet, again indicating units of measure. Students needed to realize that the volume of water in the tank, as a function of its height h, is given by $V(h) = \int_0^h f(x)\,dx$ and then use the Fundamental Theorem of Calculus to find that the rate of change of the volume of water with respect to its height is given by $V'(h) = f(h).$ Then, using the chain rule for derivatives, students needed to relate the rates of change of volume with respect to time and height and the rate of change of the water's height with respect to time. Information in the problem suffices to be able to find these rates when the water's height is 5 feet. [LO 2.3C/EK 2.3C2, LO 3.3A/EK 3.3A2] This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 1A
Score: 9

The response earned all 9 points: the units point, 2 points in part (a), 1 point in part (b), 2 points in part (c), and 3 points in part (d). The units point was earned because the units of ft^3 in parts (a) and (c) and ft^3/min in part (d) are all correct. In part (a) the left Riemann sum point was earned by the numerical expression in the first line. This expression would have also earned the approximation point without simplification. The student chooses to simplify, does so correctly, and thus earned the approximation point. In part (b) the statement "overestimate because A is a decreasing function" earned the point. In part (c) the definite integral earned the integral point, and 101.325 earned the answer point. In part (d) the second line on the right earned the 2 points for $\frac{dV}{dt}$. The third line on the right would have earned the answer point without simplification. The student chooses to give a final answer of 1.694 that is computed correctly and earned the answer point.

Question 1 (continued)

Sample: 1B
Score: 6

The response earned 6 points: the units point, 2 points in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). The units point was earned because the units of ft^3 in parts (a) and (c) and ft^3/minute in part (d) are all correct. In part (a) the left Riemann sum point was earned by the symbolic expression in the first line. The approximation point was earned by the second line. The numerical expression in the second line would have earned the approximation point without simplification to 176.3. The student chooses to simplify, does so correctly, and thus earned the approximation point. In part (b) the response of "overestimated" with the reason that "$A(h)$ is continuously decreasing" earned the point. In part (c) the expression $\int_0^{10} f(h)\,dh$ earned the integral point. The answer of 101 did not earn the answer point because the result is not accurate to three places after the decimal point. In part (d) the equation $\frac{dV}{dt} = [f(5) - f(0)]\frac{dh}{dt}$ earned 1 of the 2 $\frac{dV}{dt}$ points for using the chain rule. The student has made an error in the application of the Fundamental Theorem of Calculus. The numerical evaluation of the student's expression for $\frac{dV}{dt}$ is not eligible to earn the answer point.

Sample: 1C
Score: 3

The response earned 3 points: no units point, no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). The units of ft/min^2 in part (d) are incorrect, so the student did not earn the units point. Because the student uses a right Riemann sum, no points were earned in part (a). The confusion of left with right has implications in part (b). Because the setup in part (a) for a right Riemann sum is accurate, the student is eligible to earn the point in part (b) if the response is consistent for a right Riemann sum. In part (b) the implication that A is decreasing leads to an "under-estimate" is consistent with a right Riemann sum. Thus, the student earned the point. In part (c) $\int_0^{10} \frac{50.3}{e^{.2h} + h}\,dh$ earned the integral point, and 101.325 earned the answer point. In part (d) no points were earned because the statement of $\frac{dV}{dt} = f(h)$ is incorrect, and the remaining work only implies, incorrectly, that $\frac{dV}{dt} = f(5) = 6.517.$ The answer point was not earned because the student's expression for $\frac{dV}{dt}$ is not eligible to earn the answer point.