CollegeBoard
Advanced Placement Program

AP® Physics C: Mechanics
2009 Free-Response Questions

The College Board

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TABLE OF INFORMATION FOR 2008 and 2009

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J•s} = 4.14 \times 10^{-15}\text{ eV•s}$
	$hc = 1.99 \times 10^{-25}\text{ J•m} = 1.24 \times 10^{3}\text{ eV•nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N•m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\text{ N•m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ (T•m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 10^{-7}\text{ (T•m)/A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$

UNIT SYMBOLS								
meter,	m	mole,	mol	watt,	W	farad,	F
kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
second,	s	newton,	N	volt,	V	degree Celsius,	°C
ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2008 and 2009

MECHANICS

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$v = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1 m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Nev_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Parallelepiped

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

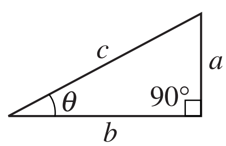
$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

Mech. 1.

A 3.0 kg object is moving along the x-axis in a region where its potential energy as a function of x is given as $U(x) = 4.0x^2$, where U is in joules and x is in meters. When the object passes the point $x = -0.50$ m, its velocity is +2.0 m/s. All forces acting on the object are conservative.

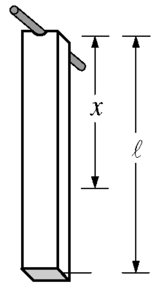
(a) Calculate the total mechanical energy of the object.

(b) Calculate the x-coordinate of any points at which the object has zero kinetic energy.

(c) Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

(d) Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

(e) On the axes below, sketch graphs of the object's position x versus time t and kinetic energy K versus time t. Assume that $x = 0$ at time $t = 0$. The two graphs should cover the same time interval and use the same scale on the horizontal axes.



Mech. 2.

You are given a long, thin, rectangular bar of known mass M and length ℓ with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance x from the end with the pivot. You are to determine the rotational inertia I_b of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of I_b, the given quantities, and fundamental constants.

(a)

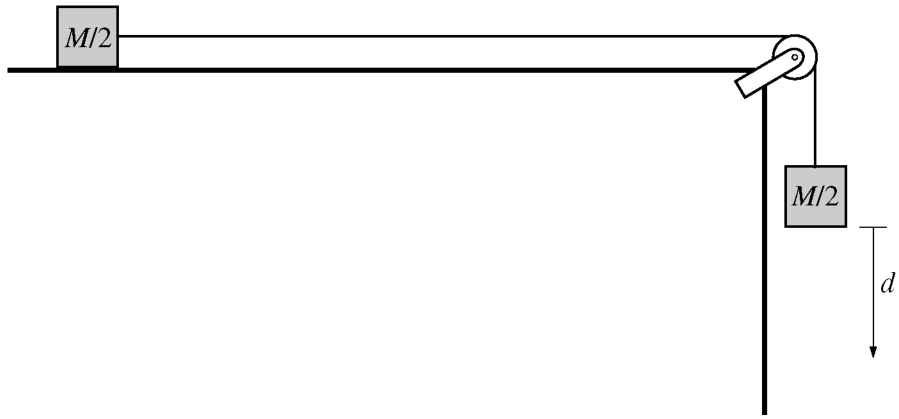
i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.

ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

(b) Describe the experimental procedure you would use to make the additional measurements needed to determine I_b. Include how you would use your measurements to obtain I_b and how you would minimize experimental error.

(c) Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.

GO ON TO THE NEXT PAGE.

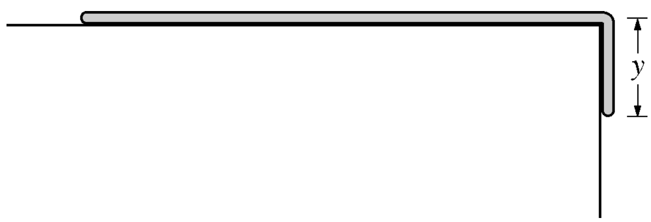


Mech. 3.

A block of mass $M/2$ rests on a frictionless horizontal table, as shown above. It is connected to one end of a string that passes over a massless pulley and has another block of mass $M/2$ hanging from its other end. The apparatus is released from rest.

(a) Derive an expression for the speed v_h of the hanging block as a function of the distance d it descends.

Now the block and pulley system is replaced by a uniform rope of length L and mass M, with one end of the rope hanging slightly over the edge of the frictionless table. The rope is released from rest, and at some time later there is a length y of rope hanging over the edge, as shown below. Express your answers to parts (b), (c), and (d) in terms of y, L, M, and fundamental constants.



(b) Determine an expression for the force of gravity on the hanging part of the rope as a function of y.

(c) Derive an expression for the work done by gravity on the rope as a function of y, assuming y is initially zero.

(d) Derive an expression for the speed v_r of the rope as a function of y.

(e) The hanging block and the right end of the rope are each allowed to fall a distance L (the length of the rope). The string is long enough that the sliding block does not hit the pulley. Indicate whether v_h from part (a) or v_r from part (d) is greater after the block and the end of the rope have traveled this distance.

____ v_h is greater. ____ v_r is greater. ____ The speeds are equal.

Justify your answer.

END OF EXAM