2022

AP CollegeBoard

AP® Calculus BC

Sample Student Responses and Scoring Commentary

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Free-Response Question 1

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Part A (AB or BC): Graphing calculator required
Question 1 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

	Model Solution	Scoring
(a)	Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).	
	The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t)\,dt$.	Answer **1 point**

Scoring notes:

- The response must be a definite integral with correct lower and upper limits to earn this point.
- Because $|A(t)| = A(t)$ for $1 \le t \le 5$, a response of $\int_1^5 \left|450\sqrt{\sin(0.62t)}\right|\,dt$ or $\int_1^5 |A(t)|\,dt$ earns the point.
- A response missing dt or using dx is eligible to earn the point.
- A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t)\,dt$.

Total for part (a) 1 point

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

$\text{Average} = \frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966$	Uses average value formula: $\frac{1}{b-a}\int_a^b A(t)\,dt$ **1 point**
The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.	Answer **1 point**

Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points: $\int_1^5 A(t)\,dt = 1502.147865$, so the average value is 375.536966.
- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t)\,dt = 1502.147865 = 375.536966$.
- The answer must be correct to three decimal places. For example, $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966 \approx 376$ earns only the first point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4}\int_1^5 A(t)\,dt = 79.416068$.
- Special case: $\frac{1}{5}\int_1^5 A(t)\,dt = 300.429573$ earns 1 out of 2 points.

Total for part (b) 2 points

(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$A'(1) = 148.947272$	Considers $A'(1)$	**1 point**
Because $A'(1) > 0,$ the rate at which the vehicles arrive at the toll plaza is increasing.	Answer with reason	**1 point**

Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1,$ the first point is earned by any of the following:
  - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
  - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

Total for part (c) 2 points

(d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time $t,$ for $a \leq t \leq 4,$ is given by $N(t) = \int_a^t (A(x) - 400)\, dx,$ where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$N'(t) = A(t) - 400 = 0$ $\Rightarrow A(t) = 400 \Rightarrow t = 1.469372,\ t = 3.597713$ $a = 1.469372$ $b = 3.597713$	Considers $N'(t) = 0$	**1 point**
	$t = a$ and $t = b$	**1 point**
t | $N(t) = \int_a^t (A(x) - 400)\, dx$ a | 0 b | 71.254129 4 | 62.338346 The greatest number of vehicles in line is 71.	Answer	**1 point**
	Justification	**1 point**

Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of "$A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$" will earn the first 2 points. A response of "$A(t) \geq 400$" along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of "$A(t) \geq 400$" by itself will not earn either of the first 2 points.
- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving t or x is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or $71.254***$ only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:

  For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.

  Thus, $N(t) = \int_a^t (A(x) - 400)\,dx$ is greatest at $t = b$.

  $N(b) = 71.254129$, and the greatest number of vehicles in line is 71.
- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

Total for part (d) 4 points

Total for question 1 9 points

Sample 1A

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (a) and (b) on this page.

Response for question 1(a)

$$\int_1^5 450\sqrt{\sin(0.62t)}\,dt$$

Response for question 1(b)

$$\frac{1}{5-1}\int_1^5 450\sqrt{\sin(0.62t)}\,dt = 375.537 \text{ vehicles per hour}$$

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$A'(1) = 148.947$ vehicles per hour per hour

The rate at which vehicles arrive at the toll plaza at 6 A.M. ($t=1$) is increasing because the rate of change of $A(t)$ at $t=1$ is positive.

Response for question 1(d)

$N'(t) = A(t) - 400 = 0$ when $t = 3.59771$ hours

$A(t) = 450\sqrt{\sin(0.62t)} = 400$ at $t = 1.46937$

t	$N(t)$
1.46937	0
3.59771	71.2541
4	62.3383

The greatest number of vehicles in line is 71 vehicles at $t = 3.59771$ hours because $N(t)$ achieves a relative maximum at $t = 3.59771$, and since $t = 3.59771$ is the only critical number on the given interval, the relative maximum is the absolute maximum.

Sample 1B

Answer QUESTION 1 parts (a) and (b) on this page.

Response for question 1(a)

$\int_1^5 A(t)\,dt$

$A(t) = 450\sqrt{\sin(0.62t)}$

Response for question 1(b)

$\frac{1}{5-1}\int_1^5 A(t)\,dt$

$= 375.537 \frac{\text{vehicles}}{\text{hour}}$

the

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$A(t) = 450\sqrt{\sin(0.62t)}$

$A'(1) = 148.947...$

At 6 AM or $t=1$ time, the rate at which vehicles arrive @ the toll plaza is increasing because $A'(1) > 0$.

Response for question 1(d)

Abs. max of # vehicles.

$N(t) = \int_a^t (A(t) - 400)\,dt$

$N'(t) = A(t) - 400$

$N'(t) = 0$ or DNE

$450\sqrt{\sin(0.62t)} = 400$

$\sqrt{\sin(0.62t)} = \frac{8}{9}$

$\sin(0.62t) = \frac{64}{81}$

$t = \frac{\arcsin\left(\frac{64}{81}\right)}{0.62}$

$t = 1.469...$

$t \to B$

$a \le t \le 4$

$\sqrt{\sin(0.62t)}$

$= 1$

$t = 1.613$

a | B | 4

vehicles $= 450\sqrt{\sin(0.62t)}$

IF $\sqrt{\sin(0.62t)} = 1$,

vehicles $= 450$

Sample 1C

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (a) and (b) on this page.

Response for question 1(a)

$$\int_1^5 A(t)\,dt$$

$$\int_1^5 440 + 5\sin(.67t)\,dt$$

Response for question 1(b)

$$\frac{A(5) - A(1)}{5 - 1} =$$

$$\frac{1}{5}\int_1^5 A'(t)\,dt$$

81.0498

Sample 1C

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

increasing because $A'(t)$ is positive @ $t=1$ ∴ the rate would be increasing

$A'(1) = 148.447$

Response for question 1(d)

$$0 = \int_{3}^{k} (A(x) - 400)\, dx$$

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The context of this problem is vehicles arriving at a toll plaza at a rate of $A(t) = 450\sqrt{\sin(0.62t)}$ vehicles per hour, with time t measured in hours after 5 A.M., when there are no vehicles in line.

In part (a) students were asked to write an integral expression that gives the total number of vehicles that arrive at the plaza from time $t = 1$ to time $t = 5$. A correct response would report $\int_1^5 A(t)\,dt$.

In part (b) students were asked to find the average value of the rate of vehicles arriving at the toll plaza over the same time interval, $t = 1$ to $t = 5$. A correct response would report $\frac{1}{4}\int_1^5 A(t)\,dt$ and then evaluate this definite integral using a calculator to find an average value of 375.537. (The units, vehicles per hour, were given in the statement of the problem.)

In part (c) students were asked to reason whether the rate of vehicles arriving at the toll plaza is increasing or decreasing at 6 A.M., when $t = 1$. A correct response would use a calculator to determine that $A'(1)$, the derivative of the function $A(t)$ at this time, is positive ($A'(1) = 148.947$) and conclude that because $A'(1)$ is positive, the rate of vehicles arriving at the toll plaza is increasing.

Finally, in part (d) students were told that a line of vehicles forms when $A(t) \geq 400$ and the number of vehicles in line is given by the function $N(t) = \int_a^t (A(x) - 400)\,dx$, where a denotes the time, $a \leq t \leq 4$, when the line first begins to form. Students were asked to find the greatest number of vehicles in line at the plaza, to the nearest whole number, in the time interval $a \leq t \leq 4$ and to justify their answer. A correct response would recognize that the greatest number of vehicles is the maximum value of $N(t)$ on the closed interval $a \leq t \leq 4$. To find this maximum, a response should first determine the times t, $0 < t \leq 4$, when the derivative of $N(t)$ is 0. This requires using the Fundamental Theorem of Calculus to find $N'(t) = A(t) - 400$ and then using a calculator to determine that $N'(t)$ is equal to zero when $t = a = 1.469372$ and when $t = b = 3.597713$. A response should then evaluate the function $N(t)$ at each of the values $t = a$, $t = b$, and $t = 4$ to determine that the greatest number of vehicles in line is $N(b) = 71$.

Sample: 1A
Score: 9

The response earned 9 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 4 points in part (d).

In part (a) the response earned the point with the definite integral presented.

In part (b) the response earned the first point for the average value expression on the left side of the given equation. The second point was earned for the number on the right side of the equation, which is correct to three decimal places.

In part (c) the response earned the first point for the left side of the equation in the first line. The second point was earned with the concluding sentence.

Question 1 (continued)

In part (d) the response earned the first point with the equation on the left side of line 1. The middle expression, "$A(t) - 400,$" of the equation is not needed to earn that point. The second point was earned with the values at the end of line 1 and line 2. The third point was earned in line 2 of the sentence on the right by identifying 71. The Candidates Test table on the left side earned the fourth point. The sentence on the right is consistent with the information in the Candidates Test table.

Sample: 1B
Score: 6

The response earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d).

In part (a) the response earned the point with the definite integral presented in line 1.

In part (b) the response earned the first point for the expression in line 1. The second point was earned for the correct value in line 2.

In part (c) the response earned the first point for the left side of the equation in line 2. The second point was earned with the concluding sentence.

In part (d) the response earned the first point with the equation on the left in line 3. The second point was not earned because the value of 3.598 is never given. The third point was not earned because there is no value given for $N(3.598)$. The fourth point was not earned because no justification is presented.

Sample: 1C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d).

In part (a) the response earned the point with the definite integral presented in line 1. The definite integral in line 2 is not necessary.

In part (b) the response did not earn the first point because the integrand given in line 1 after the crossed-out work is $A'(t)$ and not $A(t)$. Because the integrand presented is $A'(t)$ the response is not eligible to earn the second point.

In part (c) the response earned the first point with the statement "$A'(t)$ is positive @ $t = 1$" in line 1. The second point was earned in line 1 with the prior words: "increasing because $A'(t)$ is positive @ $t = 1.$"

In part (d) the response did not earn any points because no correct work is presented.