AP® Physics C: Mechanics
1999 Sample Student Responses

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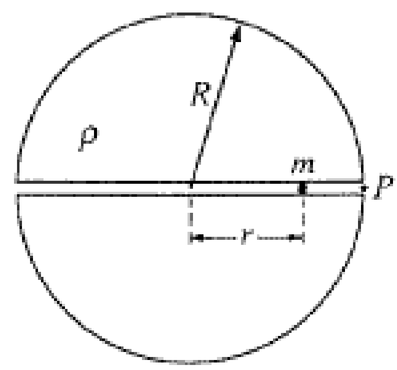


Mech 2. A spherical, nonrotating planet has a radius R and a uniform density ρ throughout its volume. Suppose a narrow tunnel were drilled through the planet along one of its diameters, as shown in the figure above, in which a small ball of mass m could move freely under the influence of gravity. Let r be the distance of the ball from the center of the planet.

(a) Show that the magnitude of the force on the ball at a distance $r < R$ from the center of the planet is given by $F = -Cr$, where $C = \frac{4}{3}\pi G\rho m$.

$\oint g \cdot dA = 4\pi G m_1$ where m_1 is mass inside Gaussian surface

$\rho = \frac{m_1}{V_1}$ $m_1 = \rho V_1$ V_1 of spherical gaussian surface is $\frac{4}{3}\pi r^3$

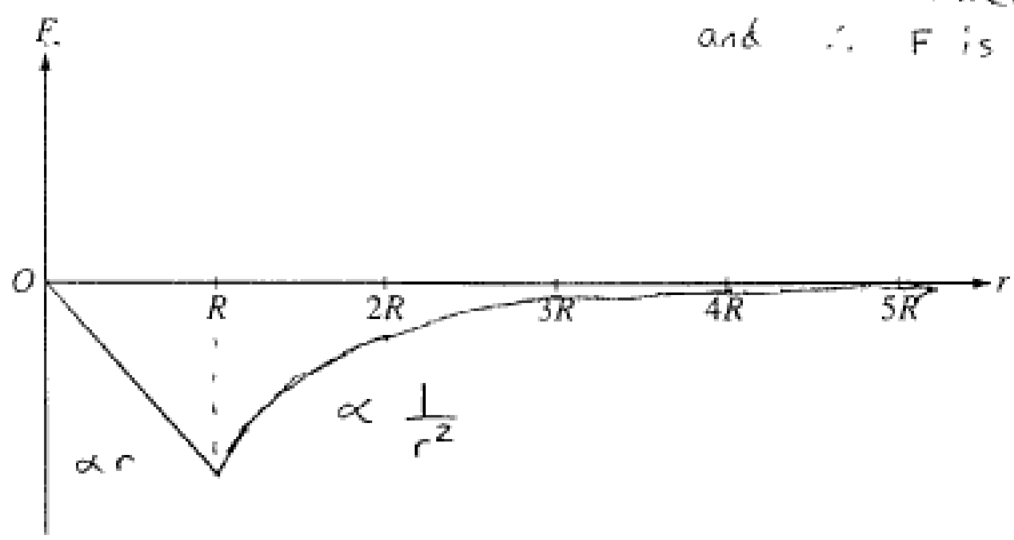
$m_1 = \frac{4}{3}\pi r^3 \rho$

$g = \frac{4\pi G m_1}{\oint dA}$ where $\oint dA$ is surface area $= 4\pi r^2$ and $m_1 = \frac{4}{3}\pi r^3 \rho$

$g = \frac{\cancel{4\pi} G(\frac{4}{3}\pi r^{\cancel{3}}\rho)}{\cancel{4\pi r^2}} = \frac{4}{3}G\rho r$ $F = mg = \frac{4}{3}G\rho r m\pi$

and since it is directed back toward the origin (center) r is in direction opposite F and $\therefore$ F is negative: $\boxed{-\frac{4}{3}G\rho r m\pi}$

(b) On the axes below, sketch the force F on the ball as a function of distance r from the center of the planet.



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The ball is dropped into the tunnel from rest at point P at the planet's surface.

(c) Determine the work done by gravity as the ball moves from the surface to the center of the planet.

$$F = -\frac{4}{3}G\rho r m\pi$$

$$U = -\int F \cdot dr = \int_R^0 \frac{4}{3}G\rho r m\pi \, dr = \frac{4}{3}G\rho m\pi \frac{r^2}{2}\Big|_R^0$$

$$= 0 - \frac{4}{3}G\rho m\pi\frac{R^2}{2} = -\frac{2}{3}G\rho m\pi R^2$$

$$\text{work done} = -\Delta U = \boxed{\frac{2}{3}G\rho m\pi R^2}$$

(d) Determine the speed of the ball when it reaches the center of the planet.

the loss in potential energy = gain in kinetic energy

$$\frac{\cancel{m}v^2}{2} = \frac{2}{3}G\rho\cancel{m}\pi R^2$$

$$v^2 = \frac{4}{3}G\rho\pi R^2$$

$$v = \boxed{2R\sqrt{\frac{G\rho\pi}{3}}}$$

(e) Fully describe the subsequent motion of the ball from the time it reaches the center of the planet.

it will have a velocity still, so it won't stop instantly. It will continue through the planet until it reaches the other side. Then it will come back. There will be simple harmonic motion with amplitude = R

(f) Write an equation that could be used to calculate the time it takes the ball to move from point P to the center of the planet. It is not necessary to solve this equation.

$$F = -\frac{4}{3}G\rho r m\pi = ma = m\frac{dv}{dt}$$ - no this won't work.

it would be $\frac{1}{4}$ the period time.

$$F = ma = \cancel{4}\pi^2 f^2 \cancel{r}\cancel{m} = \frac{\cancel{4}}{3}\cancel{\pi}G\rho\cancel{r}\cancel{m}$$

then, $$f = \sqrt{\frac{G\rho}{3\pi}}$$

$$T = \frac{1}{f} = \sqrt{\frac{3\pi}{G\rho}}$$

$$\text{time} = \frac{T}{4} = \boxed{\frac{1}{4}\sqrt{\frac{3\pi}{G\rho}}}$$

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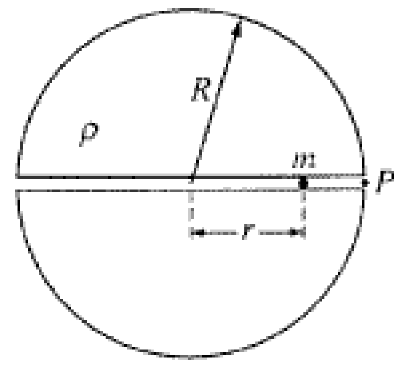

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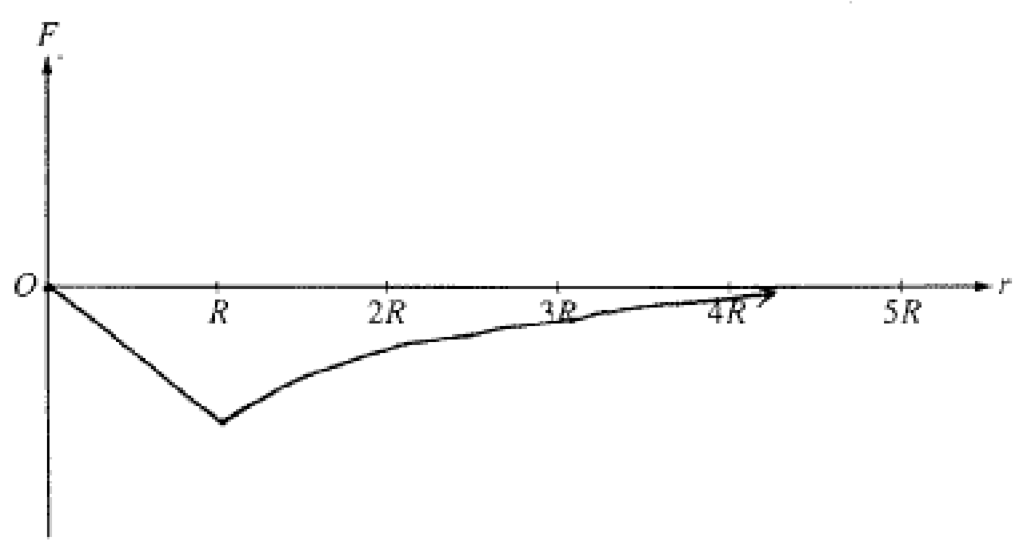
$F_g = -\frac{Gm_1m_2}{r^2}$

$m_1 = m$

$m_2 = M_{inside} = \frac{4}{3}\pi\rho r^3$

$F_g = \frac{-Gm\cdot\frac{4}{3}\pi\rho r^3}{r^2} = -Gm\frac{4}{3}\pi\rho r \rightarrow \boxed{F = -Cr}$

(b) On the axes below, sketch the force F on the ball as a function of distance r from the center of the planet.



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The ball is dropped into the tunnel from rest at point P at the planet's surface.

(c) Determine the work done by gravity as the ball moves from the surface to the center of the planet.

~~$W = F \cdot d = Cr$~~

$W = +\int F \cdot ds = +\int_R^0 -Cr = +\frac{Cr^2}{2}\Big|_R^0 = \boxed{\frac{CR^2}{2}}$ $\left(= \frac{2\pi G \rho m R^2}{3} \right)$

(d) Determine the speed of the ball when it reaches the center of the planet.

$F = ma \rightarrow a = -\frac{4}{3}\pi G \rho r$

$E_{p_1} = E_{k_2}$

$mgR = \frac{mv^2}{2} \rightarrow v^2 = 2gR \rightarrow \boxed{v = \sqrt{2gR}}$

~~$s = s_0 + v_0 t + \ldots$~~

$\cancel{v^2 = v_0^2 + 2a(s - s_0)}$

$\cancel{v^2 = 2\left(-\frac{4}{3}\pi G \rho r\right)(R)}$ $\rightarrow$ $\cancel{v = \sqrt{-\frac{8}{3}\pi G \rho r R}}$

(e) Fully describe the subsequent motion of the ball from the time it reaches the center of the planet.

The ball will exhibit simple harmonic motion with an amplitude of R

(f) Write an equation that could be used to calculate the time it takes the ball to move from point P to the center of the planet. It is not necessary to solve this equation.

$\cancel{\frac{dr}{dt} = \sqrt{-\frac{8}{3}\pi G \rho r R}}$ (from d)

$\boxed{\frac{dr}{dt} = \sqrt{2gR}}$

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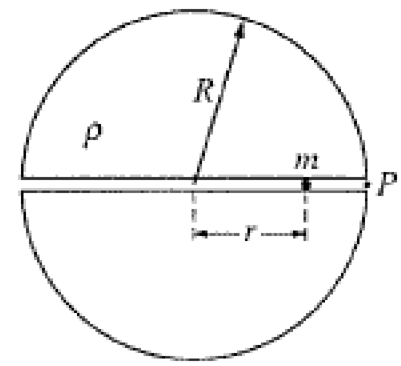

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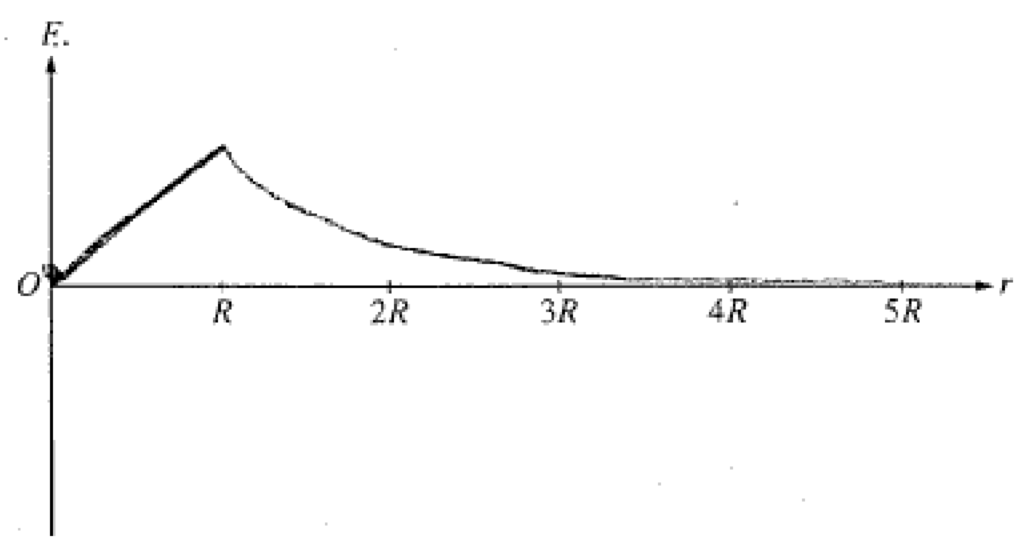
(a) Show that the magnitude of the force on the ball at a distance $r < R$ from the center of the planet is given by $F = -Cr$, where $C = \frac{4}{3}\pi G\rho m$.

$F = -\frac{G m M_{planet}}{r^2}$ $\qquad$ $M_{planet} = \rho \frac{4}{3}\pi r^3$ $\qquad$ DENSITY × VOLUME = MASS

$= -\frac{G m \frac{4}{3}\pi r^3}{\cancel{r^2}} = -\frac{4}{3}\pi G\rho m r = -Cr$

(b) On the axes below, sketch the force F on the ball as a function of distance r from the center of the planet.



The ball is dropped into the tunnel from rest at point P at the planet's surface.

(c) Determine the work done by gravity as the ball moves from the surface to the center of the planet.

$F = -Cr$

$W = \int F\,dr \qquad W = -\int_R^0 Cr\,dr = \left[\frac{-Cr^2}{2}\right]_R^0 = \boxed{\frac{1}{2}CR^2}$

(d) Determine the speed of the ball when it reaches the center of the planet.

$v_i^2 = 0$

$x = R$

$v_f^2 = v_i^2 + 2ax$

$F = ma = -\frac{4}{3}\pi G\rho m r$

$a = -\frac{4}{3}\pi G\rho r$

$v_f = r\sqrt{\frac{8\pi}{3}G\rho}$

(e) Fully describe the subsequent motion of the ball from the time it reaches the center of the planet.

THE BALL CONTINUES THROUGH THE CENTER WITH THE VELOCITY IT HAS GAINED FROM FALLING AND MOVES AWAY UNTIL THE ATTRACTIVE FORCE OF GRAVITY HAS CAUSED IT TO SLOWDOWN AND FALL "BACKWARDS" TOWARD THE CENTER (THIS HAPPENS WHEN THE BALL HAS REACHED THE SURFACE ON THE OTHER SIDE). THE BALL CONTINUES TO OSCILLATE BETWEEN THE ANTIPODES IN SIMPLE HARMONIC MOTION FOREVER.

(f) Write an equation that could be used to calculate the time it takes the ball to move from point P to the center of the planet. It is not necessary to solve this equation.

$a = \frac{-4}{3}\pi G\rho r$

$\frac{dv}{dt} = -\frac{4}{3}\pi G\rho r \qquad \int dv = \frac{4}{3}\pi G\rho r \int dt$

$v = -\frac{4}{3}\pi G\rho r t$

$\frac{dr}{dt} = -\frac{4}{3}\pi G\rho r t$

$r = -\frac{2}{3}\pi G\rho r t^2 + R \Rightarrow r = 0$

$R = \frac{2}{3}\pi G\rho r t^2$

$t = \left(\frac{3R}{2\pi G\rho r}\right)^{1/2}$

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