2018

AP CollegeBoard

AP Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 2

☑ Scoring Guideline

☑ Student Samples

☑ Scoring Commentary

Question 2

(a) $p'(25) = -1.179$ At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$
(b) $\int_0^{30} 3p(h)\,dh = 1675.414936$ There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_{30}^{K} 3f(h)\,dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters. The number of plankton cells, in millions, in the entire column of water is given by $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh.$ Because $0 \le f(h) \le u(h)$ for all $h \ge 30$, $3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{K} u(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 3 \cdot 105 = 315.$ The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \le 2000$ million.	$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$
(d) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.455862$ The total distance traveled by the boat over the time interval $0 \le t \le 1$ is 757.456 (or 757.455) meters.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$

2A
1of2

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = -1.179$

at a depth of 25 meters, the density of plankton is decreasing at a rate of $1.179 \dfrac{\text{millions of cells}}{\text{meter}^3} / \text{meter}$ as depth increases

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$3\int_0^{30} p(h)\,dh = 1675$ million plankton cells

Continue question 2 on page 7.

2A
2d2

me 1990

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

number of plankton cells $= 3\int_0^{30} p(h)\,dh + 3\int_{30}^{K} f(h)\,dh$

since K is finite, and $0 \le f(h) \le u(h)$ for all $h \ge 30$,

$3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 315$

since $3\int_0^{30} p(h)\,dh$ is approximately 1675,

$3\int_0^{30} p(h)\,dh + 3\int_{30}^{K} f(h)\,dh \le 1990$ million cells

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt =$ $\boxed{757.456 \text{ meters}}$

GO ON TO THE NEXT PAGE.

2B
1 of 2

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$$p'(h) = (.4x - .001x^3)(.99750)^{x^2}$$

$$p'(25) = -1.18$$

At a depth of 25 meters, the density of plankton is decreasing by 1.18 million cells per cubic meters per meter in depth.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$$3\int_0^{30} .2h^2 e^{-.0025h^2}\, dh$$

$\approx$ 1,675 million plankton cells

Continue question 2 on page 7.

2B
2 of 2

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$3\int_0^{30} p(h)\,dh + 3\int_{30}^{K} f(h)\,dh$$

$$1{,}675 + 3\int_{30}^{K} f(h)\,dh$$

Since $f(h)$ is less than $u(h)$, $\int_{30}^{\infty} f(h)\,dh$ is less than $\int_{30}^{\infty} u(h)\,dh$

Therefore $\int_{30}^{\infty} f(h)\,dh$ is less than 105.

$1{,}675 + 105 < 2000$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\int_0^1 \sqrt{(662\sin(5t))^2 + (880\cos(6t))^2}\,dt = 757.46 \text{ meters}$$

GO ON TO THE NEXT PAGE.

2C
1 of 2

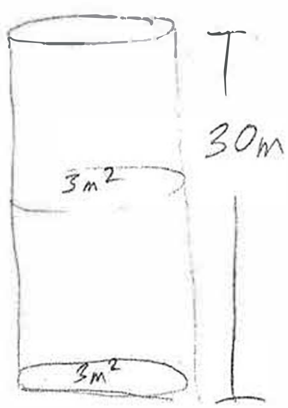
2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$$p'(25) = \frac{d}{dh}\left(0.2h^2e^{-0.0025h^2}\right) = \boxed{-1.179 \text{ million cells}/m^3/s}$$

p'(25) is the rate of change of the dencity of the plankton. In this case, the density of the plankton is decreasing at a rate of 1.179 million cells/m³/s.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?



$$\#cells = \int_0^{30} p(h)\,dh = \boxed{558 \text{ million cells}}$$

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Continue question 2 on page 7.

2C
2 of 2

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$\int_0^{30} p(h)\, dh + \int_{30}^{\infty} u(h)\, dh = \text{\# plankton cells in column}$$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\text{dist} = \int_0^1 \sqrt{(880\cos(6t))^2 + (662\sin(5t))^2}\, dt$$

GO ON TO THE NEXT PAGE.

Question 2

Overview

The context of this problem is an investigation of plankton cells in a sea. The density of plankton cells at a depth of h meters is modeled by $p(h) = 0.2h^2 e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30.$ The density is measured in millions of cells per cubic meter, and the function f is stated to be continuous but is not explicitly given. In part (a) students were asked for the value of $p'(25)$ and to interpret the meaning of $p'(25)$ in the context of the problem. A correct response should give the derivative value as obtained from a graphing calculator and interpret this value as the rate of change of the density of plankton cells, in million cells per cubic meter per meter, at a depth of 25 meters. In part (b) students were asked for the number of plankton cells (to the nearest million) contained in the top 30 meters of a vertical column of water that has horizontal cross sections of constant area 3 square meters. A correct response should combine the density of the plankton, $p(h)$ million cells per cubic meter, and the cross-sectional area of the vertical column to obtain that the number of plankton cells changes at a rate of $3p(h)$ million cells per meter of depth. Thus the number of plankton cells (in millions) in the top 30 meters of the column is the accumulation of this rate for $0 \le h \le 30,$ given by the integral $\int_0^{30} 3p(h)\,dh.$ This integral should be evaluated using a graphing calculator and rounded to the nearest integer. In part (c) a function u is introduced that satisfies $0 \le f(h) \le u(h)$ for $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105.$ Given that the column of water in part (b) is K meters deep, where $K > 30,$ students were asked to write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column, and to explain why the number of plankton cells in the column is at most 2000 million. Using the idea from part (b), a correct response should realize the number of plankton cells in the column is a definite integral of 3 times the density from $h = 0$ to $h = K.$ Because $K > 30,$ and the density is given by $f(h)$ at depths $h \ge 30,$ the number of plankton cells, in millions, in the entire column is $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh.$ The first term was found in part (b); the second term can be bounded by $3 \cdot 105 = 315$ using the given information about the functions f and u, together with properties of integrals. Summing the answer from part (b) with the upper bound of 315 for the second term shows that the number of plankton cells in the entire column of water is less than 2000 million. In part (d) the position of a research boat on the sea's surface is described parametrically by $(x(t), y(t))$ for $t \ge 0,$ where $x'(t) = 662\sin(5t),$ $y'(t) = 880\cos(6t),$ t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Students were asked to find the total distance traveled by the boat over the time interval $0 \le t \le 1.$
A correct response should find the total distance traveled by the boat as the integral of its speed, $\sqrt{(x'(t))^2 + (y'(t))^2},$ across the time interval $0 \le t \le 1$ and evaluate this integral using a graphing calculator.

For part (a) see LO 2.3A/EK 2.3A1, LO 2.3D/EK 2.3D1. For part (b) see LO 3.3B(b)/EK 3.3B2, LO 3.4E/EK 3.4E1. For part (c) see LO 3.2C/EK 3.2C2, LO 3.2D (BC)/EK 3.2D2 (BC), LO 3.4E/EK 3.4E1. For part (d) see LO 3.3B(b)/EK 3.3B2, LO 3.4C/EK 3.4C2 (BC). This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 2A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 3 points in part (c), and 2 points in part (d). In part (a) the response earned the first point for presenting $-1.179.$ The response earned the second point by addressing the value of 25 and identifying that the density is decreasing at a rate of 1.179 millions of

Question 2 (continued)

cells per cubic meter per meter. In part (b) the response earned the first point for presenting a definite integral with the correct integrand. The response earned the second point for computing 1675. In part (c) the response earned the first point for presenting the correct integral expression for the number of plankton cells, in millions, in line 1. The response earned the second point for comparing $3\int_{30}^{K} f(h)\, dh$ to $3\int_{30}^{\infty} u(h)\, dh.$ The response earned the third point for the explanation with an upper bound of 1990. In part (d) the response earned the first point for the integral setup for the distance traveled. The response earned the second point for the answer presented accurately to three places after the decimal point.

Sample: 2B
Score: 6

The response earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response presents $-1.18,$ the answer to only two decimal places, and did not earn the first point. Note that the symbolic expression for $p'(h)$ is correct; the point would still be earned with the use of x instead of h if the numerical answer were correct. The meaning presented does not express the concept of rate explicitly, so the second point was not earned. In part (b) the response earned the first point for presenting a definite integral with the correct integrand. The response earned the second point for computing 1,675. In part (c) the response earned the first point for presenting the correct integral expression for the number of plankton cells, in millions, in line 1. The response earned the second point for "$\int_{30}^{\infty} f(h)\, dh$ is less than $\int_{30}^{\infty} u(h)\, dh$" in line 4. The bound of 105 does not address the factor of 3 that is needed for the second integral in line 4, so the third point was not earned. In part (d) the response earned the first point for the integral setup for the distance traveled. Because the first point in part (a) was not earned due to a decimal presentation error, the second point in part (d) was earned, even though the answer of 757.46 is presented accurately to only 2 places after the decimal point. In any response, no more than 1 point may be impacted by decimal presentation errors.

Sample: 2C
Score: 3

The response earned 3 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 1 point in part (d). In part (a) the response earned the first point for presenting $-1.179.$ The meaning presents incorrect units, so the second point was not earned. In part (b) the response earned the first point for presenting a definite integral with the correct integrand. The response does not address the factor of 3 that is needed, so the second point was not earned. In part (c) the response presents an incorrect integral expression, so the first point was not earned. There is no additional work, so neither of the last 2 points in part (c) were earned. In part (d) the response earned the first point for the integral setup for the distance traveled. The absolute values in the integrand are acceptable. Because the integral is not evaluated, the second point was not earned.