AP® PHYSICS C: MECHANICS
2007 SCORING GUIDELINES

General Notes About 2007 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive and expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics Course Description* .

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

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Question 2

15 points total **Distribution of points**

(a) 2 points

For a correct expression of the relationship among T, R, and υ — 1 point

$$T = \frac{2\pi R}{\upsilon}$$

$$R = \frac{\upsilon T}{2\pi}$$

$$R = \frac{\left(3.40 \times 10^3 \text{ m/s}\right)\left(7.08 \times 10^3 \text{ s}\right)}{2\pi}$$

For the correct numerical answer — 1 point

$$R = 3.83 \times 10^6 \text{ m}$$

(b) 2 points

For correctly equating centripetal force and gravitational force — 1 point

$$\frac{m_s \upsilon^2}{R} = \frac{G m_s M_M}{R^2}$$

$$M_M = \frac{\upsilon^2 R}{G}$$

For substituting the value of R from (a) into either the original equation or the simplified expression for M_M above — 1 point

$$M_M = \frac{\left(3.40 \times 10^3 \text{ m/s}\right)^2 \left(3.83 \times 10^6 \text{ m}\right)}{6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2}$$

$$M_M = 6.64 \times 10^{23} \text{ kg}$$

Question 2 (continued)

Distribution of points

(c) 4 points

For a correct expression that equates E_{tot} to the sum of kinetic and gravitational potential energies — 1 point

$E_{tot} = K + U$

For a negative sign on a correct expression for U_G — 1 point

$$E_{tot} = \frac{1}{2}m_s\upsilon^2 - \frac{Gm_sM_M}{R}$$

For explicit substitution of the value of R from (a) and the value of M_M from (b) in the equation above <u>or</u> for correct numerical answer if worked as follows: — 1 point

From (b), $M_M = \dfrac{\upsilon^2 R}{G}$

$$E_{tot} = \frac{1}{2}m_s\upsilon^2 - \frac{Gm_s}{R}\frac{\upsilon^2 R}{G} = \frac{1}{2}m_s\upsilon^2 - m_s\upsilon^2 = -\frac{1}{2}m_s\upsilon^2$$

$$E_{tot} = -\frac{1}{2}(930\text{ kg})\left(3.40\times10^3\text{ m/s}\right)^2$$

For a negative sign on the final answer — 1 point

$E_{tot} = -5.38\times10^9\text{ J}$

(d) 3 points

For correct selection of "Less than" check space — 1 point

For a correct justification — 2 points

Example 1: From Kepler's third law ($r^3/T^2 = \text{constant}$), if r decreases, then T must also decrease

Example 2: Use relationships among R, υ, and T with no incorrect physics such as the following: From (b), $M_M = \dfrac{\upsilon^2 R}{G}$, so as R decreases, υ must increase. From (a), $T = \dfrac{2\pi R}{\upsilon}$, so both a decrease in R and an increase in υ contribute to a decrease in T.

<u>Note</u>: *1 point partial credit was awarded for using only $T = \dfrac{2\pi R}{\upsilon}$, unless it was stated that υ was constant, in which case no credit was awarded.*

Question 2 (continued)

Distribution of points

(e) 2 points

For a correct expression of conservation of angular momentum — 1 point

$m_s v_1 r_1 = m_s v_2 r_2$ or equivalent such as $I_1 \omega_1 = I_2 \omega_2$ or $v_1 r_1 = v_2 r_2$

$v_2 = v_1 \dfrac{r_1}{r_2} = v_1 \dfrac{R_C + R_M}{R_F + R_M}$, where R_C and R_F are the distances of closest and farthest approaches, respectively, and R_M is the radius of Mars

For explicit substitution of radii (not altitudes) into the equation <u>or</u> for the correct numerical answer — 1 point

$$v_2 = \left(3.40 \times 10^3 \text{ m/s}\right) \frac{3.71 \times 10^5 \text{ m} + 34.3 \times 10^5 \text{ m}}{4.36 \times 10^5 \text{ m} + 34.3 \times 10^5 \text{ m}}$$

$$v_2 = 3.34 \times 10^3 \text{ m/s}$$

Alternatively, if the longer approach using conservation of energy was taken, 1 point was awarded for a correct statement of conservation of energy if explicitly written as $\frac{1}{2} m_s v_1^2 - \frac{G m_s M_M}{r_1} = \frac{1}{2} m_s v_2^2 - \frac{G m_s M_M}{r_2}$*, and 1 point was awarded for the explicit substitution of radii (not altitudes)* <u>*or*</u> *for a correct numerical answer.*

Units point

For including correct units on at least three numerical answers — 1 point

Significant figures point

For including less than five significant digits on at least three numerical answers for which a calculation was shown — 1 point

Mech. 2.

In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes $= 7.08 \times 10^3$ s and orbital speed of 3.40×10^3 m/s. The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

(a) Calculate the radius of the GS orbit.

$$v = \frac{2\pi R}{T}$$

$$Tv = 2\pi R$$

$$R = \frac{Tv}{2\pi}$$

$$= \frac{(7.08 \times 10^3 \text{ s})(3.40 \times 10^3 \text{ m/s})}{2\pi} = \boxed{3.83 \times 10^6 \text{ m}}$$

(b) Calculate the mass of Mars.

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{GM}{R} = v^2$$

$$GM = Rv^2$$

$$M = \frac{Rv^2}{G}$$

$$M = \frac{(3.83 \times 10^6 \text{ m})(3.40 \times 10^3 \text{ m/s})^2}{6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2}$$

$$= \boxed{6.64 \times 10^{23} \text{ kg}}$$

(c) Calculate the total mechanical energy of the GS in this orbit.

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$= \frac{1}{2}(930 \text{ kg})(3.40 \times 10^3 \text{ m/s})^2 - \frac{(6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2)(6.64 \times 10^{23} \text{ kg})(930 \text{ kg})}{3.83 \times 10^6 \text{ m}}$$

$$= \boxed{-5.38 \times 10^9 \text{ J}}$$

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(d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_______Greater than ___✓___Less than

Justify your answer.

The period rule for a circular orbit states that

$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$, where R is the orbital radius, so if the orbital radius decreases, the period of orbit will decrease as well.

(e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

$$L_o = I\omega_o$$
$$= I\left(\frac{V_o}{R_o}\right)$$
$$= mR_o^2\left(\frac{V_o}{R_o}\right)$$
$$= mv_oR_o$$
$$= (930\text{ kg})(3.40\times10^3\text{ m/s})(3.71\times10^5\text{ m} + 3.43\times10^6\text{ m})$$
$$= 1.20\times10^{13}\text{ kg-m}^2/\text{s}$$

$$L_f = L_o = I\omega_f$$
$$L_o = mR_f^2\left(\frac{V_f}{R_f}\right)$$
$$L_o = mv_fR_f$$
$$V_f = \frac{L_o}{mR_f}$$
$$= \frac{1.20\times10^{13}\text{ kg-m}^2/\text{s}}{(930\text{ kg})(4.36\times10^5\text{ m} + 3.43\times10^6\text{ m})}$$
$$= \boxed{3.34\times10^3\text{ m/s}}$$

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Mech. 2.

In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes $= 7.08 \times 10^3$ s and orbital speed of 3.40×10^3 m/s. The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

(a) Calculate the radius of the GS orbit.

$$\cancel{G\frac{Mm}{r^2} = m\frac{v^2}{r}}$$

$$\cancel{6.67 \times 10^{-11} \times}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

$$\therefore\ r = \frac{Tv}{2\pi} = 3.831177 \times 10^6 \text{ m}$$

(b) Calculate the mass of Mars.

$$G\frac{M\cancel{m}}{r^2} = \cancel{m}\frac{v^2}{r}$$

$$GM = v^2 r$$

$$M = \frac{v^2 r}{G} = 6.6399 \times 10^{23} \text{ kg}$$

(c) Calculate the total mechanical energy of the GS in this orbit.

$$E = \frac{1}{2} m v^2$$

$$= 5375400000 \text{ J}$$

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(d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_______Greater than   __✓__Less than

Justify your answer.

$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

$$GM = v^2 r$$

$$\because v = \frac{2\pi r}{T} \qquad \therefore GM = \frac{4\pi r^3}{T^2}$$

$$T = \sqrt{\frac{4\pi r^3}{GM}}$$

when r decreases, T decreases

(e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s , calculate the speed at the furthest point of the orbit.

$$m v_1 L_1 = m v_2 L_2$$

$$(3.71 \times 10^5 + 3.43 \times 10^6) \times 3.4 \times 10^3$$

$$= (4.36 \times 10^5 + 3.43 \times 10^6) \times v_2$$

$$\therefore v_2 = 3342.835 \quad m/s$$

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Mech. 2.

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(a) Calculate the radius of the GS orbit.

$T = \frac{2\pi}{\omega}$ $\omega = \frac{2\pi}{T}$

$\omega \cdot r = v$

$T = \frac{2\pi r}{v}$ $= 3.4 \times 10^3$ m/s

$r = \frac{Tv}{2\pi}$ $\frac{(7.08 \times 10^3 s)(3.40 \times 10^3 m)}{2\pi} = r = \boxed{3.831 \times 10^6 m}$

(b) Calculate the mass of Mars.

$a = \frac{v^2}{r}$

$\frac{v^2}{r} = G\frac{M_{mars}}{r^2}$

$\frac{rv^2}{G} = M_{mars} = \boxed{1.953 \times 10^{20} kg}$

(c) Calculate the total mechanical energy of the GS in this orbit.

0 J

~~Beca~~
no work is performed in orbit.

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(d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_______Greater than ____✓____Less than

Justify your answer.

$$T = \frac{2\pi r}{v}$$

If all other constants remain the same, a decrease in radius will give a smaller period.

(e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

$$PE = G\frac{Mm}{r}$$

$$G\frac{M_{mars} M_{GS}}{3.71 \times 10^5} + \frac{1}{2}(930)(3.4 \times 10^3)^2 = G\frac{M_{mars} M_{GS}}{4.36 \times 10^5} + \frac{1}{2}(930)v^2$$

$$\frac{1}{3.71 \times 10^5} + 5.3754 \times 10^9 = \frac{1}{4.39 \times 10^5} + 465 v^2$$

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Question 2

Overview

This question evaluated students' understanding of orbital motion and energetics. The specific system was an artificial satellite of the planet Mars. The orbital period and the satellite's orbital speed were given, as were the satellite's mass and Mars's radius. Part (a) asked students to calculate the radius of the satellite orbit, and part (b) asked them to calculate the mass of Mars. Part (c) required them to calculate the total mechanical energy. In part (d) students were told to suppose that the satellite was placed in a lower circular orbit and to decide whether the resultant orbital period would be greater than or less than the original period. They were expected to justify their answers. In part (e) they were informed that the satellite orbit was slightly elliptical and were asked a question about the variation of orbital speed in such an orbit.

Sample: M2A
Score: 15

Full credit was awarded to all parts of this clearly written and well organized response. Kepler's third law is cited (though not by name) as the justification for part (d). The student takes a longer approach than necessary to answer part (e) by actually calculating a value for the angular momentum but answers this part correctly.

Sample: M2B
Score: 10

Full credit was awarded for parts (a) and (b). However, all 4 points were lost in part (c) for showing an incorrect equation for total mechanical energy that does not include a potential energy term. Part (d), in which Kepler's law is derived, received full credit. Part (e) also received full credit. The units point was earned but not the point for significant digits, since no calculated answer shows less than five digits.

Sample: M2C
Score: 5

Full credit was awarded for part (a). Part (b) earned 1 point for the correct equation but did not earn the point for substituting the value of R from part (a). The substitutions in part (b) are not shown, so it is not clear where the error in calculating the final answer is made. Parts (c) and (e) received no credit. Part (d) was awarded 1 point for the correct check mark, but the incomplete justification that implies a constant velocity received nothing. The units point was given but not the significant figures point because the answer to (c) is not from a calculation, leaving only two calculated answers.