2021

AP® CollegeBoard

AP® Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 2

☑ Scoring Guideline

☑ Student Samples

☑ Scoring Commentary

Part A (BC): Graphing calculator required

Question 2 — 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

For time $t \geq 0,$ a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right) \right\rangle.$ At time $t = 0,$ the position of the particle is $(-2, 5).$

	Model Solution	Scoring	
(a)	Find the speed of the particle at time $t = 1.2.$ Find the acceleration vector of the particle at time $t = 1.2.$		
	$\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271488$ At time $t = 1.2,$ the speed of the particle is $1.271.$	Speed	**1 point**
	$\langle x''(1.2), y''(1.2) \rangle = \langle 6.246630, 0.405125 \rangle$ At time $t = 1.2,$ the acceleration vector of the particle is $\langle 6.247 \text{ (or } 6.246), 0.405 \rangle.$	Acceleration vector	**1 point**

Scoring notes:

- Unsupported answers do not earn any points in this part.
- The acceleration vector may be presented with other symbols, for example $(\, , \,)$ or $[\, , \,],$ or the coordinates may be listed separately, as long as they are labeled.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, speed $= 0.844$ and $y''(1.2) = 0.023$ (or 0.022).

Total for part (a) 2 points

(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 1.2$.

$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.009817$	Integrand	**1 point**
The total distance traveled by the particle over the time interval $0 \le t \le 1.2$ is 1.010 (or 1.009).	Answer	**1 point**

Scoring notes:

- The first point is earned by presenting the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$ in a definite integral with any limits. A definite integral with incorrect limits is not eligible for the second point.
- Once earned, the first point cannot be lost. Even in the presence of subsequent copy errors, the correct answer will earn the second point.
- If the first point is not earned because of a copy error, the second point is still earned for a correct answer.
- Unsupported answers will not earn either point.
- Degree mode: distance $= 0.677$ (or 0.676). (See degree mode statement in part (a).)

Total for part (b) 2 points

(c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

$x'(t) = (t-1)e^{t^2} = 0 \Rightarrow t = 1$	Sets $x'(t) = 0$	**1 point**
Because $x'(t) < 0$ for $0 < t < 1$ and $x'(t) > 0$ for $t > 1$, the particle is farthest to the left at time $t = 1$.	Explains leftmost position at $t = 1$	**1 point**
$x(1) = -2 + \int_0^1 x'(t)\,dt = -2.603511$ $y(1) = 5 + \int_0^1 y'(t)\,dt = 5.410486$	One coordinate of leftmost position	**1 point**
The particle is farthest to the left at point $(-2.604 \text{ (or } -2.603), 5.410)$.	Leftmost position	**1 point**
Because $x'(t) > 0$ for $t > 1$, the particle moves to the right for $t > 1$. Also, $x(2) = -2 + \int_0^2 x'(t)\,dt > -2 = x(0)$, so the particle's motion extends to the right of its initial position after time $t = 1$. Therefore, there is no point at which the particle is farthest to the right.	Explanation	**1 point**

Scoring notes:

- The second point is earned for presenting a valid reason why the particle is at its leftmost position at time $t = 1$. For example, a response could present the argument shown in the model solution, or it could indicate that the only critical point of $x(t)$ occurs at $t = 1$ and $x'(t)$ changes from negative to positive at this time.
- Unsupported positions $x(1)$ and/or $y(1)$ do not earn the third (or fourth) point(s).
- Writing $x(1) = \int_0^1 x'(t) - 2 = -2.603511$ does not earn the third (or fourth) point, because the missing dt makes this statement unclear or false. However, $x(1) = -2 + \int_0^1 x'(t) = -2.603511$ does earn the third point, because it is not ambiguous. Similarly, for $y(1)$.
- For the fourth point the coordinates of the leftmost point do not have to be written as an ordered pair as long as they are labeled as the x- and y-coordinates.
- To earn the last point a response must verify that the particle moves to the right of its initial position (as well as moves to the right for all $t > 1$). Note that there are several ways to demonstrate this.
- Degree mode: y-coordinate $= 5.008$ (or 5.007). (See degree mode statement in part (a).)

Total for part (c) **5 points**

Total for question 2 **9 points**

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$

$\sqrt{((1.2-1)e^{(1.2)^2})^2 + (\sin((1.2)^{1.25}))^2}$

accel: $\langle (t-1)(2te^{t^2}) + (1)(e^{t^2}),\ \cos(t^{1.25})\cdot 1.25t^{.25} \rangle$

speed $= \boxed{1.271}$

at $t=1.2$

$\boxed{\text{accel} = \langle 6.247,\ .405 \rangle}$

Response for question 2(b)

$\int_0^{1.2} \sqrt{((t-1)e^{t^2})^2 + (\sin(t^{1.25}))^2}\,dt$

$= \boxed{1.010}$

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Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

$x'(t)$ is negative on $(0,1)$ and is positive on $(1,\infty)$. ~~This~~ $x'(1)=0$. Since $x'(t)$ goes from negative to positive at $t=1$ and $x'(1)=0$, $x(t)$ has a rel. minimum at $t=1$, Since $x(t)$ decreases until $t=1$ and always increases afterwards, $x(1)$ is an abs. min /that is the location of furthest to the left,

$-2+\int_0^1 (t-1)e^{t^2}\,dt = x(1)$

$x(1)=-2.604$

$5+\int_0^1 (\sin(t^{1.25}))\,dt = y(1)$

$y(1)=5.410$

$\boxed{(-2.604,\ 5.410)}$

There is no point farthest to the right because $x'(t)$ increases towards infinity on $t\geq 0$, meaning there is no abs max of $x(t)$ (since $x(t)$ will ~~thus~~ be increasing towards ∞ on $t>1$). This means the particle will continue to the right for $t>1$, creating no furthest right point,

Sample 2B

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

a. $\sqrt{((t-1)e^{t^2})^2 + \sin(t^{1.25})^2}$, $t = 1.2$

$\sqrt{(0.744139)^2 + (0.95084775)^2}$

$= 1.271$

Response for question 2(b)

b. $\int_0^{1.2} \sqrt{((t-1)e^{t^2})^2 + (\sin(t^{1.25}))^2}\, dt$

$= 1.010$

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

$x'(t) =$ velocity of x

$y'(t) =$ velocity of y

c. farthest left, minimum x position

$(t-1)e^{t^2} = 0$

Critical point: $t = 1$ is a minimum because $x''(1) = 2.718 > 0$.

$x(1) = -2 + \int_0^1 x'(t)\,dt$

$= -2.604$

$y(1) = 5 + \int_0^1 y'(t) = 5.405$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$S = \sqrt{\left((1.2-1)e^{(1.2)^2}\right)^2 + \left(\sin\left(1.2^{1.25}\right)\right)^2}$$

$$\text{Acc.} = \langle 6.2467,\ 0.4051 \rangle$$

Response for question 2(b)

$$\text{TDT} = \int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$= 1.0098$$

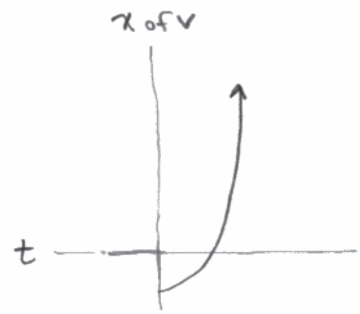
Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

Right − Left → use $x(t)$

$(t-1)e^{t^2}$ @ $t=0$

$x=-2$



x componet of velocity is negative from $t=0$ to $t=1$ therefore since x starts at -2 the furthest left point will be at $t=1$, $(-2.6035, 5.4105)$

There is no point at which the particle is furthest to the left because the x value continues to increase to infinity. when $t \geq 0$

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this problem a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right)\right\rangle$; the position is $(-2, 5)$ at time $t = 0$.

In part (a) students were asked to find the speed and the acceleration vector of the particle at time $t = 1.2.$ A correct response would show the speed setup, $\sqrt{(x'(1.2))^2 + (y'(1.2))^2}$, and the acceleration setup, $\langle x''(1.2), y''(1.2)\rangle$, and then use a graphing calculator to find both values.

In part (b) students were asked to determine the total distance traveled by the particle over the time interval $0 \le t \le 1.2.$ A correct response would present the integral $\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ and determine the numerical value using a graphing calculator.

In part (c) students were asked to find the coordinates of the point (for $t \ge 0$) when the particle is farthest to the left. They were also asked to explain why there is no point at which the particle is farthest to the right. A correct response would determine that the particle changes direction when $x'(t)$ changes sign at time $t = 1,$ by setting $x'(t) = 0.$ Because $x'(t) < 0$ for $0 < t < 1$ and $x'(t) > 0$ for $t > 1,$ the left-most position of the particle would be computed by adding the initial position to the net change, found by integrating the velocity function from $t = 0$ to $t = 1,$ for each coordinate position of the particle. Finally, the response should argue that the particle's initial x-coordinate at time $t = 0$ is to the right of the particle's position at time $t = 1,$ and from this time on, the particle is moving to the right. Therefore, there is no point at which the particle is farthest to the right.

Sample: 2A
Score: 9

The response earned 9 points: 2 points in part (a), 2 points in part (b), and 5 points in part (c). In part (a) the response earned the first point with the radical expression on the second line. Note that the response continues by simplifying and rounding correctly to the boxed answer of 1.271. While simplifying is not necessary, given that it is presented, it must be correct. The response earned the second point on the last line with the boxed answer of $\langle 6.247, .405\rangle$ and the supporting work found in the vector expression $\left\langle (t-1)\left(2te^{t^2}\right) + (1)e^{t^2}, \cos\left(t^{1.25}\right)\cdot 1.25t^{.25}\right\rangle$ on the lines above.

In part (b) the response earned the first point on the first line with the integrand, noting that it is contained within a definite integral with numeric limits. As the limits on the integral are correct and the boxed answer 1.010 is correct, the response earned the second point. In part (c) the response earned the first and second points by noting that "$x'(t)$ is negative on $(0, 1)$ and is positive on $(1, \infty)$." The second point is reinforced on lines two through six by explaining that due to the fact that $x(t)$ decreases until $t = 1$ and always increases afterward, it follows that $x(1)$ is an absolute minimum. The response earned the third point with the equations $-2 + \int_0^1 (t-1)e^{t^2}\, dt = x(1)$ and $x(1) = -2.604.$ The response earned the fourth point with the boxed answer $(-2.604, 5.410)$ and the supporting equation $5 + \int_0^1 \left(\sin\left(t^{1.25}\right)\right) dt = y(1)$ two lines above. The response earned the fifth point in the final paragraph by noting that "$x'(t)$ increases toward infinity on $t > 1$" and correctly concluding that "there is no abs max of $x(t)$ (since $x(t)$ will thus be increasing towards ∞ on $t > 1$)."

Question 2 (continued)

Sample: 2B
Score: 6

The response earned 6 points: 1 point in part (a), 2 points in part (b), and 3 points in part (c). In part (a) the response earned the first point with the first two lines. The third line correctly simplifies the expression in the second line, while the first line supports the work leading to the answer. The response did not earn the second point as there is no answer or work presented for the acceleration vector. In part (b) the response earned the first point in the first line with the integrand, as it is within a definite integral. The response earned the second point with correct limits on the integral and the answer 1.010. In part (c) the response earned the first point with the equation on the third line. The response goes on to make an argument that the x-coordinate is minimized due to the fact that $x''(t) > 0$. This, however, is a local argument. Thus, the response did not earn the second point. The response earned the third point with the equations on the fifth and seventh lines. The response earned the fourth point with the correct value of $x(1)$ and with the equation on the last line. The response did not earn the final point as no answer or argument is given.

Sample: 2C
Score: 3

The response earned 3 points: 1 point in part (a), 2 points in part (b), and no points in part (c). In part (a) the response earned the first point with the radical expression on the first line. While the vector presented on the second line is correct, there is no supporting work; therefore, the response did not earn the second point. In part (b) the response earned the first point on the first line with an appropriate integrand within a definite integral with numeric limits. As the limits of this definite integral are correct, the response earned the second point on the second line with the answer 1.0098. In part (c) the response did not earn the first point. While the response notes in the middle paragraph that the "x componet of velocity is negative from $t = 0$ to $t = 1$," no connection is made between the x-component of "velocity" and $x'(t)$. The response did not earn the second point as the argument given is local and does not support the particle being furthest to the left when $t = 1$. While the coordinates presented at the end of the middle paragraph are correct for the point at $t = 1$, the response earned neither the third nor the fourth points as no supporting work for either of these coordinates is presented. The response did not earn the fifth point because the statement "the x value continues to increase to infinity when $t \geq 0$" contradicts that $x(t)$ is decreasing from $t = 0$ to $t = 1$.