AP® Physics C: Mechanics
1999 Sample Student Responses

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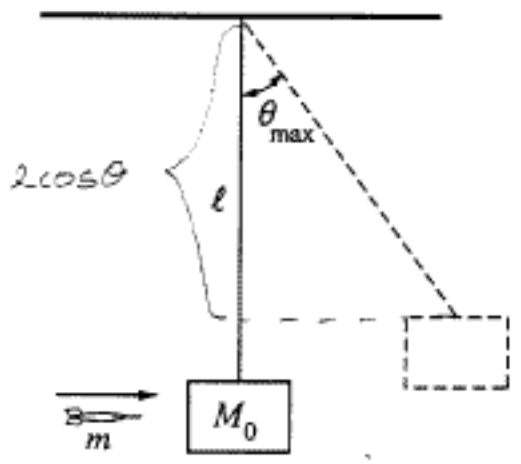
PHYSICS C

SECTION II, MECHANICS

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.



Mech 1. In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M_0, which hangs from a cord of length ℓ and negligible mass, as shown above. Assume the size of the block is negligible compared to ℓ, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle θ_{max} from the vertical. Express your answers to the following in terms of m, M_0, ℓ, θ_{max}, and g.

(a) Determine the speed v_0 of the dart immediately before it strikes the block.

$U_f = K_i$

$U_f = (M_0+m)g(\ell-\ell\cos\theta)$

$K_i = \frac{1}{2}(M_0+m)v^2 = (M_0+m)g(\ell-\ell\cos\theta)$

$\Rightarrow v = \sqrt{2g\ell(1-\cos\theta)}$

$P_m = P_{m+m}$

$P_{m+m} = (M_0+m)v = (M_0+m)\sqrt{2g\ell(1-\cos\theta)}$

$P_m = mV_0 = (M_0+m)\sqrt{2g\ell(1-\cos\theta)}$

$\Rightarrow \boxed{V_0 = \frac{M_0+m}{m}\sqrt{2g\ell(1-\cos\theta)}}$

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(b) The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.

at bottom, $v^2 = 2g\ell(1-\cos\theta)$

$$F_c = \frac{(M_0+m)v^2}{\ell} = (M_0+m)\cdot 2g(1-\cos\theta)$$

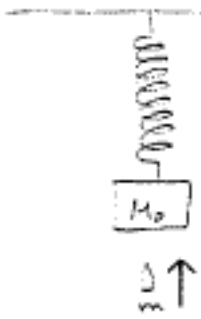
$$F_c = T - (M_0+m)g \quad \Rightarrow \quad T = F_c + (M_0+m)g$$

$$\Rightarrow T = (M_0+m)g\left[2(1-\cos\theta)+1\right]$$

(c) At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses
Protractor	5 m of string	Five more blocks of mass M_0
Spring		

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.



Hang a known mass on the spring and measure how far it stretches downward. This will determine k because $kx = mg \Rightarrow k = \frac{mg}{x}$.

Now fire the dart at the bottom of a mass M_0 hanging from the spring. Measure how far the spring is compressed. From this, the potential energy due to gravity (call h=0 the equilibrium point) and due to the compressed spring can be calculated. The initial kinetic energy of the M_0 and m can then be found (equal to?) and m's initial K can be found as in above example.

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(d) The dart is now shot into a block of wood that is fixed in place. The block exerts a force $\mathbf{F}$ on the dart that is proportional to the dart's velocity $\mathbf{v}$ and in the opposite direction, that is $\mathbf{F} = -b\mathbf{v}$, where b is a constant. Derive an expression for the distance L that the dart penetrates into the block, in terms of m, v_0, and b.

$$a = \frac{F}{m} \quad \Rightarrow \quad a = \frac{-bv}{m}$$

$$\frac{dv}{dt} = a \quad \Rightarrow \quad dt = \frac{dv}{a} \quad \Rightarrow \quad t = \int \frac{dv}{a}$$

$$t = \int_{v_0}^{v} -\frac{m}{bv}\, dv = -\frac{m}{b}\ln v + \frac{m}{b}\ln v_0$$

$$\Rightarrow v = v_0 e^{-bt/m}$$

$$L = \frac{dv}{dt} = v_0 \frac{b}{m} e^{-bt/m}$$

$v = 0$ when $v_0 e^{-bt/m} = 0 \quad \Rightarrow t \to \infty$

$$\lim_{t\to\infty} L = \lim_{t\to\infty} \left(v_0 \frac{b}{m} e^{-bt/m}\right) = \boxed{v_0 \frac{b}{m}}$$

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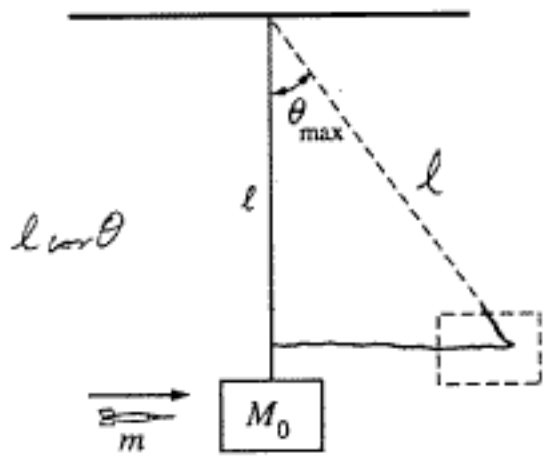

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(a) Determine the speed v_0 of the dart immediately before it strikes the block.

$$m v_0 = (m + M_0) v$$

$$v_0 = \frac{(m+M_0)v}{m}$$

$$\frac{1}{2}(m+M_0)v^2 = (m+M_0)gh$$

$$v_0 = \frac{m+M_0}{m}\sqrt{2g\ell(1-\cos\theta)}$$

$$\frac{1}{2}v^2 = g(\ell - \ell\cos\theta)$$

$$v = \sqrt{2g\ell(1-\cos\theta)}$$

(b) The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.

$$T = (M_0+m)g + \frac{(M_0+m)v^2}{\ell}$$

$$= (M_0+m)\left(g + 2g(1-\cos\theta)\right)$$

$$= g(M_0+m)[3-2\cos\theta]$$

(c) At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses
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Spring		

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.

Determine the spring constant of the spring by hanging a known mass on the end and measuring the displacement with the protractor. As $kx = mg$, $k = \frac{mg}{x}$.

Once the spring constant is known, from ground level

shoot the dart straight up. Using a stopwatch, measure the time elapsed from launch to hitting the ground. Then, perform a simple kinematics problem (air resistance is negligible)

$v_0 = ?$

$v_f = -v_0$

$a = g$

$t =$ measurable

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(d) The dart is now shot into a block of wood that is fixed in place. The block exerts a force $\mathbf{F}$ on the dart that is proportional to the dart's velocity $\mathbf{v}$ and in the opposite direction, that is $\mathbf{F} = -b\mathbf{v}$, where b is a constant. Derive an expression for the distance L that the dart penetrates into the block, in terms of m, v_0, and b.

$K = W_{\text{of friction}}$

$$\frac{1}{2}mv_0^2 = \int_0^L -bv\,dl$$

$$\frac{1}{2}mv_0^2 = -bv(L)$$

$$L = \frac{mv_0^2}{bv}$$