2018

AP CollegeBoard

AP Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 3

☑ Scoring Guideline

☑ Student Samples

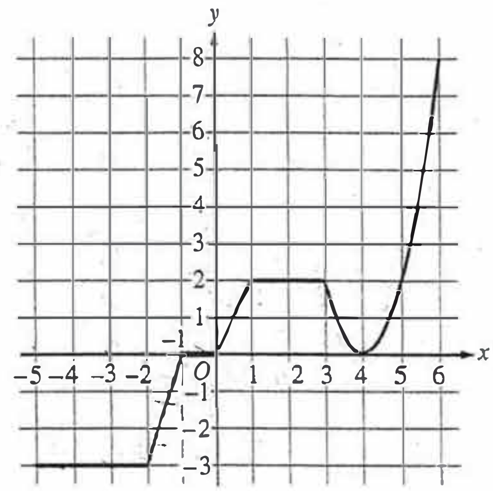
☑ Scoring Commentary

Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$ $= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$ $= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$ $= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$	$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$
(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.	$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$
(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

NO CALCULATOR ALLOWED

3A
1 of 2



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f'(x) = g(x)$

$f(x) = \int_1^x g(t)\,dt + 3$

$f(-5) = -\int_{-5}^1 g(t)\,dt + 3$

$f(-5) = -\left(\frac{1}{2}(1)(2) - \frac{1}{2}(1)(3) - (3)(3)\right) + 3$

$f(-5) = -\left(1 - \frac{3}{2} - 9\right) + 3$

$f(-5) = 8 + \frac{3}{2} + 3$

$f(-5) = 11 + \frac{3}{2}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$18/3 = 6$

$\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$

$\int_1^6 g(x)\,dx = (2)(2) + \int_3^6 2(x-4)^2\,dx$

$\int_1^6 g(x)\,dx = 4 + \left(\frac{2}{3}(x-4)^3\right)\Big|_3^6$

$\int_1^6 g(x)\,dx = 4 + \left(\frac{2}{3}(8) - \frac{2}{3}(-1)\right)$

$\int_1^6 g(x)\,dx = 4 + \frac{16}{3} + \frac{2}{3}$

$\int_1^6 g(x)\,dx = 10$

Continue question 3 on page 15.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

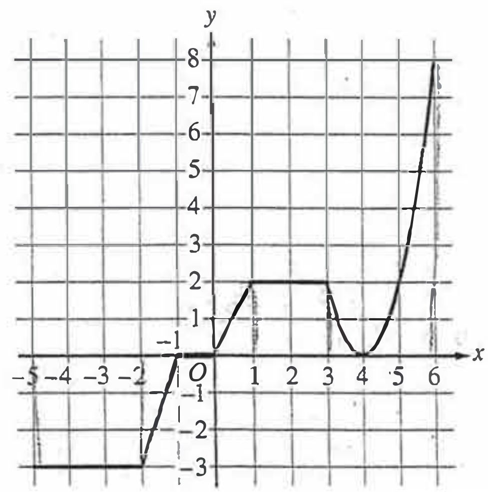
On the interval $(0,1) \cup (4,6)$ f is both increasing and concave up since $f'(x) = g(x)$ and g is positive on the that interval meaning f is increasing on that interval, and g is increasing ~~on~~ that interval, meaning $f''(x) > 0$ on that interval, therefore f is concave up on that interval

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer. A

f has a point of inflection at $x=4$ since $f'(x) = g(x)$ and since g switches from decreasing to increasing at $x=4$, therefore $f''(4) = 0$ at that point and would change signs from $\ominus$ to $\oplus$ at $x=4$, therefore $x=4$ is an inflection point.

NO CALCULATOR ALLOWED

3B
1 of 2



Graph of g

$g(x) = f'(x) \quad f(x) = \int g(x)dx$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$\frac{6}{2} + \frac{19}{2} \quad -\frac{19}{2} - \frac{3}{2}$

$$\int_{-5}^{1} g(x)\,dx = f(1) - f(-5)$$
$$f(-5) = 3 - \int_{-5}^{1} g(x)\,dx$$
$$= 3 - \left[-3(3) - \tfrac{1}{2}(1)(3) + \tfrac{1}{2}(1)(2)\right]$$
$$= 3 - \left[\left(-9 - \tfrac{3}{2}\right) + 1\right]$$
$$= 3 - \left[\tfrac{-21}{2} + \tfrac{2}{2}\right] = 3 + \tfrac{19}{2} = \tfrac{25}{2}$$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$u = x - 4$
$du = dx$

$$\int_1^6 g(x)dx = 3(3) + \int_3^6 g(x)\,dx$$
$$= 9 + \int_3^6 2(x-4)^2\,dx \qquad \tfrac{27}{3}$$
$$= 9 + 2\int_{-1}^{2} u^2\,du$$
$$= 9 + 2 \cdot \tfrac{1}{3}u \Big|_{-1}^{2}$$
$$= 9 + 2\left(\tfrac{2}{3} - \tfrac{1}{3}\right) = 9 + \tfrac{2}{3} = \tfrac{29}{3}$$

Continue question 3 on page 15.

NO CALCULATOR ALLOWED

3B
2 of 2

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

f' is pos f'' pos

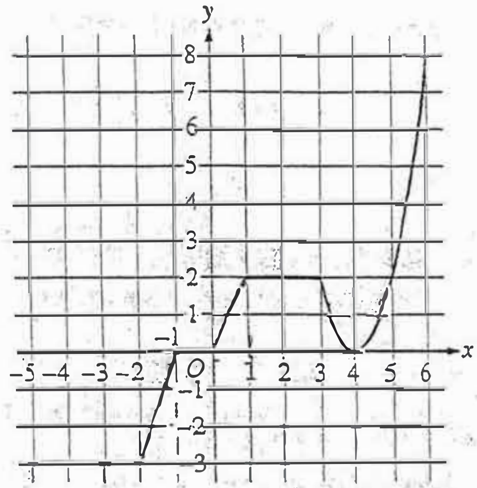
f is Increasing when $f'(x) = g(x)$ is positive
f is Concave up when $f'(x) = g(x)$ is increasing

$\Rightarrow$ The graph of f is concave up and increasing on $(0,1) \cup (4,6)$.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer. a

f has a point of inflection when $f'(x) = g(x)$ has a maximum or minimum (local)

$\Rightarrow x = 4$



Graph of g f'(x) Œ

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$? $f'(x) = g(x)$

$f'(x) = 2(x-4)^2$

$f(x) = \frac{2(x-4)^3}{3}$

$f(-5) = \frac{2(-5-4)^3}{3}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$\int_1^6 g(x)\,dx$

$\left.\frac{2(x-4)^3}{3}\right]_1^6 \quad \frac{16}{3} + \frac{54}{3} = \boxed{\frac{70}{3}}$

Continue question 3 on page 15.

(c) bFor $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

from $0 < x < 1$ and $4 < x < 6$
the graph of f is both inc.
and concave up because
f' is above the x-axis (positive)
and has an increasing slope

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

there is a point of inflection at
$x = 4$ because the slope of f'
changes from decreasing to increasing
(-) (+)

2018 SCORING COMMENTARY

Question 3

Overview

In this problem the graph of the continuous function g is provided; g is piecewise linear for $-5 \le x < 3,$ and $g(x) = 2(x-4)^2$ for $3 \le x \le 6.$ It is also given that g is the derivative of the function f. In part (a) students were given that $f(1) = 3$ and asked for the value of $f(-5).$ A correct response should demonstrate knowledge that f is an antiderivative of g, so that $f(-5) = f(1) + \int_1^{-5} g(x)\,dx.$ The integral $\int_1^{-5} g(x)\,dx$ should then be evaluated using properties of definite integrals and computation of areas of the regions between the graph of g and the x-axis using geometry. In part (b) students were asked to evaluate $\int_1^6 g(x)\,dx.$ A correct response should use the property of integrals to split the interval of integration into the sum of integrals across adjacent intervals $[1, 3]$ and $[3, 6].$ One of the resulting integrals can be computed using geometry and the other using an antiderivative of $g(x) = 2(x-4)^2$ on the interval $3 \le x \le 6.$ In part (c) students were asked for the open intervals on $-5 < x < 6$ where the graph of f is both increasing and concave up and to give a reason for their answer. A correct response should demonstrate the connection between properties of the derivative of f and the properties of monotonicity and concavity for the graph of f. The graph of f is strictly increasing where $g = f'$ is positive, and the graph of g is concave up where the graph of $g = f'$ is increasing. In part (d) students were asked for the x-coordinate of each point of inflection of the graph of f and to give a reason for their answer. A correct response should convey that a point of inflection of the graph of f occurs at a point where the derivative of f changes from increasing to decreasing, or from decreasing to increasing. This can be obtained from the supplied graph of $g = f',$ which changes from decreasing to increasing at $x = 4.$

For part (a) see LO 3.2C/EK 3.2C1, LO 3.2C/EK 3.2C2. For part (b) see LO 3.2C/EK 3.2C1, LO 3.2C/EK 3.2C2, LO 3.3B(b)/EK 3.3B2, LO 3.3B(b)/EK 3.3B5. For parts (c) and (d), see LO 2.2A/EK 2.2A1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 3A
Score: 9

The response earned all 9 points: 2 points in part (a), 3 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point with the expression $-\int_{-5}^{1} g(t)\,dt$ in line 3 on the left. The second point would have been earned by the numerical expression in line 4 with no simplification. In this case, correct simplification to $11 + \frac{3}{2}$ earned the second point. In part (b) the response earned the first point with the sum of the two integrals in line 1 on the left. The second point was earned with the antiderivative expression $\left(\frac{2}{3}(x-4)^3\right)$ in line 3. The second point would have been earned by the numerical expression in line 4 with no simplification. In this case, correct simplification to 10 earned the third point. In part (c) the union of intervals "$(0, 1) \cup (4, 6)$" earned the first point. The second point was earned with the reason $f'(x) = g(x),$ "g is positive," and "g is increasing on that interval." In part (d) the first point was earned by identifying the x-coordinate of a point of inflection at $x = 4.$ The second point was earned with the reason $f'(x) = g(x)$ and "g switches from decreasing to increasing at $x = 4.$"

Question 3 (continued)

Sample: 3B
Score: 6

The response earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point with the expression $\int_{-5}^{1} g(x)\,dx$ in line 1. The second point would have been earned by the numerical expression in line 3 with no simplification. In this case, correct simplification to $\frac{25}{2}$ earned the second point. In part (b) the integral expression $3(3) + \int_{3}^{6} g(x)\,dx$ in line 1 did not earn the first point because $\int_{1}^{3} g(x)\,dx = 4,$ not 9. This response used substitution of variables to write $\int_{3}^{6} 2(x-4)^2\,dx$ in an equivalent form. The antiderivative in line 4 is incorrect, and the response did not earn the second point. As a result of this error, the response is not eligible for the answer point. In part (c) the union of intervals "$(0, 1) \cup (4, 6)$" earned the first point. The second point was earned with the reason "$f'(x) = g(x)$ is positive" in line 1 and "$f'(x) = g(x)$ is increasing" in line 2. In part (d) the first point was earned by identifying the x-coordinate of a point of inflection at $x = 4.$ The second point was earned with the reason "$f'(x) = g(x)$ has a maximum or minimum (local)."

Sample: 3C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). In part (a) an integral expression is not presented nor is its numerical equivalent, so the first point was not earned. The value given for $f(-5)$ is incorrect, so the second point was not earned. In part (b) the response did not earn the first point because $\int_{1}^{6} g(x)\,dx$ is not written as the sum of two integrals or the equivalent. The second point was earned with the antiderivative expression in line 2. Because the second point was earned, the response is eligible for the third point. The answer is incorrect, however, so the third point was not earned. In part (c) the first point was earned with the intervals "$0 < x < 1$ and $4 < x < 6.$" Although "f' is above the x-axis" is a valid reason for why f is increasing on those intervals, f' "has an increasing slope" is not a valid reason to explain why the graph of f is concave up on those intervals. The second point was not earned. In part (d) the first point was earned by identifying the x-coordinate of a point of inflection at $x = 4.$ "The slope of f' changes from decreasing to increasing" is not a valid reason to explain why the graph of f has a point of inflection at $x = 4,$ so the second point was not earned.