AP Calculus BC Cheat Sheet

Practice AP Calculus BC

Unit 1: Limits & Continuity

- **Algebraic simplifications for limits:** Completing the square, rationalization, factoring.
- **Intermediate Value Theorem (IVT):** If f(x) is continuous on [a,b], and f(c) is between f(a and f(b), then there is a c in (a,b) such that f(c) = 0.
- **Limits formulas:**
  - $\lim_{x \to c}[af(x)] = a \cdot \lim_{x \to c} f(x)$
  - $\lim_{x \to c}[f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$
  - $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)} \quad \text{(L'Hopital's Rule)}$

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Unit 2: Differentiation: Definition and Fundamental Properties

- Differentiation is a fundamental concept in calculus that deals with finding the rate at which a function changes at any given point. Essentially, it measures how a function's output value changes as its input value changes. The derivative of a function at a particular point provides the slope of the tangent line to the curve at that point.
- In mathematical terms, if y = f(x), the derivative of f(x) with respect to x, denoted by f′(x) or $\frac{dy}{dx}$ is the rate of change of y with respect to x.
- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
- **Sum/Difference Rule:** $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
- **Product Rule:** $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$
- **Quotient Rule:** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **Chain Rule:** $\frac{d}{dx}f(g(x)) = g'(x) \cdot f'(g(x))$
- **Implicit Differentiation:** Differentiate both sides with respect to the variables.
- **Inverse Trig Functions:**
  - $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$
  - $\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$

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Unit 3: Differentiation: Composite, Implicit, and Inverse Functions

- **Chain Rule for Composite Functions:**
  - **The derivative of a composite function f(g(x)) is:** $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$
  - **This applies to any combination of nested functions. For example:** Chain Rule $\frac{d}{dx}\sin(2x) = \cos(2x) \cdot 2$
- **Derivatives of Inverse Trigonometric Functions:**
  - $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
  - $\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$
  - $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
  - $\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$
  - $\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$
  - $\frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$

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Unit 4: Contextual Applications of Differentiation

- **Particle Motion:**
- **Position:** s(t)
- **Velocity:** v(t) = s′(t)
- **Acceleration:** a(t) = v′(t) = s″(t)
  - If velocity is negative, the particle is moving to the left.
  - If velocity is positive, the particle is moving to the right.
  - If velocity and acceleration have the same sign, the particle is speeding up.
  - If velocity and acceleration have different signs, the particle is slowing down.
- **Steps for Related Rates:**
  - Draw a picture and label it, assigning variables.
  - List known and unknown values.
  - Differentiate both sides with respect to time (use d/dt).
  - Plug in known values and solve for the desired value. Don’t forget units!
- **Linearization:**
  - Linear approximation of f(x) at x = a is L(x) = f(a) + f′(a) (x-a).
- **L’Hopital’s Rule:**
  - Use $\frac{f(x)}{g(x)}$ when is indeterminate (0/0 or ∞/∞).
  - $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$

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Unit 5: Analytical Applications of Differentiation

- **Mean Value Theorem (MVT):** If f(x) is continuous on [a,b] and differentiable on (a,b), there is a ccc in (a,b) such that:
  - $f'(c) = \frac{f(b) - f(a)}{b - a}$
- **Extreme Value Theorem (EVT):** If f(x) is continuous on [a,b], there exists at least one local maximum and one local minimum on [a,b].
- **Critical Points:** Occur where f′(x) = 0 or does not exist.
- **First Derivative Test:**
  - If f′(x) changes from positive to negative at c, f(x) has a local maximum at c.
  - If f′(x) changes from negative to positive at c, f(x) has a local minimum at c.
- **Concavity:**
  - f″(x) > 0: Concave up.
  - f″(x) < 0: Concave down.
  - f″(x) = 0: Possible inflection point.
- **Second Derivative Test:**
  - If f′(x) = 0 and f″(x) > 0, then f(x) has a local minimum.
  - If f′(x)=0 and f″(x)<0, then f(x) has a local maximum.
- **Steps for Optimization:**
  - Draw and label a picture.
  - Assign variables and write an equation.
  - Find relationships among the variables.
  - Differentiate and find extrema (min/max).

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Unit 6: Integration & Accumulation of Change

- **Fundamental Theorem of Calculus (FTC):** $\int_a^b f(x)dx = F(b) - F(a)$ where F(x) is an antiderivative of f(x).
- **Integration by Parts:** ∫u dv = uv -∫vdu
- **Riemann Sum:** Approximation of area under a curve using left, right, midpoint, or trapezoidal sums.

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Unit 7: Differential Equations

- **Logistic Differential Equation:** $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$, where P is the population, L is the carrying capacity, and k is a constant.
- **Slope Fields:** Graphical representation of a differential equation $\frac{dy}{dx} = f(x, y)$
- **Euler's Method:** Used to numerically approximate solutions of differential equations.

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Unit 8: Applications of Integration

- **Volumes:**
  - **Washer Method:** $V = \pi \int_a^b (R(x)^2) dx$
  - **Disc Method:** $V = \pi \int_a^b (R(x)^2 - r(x)^2) dx$
- **Arc Length:**
  - **Parametric:** $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
  - **Polar:** $A = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$
- **Area under a Polar Curve:** $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

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Unit 9: Parametric Equations, Polar Coordinates, & Vector Valued Functions

- **Parametric Equations:** $x = f(t)$,$y = g(t)$
- **Second Derivative:** $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$
- **Area under Polar Curves:** $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

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Unit 10: Infinite Sequences and Series

- **Taylor Series:** $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$
- **Power Series:** Converges when p>1; diverges otherwise.
- **Convergence Tests:**
  - Nth-Term Test: Series diverges if $(\lim_{n\to\infty} a_n)0$
  - Ratio Test: Converges if $(\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1)$

Additional Notes: