AP® Physics C: Mechanics
2005 Sample Student Responses

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,700 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three and a half million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

Visit the College Board on the Web: www.collegeboard.com.
AP Central is the official online home for the AP Program and Pre-AP: apcentral.collegeboard.com.

1A

PHYSICS C
Section II, MECHANICS
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

Mech. 1.

A ball of mass M is thrown vertically upward with an initial speed of v_0. It experiences a force of air resistance given by $\mathbf{F} = -k\mathbf{v}$, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_0, and fundamental constants.

(a) Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases X decreases ____ remains the same

Justify your answer.

As velocity decreases with increased height, F decreases, and F is in the negative direction, so acceleration decreases

Ball
F ↓↓ mg

(b) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t as the ball moves upward.

F=-kv ↓↓ mg

$$\Sigma F_y = -kv + mg = ma \quad \text{where } a = \frac{dv}{dt}$$

$$mg - kv = m\frac{dv}{dt}$$

$$-\frac{mg}{k} + v = \frac{-m}{k}\frac{dv}{dt}$$

$$v - \frac{mg}{k} = -\frac{m}{k}\frac{dv}{dt}$$

$$\frac{dv}{v - \frac{mg}{k}} = -\frac{k}{m}\,dt$$

GO ON TO THE NEXT PAGE.

(c) Determine the terminal speed of the ball as it moves downward.

terminal speed occurs when $a = 0$

$$mg - kv = ma^{0}$$

$$mg = kv$$

$$v = \frac{mg}{k}$$

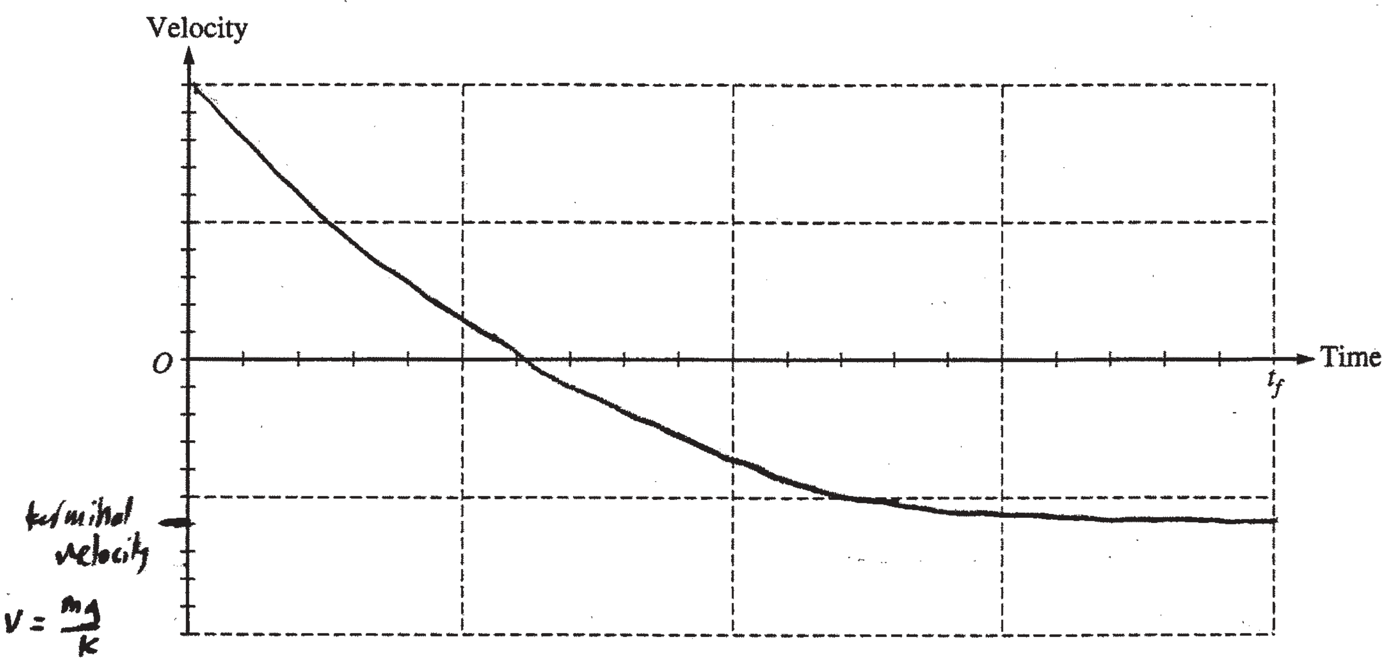
(d) Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____ longer to rise ✗ longer to fall

Justify your answer.

It reaches terminal speed and travels at this terminal speed for a longer time while on the way down.

(e) On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



GO ON TO THE NEXT PAGE.

1B

PHYSICS C
Section II, MECHANICS
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

Mech. 1.

A ball of mass M is thrown vertically upward with an initial speed of v_0. It experiences a force of air resistance given by $\mathbf{F} = -k\mathbf{v}$, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_0, and fundamental constants.

(a) Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases X decreases ____ remains the same

Justify your answer.

V is decreasing as ball moves up
thus magnitude of F (defined by $F = -kv$) is decreasing
thus the magnitude of acceleration (given by $a = \frac{F}{m}$) is decreasing

(b) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t as the ball moves upward.

$-Mg - kv = Ma$

$$-Mg - kv = M\frac{dv}{dt}$$

GO ON TO THE NEXT PAGE.

1B

(c) Determine the terminal speed of the ball as it moves downward.

$Mg = Kv$

$$v = \frac{Mg}{K}$$

(d) Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

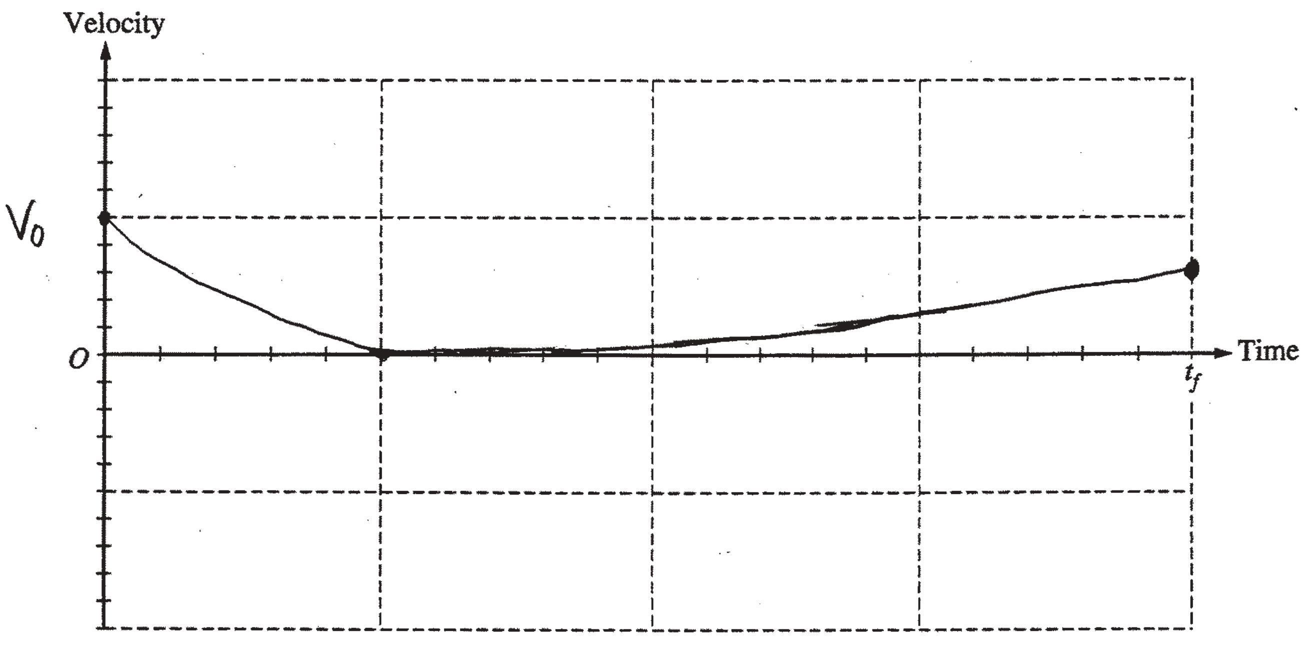
____longer to rise   X longer to fall

Justify your answer.

the force of friction does work ~~that~~ takes away from the Kinetic energy of the ball

initial v on the way down is 0 (which was positive) on the way up it was V_0 boost by V_0 will make up quicker

(e) On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



GO ON TO THE NEXT PAGE.

1C

PHYSICS C
Section II, MECHANICS
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

Mech. 1.

A ball of mass M is thrown vertically upward with an initial speed of v_0. It experiences a force of air resistance given by $\mathbf{F} = -k\mathbf{v}$, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_0, and fundamental constants.

(a) Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases ✓____ decreases ____ remains the same

Justify your answer.

~~The drag force decreases gradually, opposing the gravitational force, the ΣF on the ball goes toward 0 and because $\Sigma F = ma$, so does acceleration~~

$\Sigma F = ma$

As the ball moves upward, its acceleration decreases because the effect of the drag force on it decreases and ΣF decreases.

(b) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t as the ball moves upward.

$\Sigma F = ma = kv - mg$

$a = \frac{kv - mg}{m}$

$\frac{dv}{dt} = \frac{kv - mg}{m}$

GO ON TO THE NEXT PAGE.

1C

(c) Determine the terminal speed of the ball as it moves downward.

$\Sigma F = ma = 0$

$kv = mg$

$v = \frac{mg}{k}$

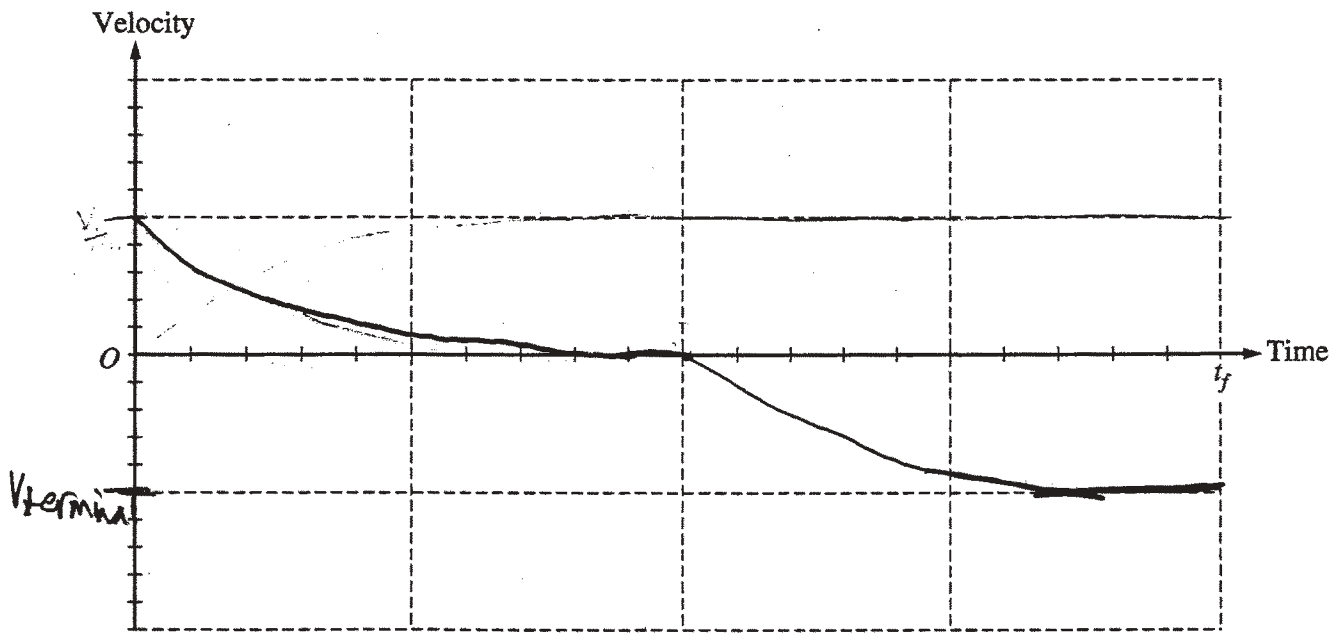
(d) Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____longer to rise __✓__longer to fall

Justify your answer.

the ball reaches a point where in stops accelerating on the way down, on the way up it accelerates throughout and therefore the trip down is longer

(e) On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



GO ON TO THE NEXT PAGE.

Mech. 2.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	$4\pi^2 R^3$ m^3	GT^2 s^2
8.14×10^4	1.85×10^8	2.499×10^{26}	0.44195
1.18×10^5	2.38×10^8	5.32×10^{26}	0.929
1.63×10^5	2.95×10^8	1.01×10^{27}	1.772
2.37×10^5	3.77×10^8	2.12×10^{27}	3.746

(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

$$F_G = G\frac{M_S m_{moon}}{R^2}$$

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

$C = 2\pi R$

$vt = D$

$$F_G = G\frac{M_S m_{moon}}{R^2} = \frac{m_{moon} v^2}{R}$$

$$v^2 = \frac{GM_S}{R} \quad v = \sqrt{\frac{GM_S}{R}} : \frac{m}{s}$$

$$\sqrt{\frac{GM_S}{R}}(T) = 2\pi R$$

$$T = \frac{2\pi R}{\sqrt{\frac{GM_S}{R}}}$$

$$T = \frac{2\pi R\sqrt{R}}{\sqrt{GM_S}}$$

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

$$T\sqrt{GM_S} = 2\pi R\sqrt{R}$$

$$T^2(GM_S) = 4\pi^2 R^3$$

$$M_S = \frac{4\pi^2}{G}\frac{R^3}{T^2}$$

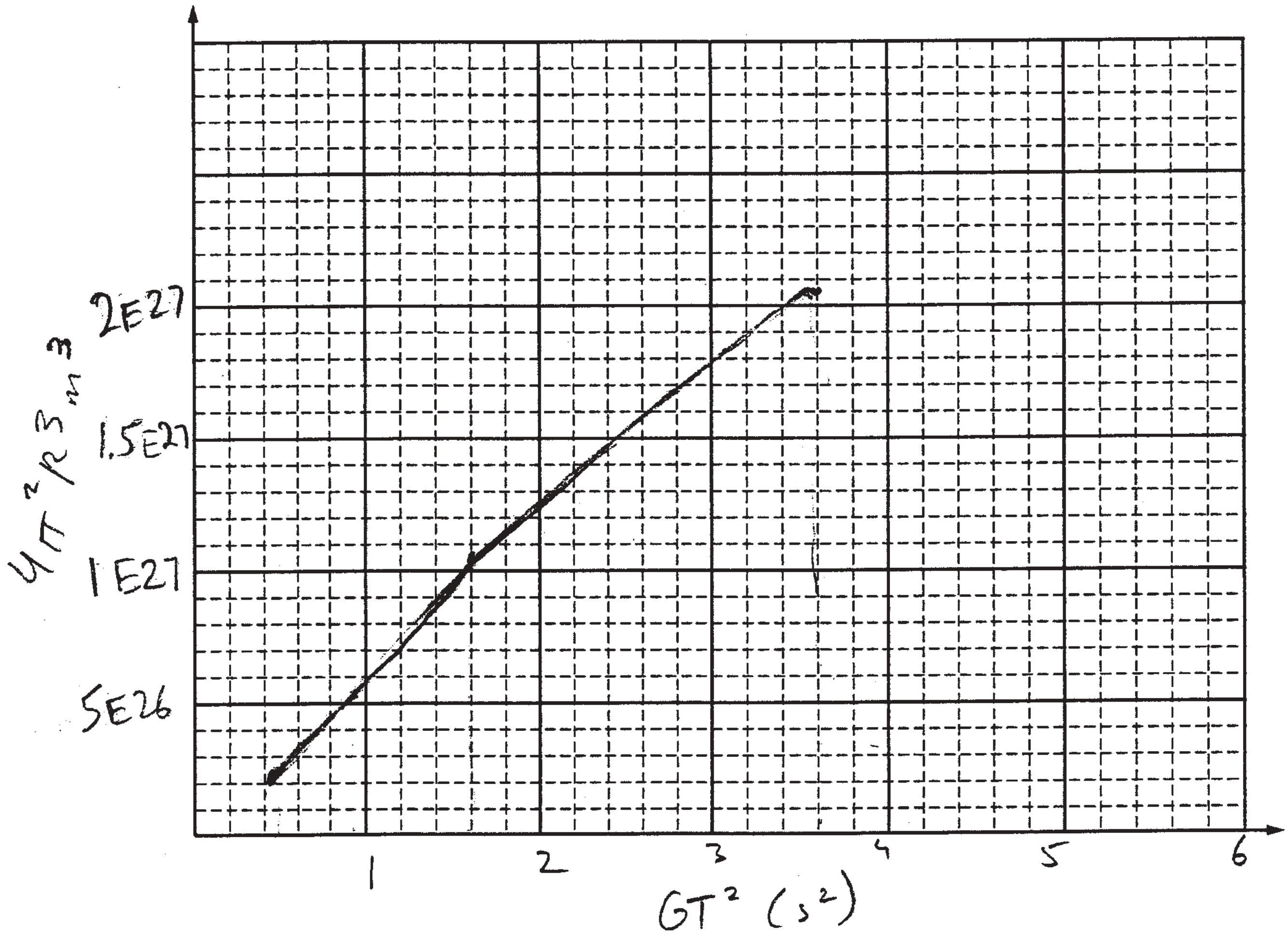
Quantities $4\pi^2 R^3$ in the y and GT^2 in x

GO ON TO THE NEXT PAGE.

2A

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

$$\frac{\Delta y}{\Delta x} = \frac{2.12\times10^{27} - 2.499\times10^{26}}{3.746 - 0.44185} = 5.66\times10^{26}\ \text{kg}$$

GO ON TO THE NEXT PAGE.

Mech. 2.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (seconds^2)	R^3 (meters^3)
8.14×10^4	1.85×10^8	6.63×10^9	6.33×10^{24}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.56×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

$T = \frac{2\pi}{\omega}$ $v = r\omega$ $\omega = \frac{v}{r}$

$m = \frac{1}{R}\left(\frac{T}{2\pi}\right)^2$ $F = -\frac{G m_1 m_2}{r^2}$ $T = \frac{2\pi r}{v}$ $\frac{m}{k} = \frac{l}{g}$

$F = -\frac{G M_S \frac{1}{R}\left(\frac{T}{2\pi}\right)^2}{R^2}$ $T = 2\pi\sqrt{\frac{l}{g}}$ $g = \frac{lk}{m}$

$F = -\frac{G M_S T^2}{R^3 4\pi^2}$ $T = 2\pi\sqrt{lm}$ $\left(\frac{T}{2\pi}\right)^2 = lm$

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

$F = -\frac{G M_E M_m}{R^2}$

$F = -\frac{G M_E T^2}{R^3 4\pi^2}$

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

T^2 AND R^3

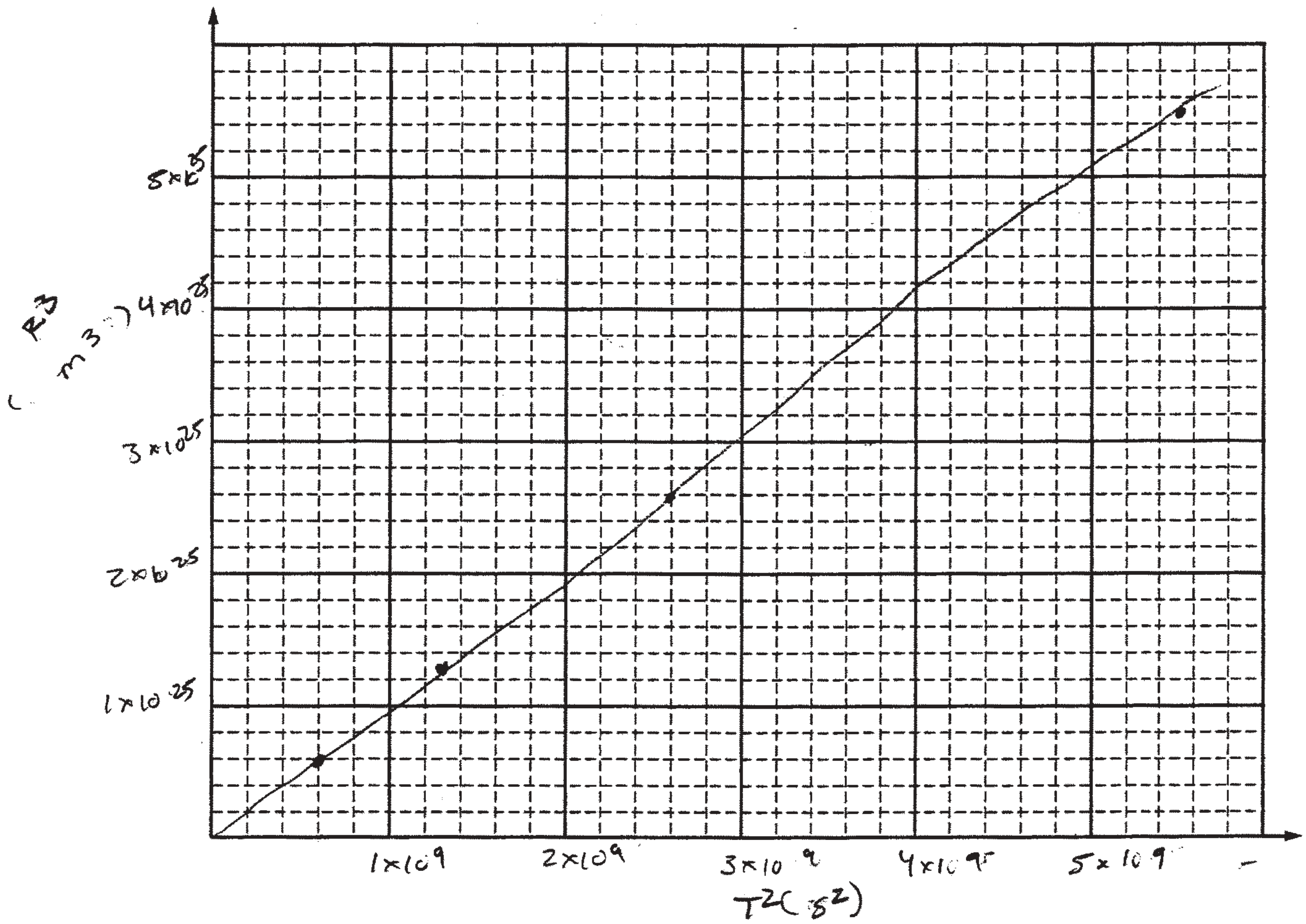
GO ON TO THE NEXT PAGE.

2B

M M M M M M M M M M M M M

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

$$\frac{5.36\times10^{25}\,m^3 - 2.26\times10^{25}\,m^3}{5.62\times10^{10}\,s^2 - 2.66\times10^{10}\,s^2} = M_S$$

$$1.05\times10^{26}\,kg = M_S$$

GO ON TO THE NEXT PAGE.

2C

Mech. 2.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	Orbital Velocity, v (meters/sec)	Grav. Pot. E, U_G (J)
8.14×10^4	1.85×10^8	1.43×10^4	[illegible]
1.18×10^5	2.38×10^8	1.27×10^4	[illegible]
1.63×10^5	2.95×10^8	1.14×10^4	[illegible]
2.37×10^5	3.77×10^8	9.99×10^3	[illegible]

(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

$$F = \frac{G m_s m_m}{R}$$

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

$\Sigma F = ma$

$$\frac{G m_s m_m}{R} = \frac{m v^2}{R}$$

$$\sqrt{G m_s} = v$$

$$T = \frac{2\pi R}{v}$$

$$T = \frac{2\pi R}{\sqrt{G m_s}}$$

$2\pi R = L$

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

orbital velocity and gravitational potential Energy should be graphed

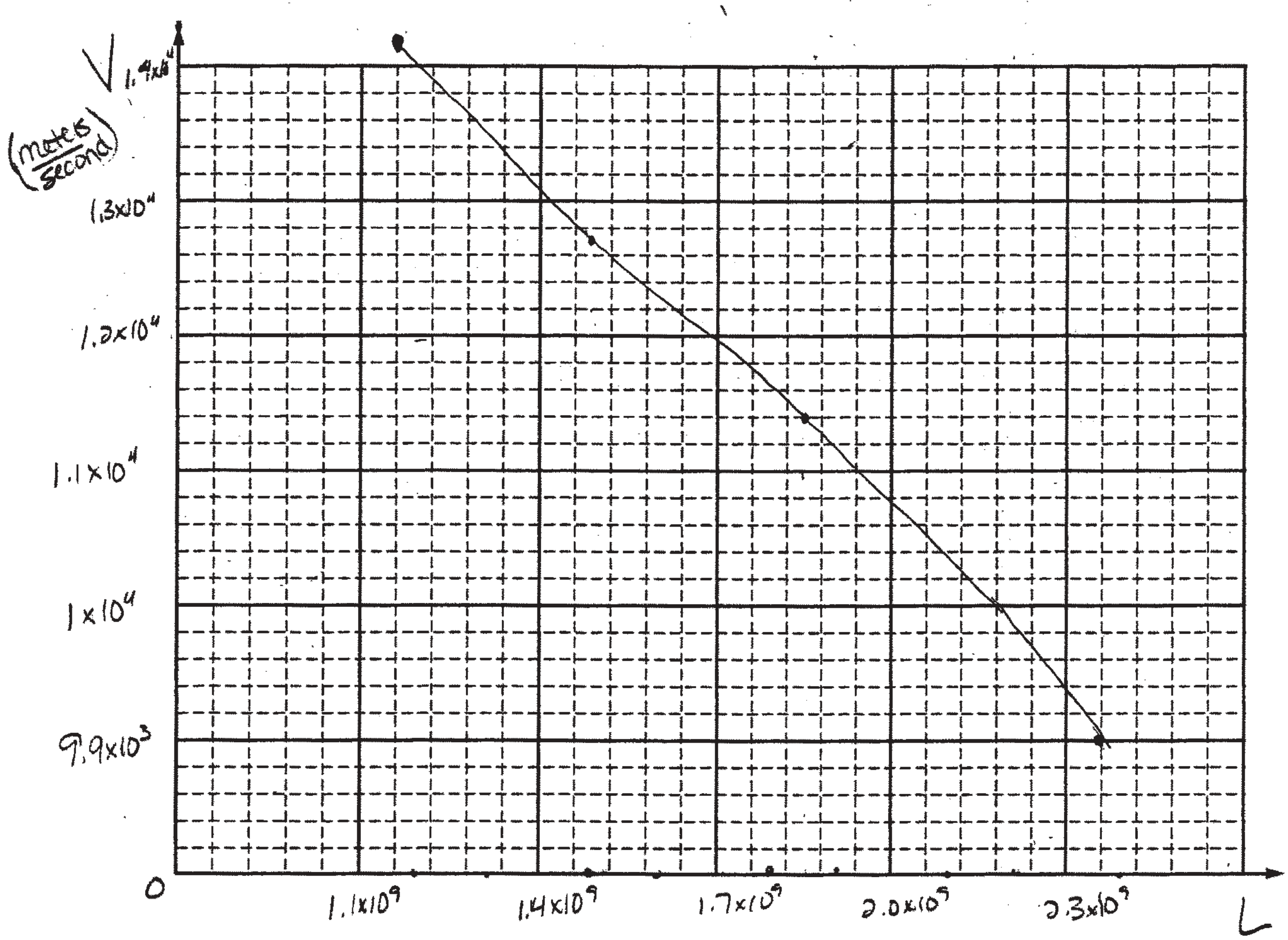
(U_G) = grav. potential E

(v) = orbital velocity = $\sqrt{G m_s}$

GO ON TO THE NEXT PAGE.

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

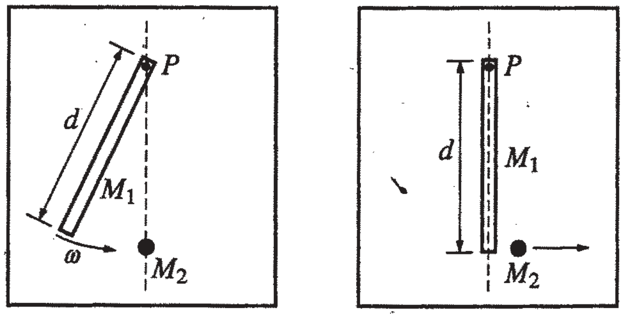
(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

$m_s = 3.066 \times 10^{18}$

GO ON TO THE NEXT PAGE.



Before Collision After Collision

TOP VIEWS

Mech. 3.

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

$L = I\omega$

$L = \frac{1}{3} M_1 d^2 \omega$

(b) Derive an expression for the speed v of the ball after the collision.

$L_i = L_f$

$\frac{1}{3} M_1 d^2 \omega = d M_2 v$

$v = \frac{M_1 d \omega}{3 M_2}$

GO ON TO THE NEXT PAGE.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.

$\frac{1}{2} I \omega^2 = \frac{1}{2} m v^2$

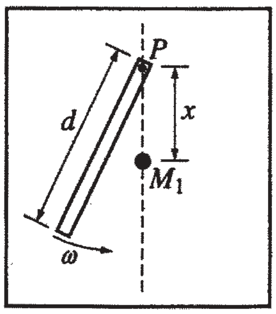
$\frac{1}{2} \cdot \frac{1}{3} M_1 d^2 \omega^2 = \frac{1}{2} M_2 v^2$

$\frac{1}{6} M_1 d^2 \omega^2 = \frac{1}{2} M_2 \frac{M_1^2 d^2 \omega^2}{9 M_2^2}$

$\frac{1}{6} = \frac{1}{18} \frac{M_1}{M_2}$

$\frac{1}{6} = \frac{1}{18} \cdot \frac{M_1}{M_2}$

$\frac{M_1}{M_2} = 3$



Before Collision

(d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

$\frac{1}{3} M_1 d^2 \omega = M_1 x v$

$\frac{d^2 \omega}{3x} = v$

$\frac{1}{2} I \omega^2 = \frac{1}{2} m v^2$

$\frac{1}{2}\left(\frac{1}{3} M_1 d^2 \omega^2\right) = \frac{1}{2} M_1 \frac{d^4 \omega^2}{9x^2}$

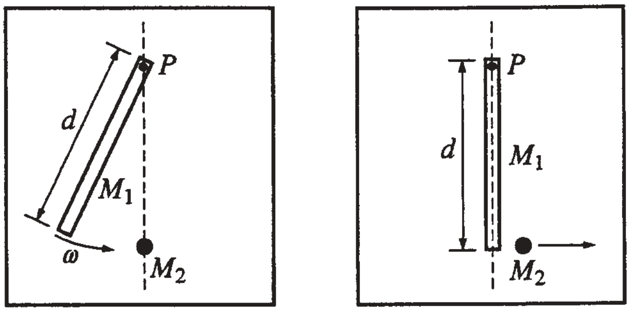
$\frac{1}{6} = \frac{1}{2} \frac{d^2}{9x^2}$

$\frac{1}{6} = \frac{d^2}{18x^2}$

$18x^2 = 6d^2$

$x = \sqrt{\frac{1}{3} d^2}$

GO ON TO THE NEXT PAGE.



Before Collision After Collision

TOP VIEWS

Mech. 3.

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

$$L = I\omega = \frac{1}{3}M_1 d^2 \cdot \omega$$

(b) Derive an expression for the speed v of the ball after the collision.

$$I_1\omega_i + I_2\omega_i = I_1\omega_f + I_2\omega_f$$

$$\frac{1}{3}M_1 d^2 \cdot \omega + 0 = M_2 d^2 \cdot \left(\frac{v}{d}\right) + 0$$

$$v = \frac{M_1 d^2 \omega}{M_2 d} = \frac{M_1 d \omega}{M_2}$$

GO ON TO THE NEXT PAGE.

3B

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.

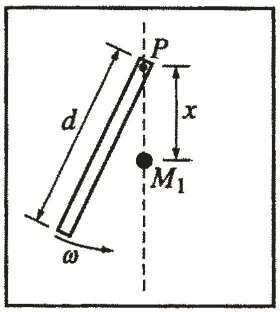
Ke is conserved

$$KE_i = \frac{1}{2} \cdot \left| \frac{1}{3} M_1 d^2 \right| \omega^2$$

$$KE_f = \frac{1}{2} M_2 \left(\frac{M_1 d\omega}{M_2} \right)^2$$

$$\omega^2 \cdot \frac{1}{3} M_1 d^2 = M_2 \cdot \frac{M_1^2}{M_2^2} \cdot d^2 \cdot \omega^2$$

$$\boxed{\frac{M_1}{M_2} = \frac{1}{3}}$$



Before Collision

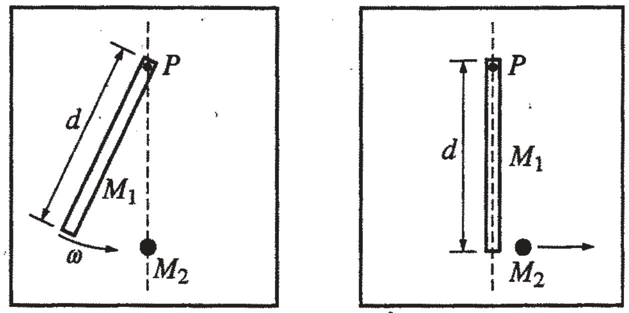
(d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

$$I_1 \omega_i + I_2 \omega_i = I_1 \omega_f + I_2 \omega_f$$

$$\frac{1}{3} M_1 d^2 \cdot \omega + 0 = M_1 x^2 \left(\frac{V}{x} \right) + 0$$

$$\frac{1}{3} M_1 d^2 \omega = M_1 x V$$

GO ON TO THE NEXT PAGE.



Before Collision — After Collision

TOP VIEWS

Mech. 3.

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

$$P = \left(\frac{1}{3}M_1 d^2\right)(\omega)$$

(b) Derive an expression for the speed v of the ball after the collision.

$$\left(\frac{1}{3}M_1 d^2\right)(\omega) = M_2 V_0$$

$$\boxed{V_0 = \frac{M_1 d^2 \omega}{3M_2}}$$

GO ON TO THE NEXT PAGE.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.

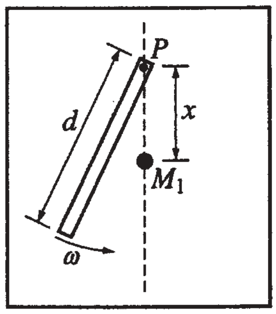
$$\frac{1}{2} I \omega^2 = \frac{1}{2} M_2 v^2$$

$$\frac{1}{2}\left(\frac{1}{3} M_1 d^2\right)\omega^2 = \frac{1}{2} M_2 \left(\frac{M_1 d^2 \omega}{3 M_2}\right)^2$$

$$\frac{1}{3} M_1 d^2 \omega^2 = \frac{M_2 M_1^2 d^4 \omega^2}{6 M_2^4}$$

$$\frac{M_1}{2 M_2^3} = 1$$

$$\frac{M_1}{M_2^3} = 2$$



Before Collision

(d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

$$\frac{1}{2} I \omega^2 = \frac{1}{2} M_2 v^2$$

GO ON TO THE NEXT PAGE.