

AP[®] Physics C: Mechanics 2010 Free-Response Questions

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TABLE OF INFORMATION FOR 2010 and 2011

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol ⁻¹	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m ³ /kg·s ²
Universal gas constant, $R = 8.31$ J/(mol·K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s ²
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	
1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27}$ kg = $931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}$ J·s = 4.14×10^{-15} eV·s
	$hc = 1.99 \times 10^{-25}$ J·m = 1.24×10^3 eV·nm
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}$ C ² /N·m ²
Coulomb's law constant, $k = 1/4\pi\epsilon_0 = 9.0 \times 10^9$ N·m ² /C ²	
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T·m)/A
Magnetic constant, $k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T·m)/A	
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5$ N/m ² = 1.0×10^5 Pa

UNIT SYMBOLS	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10 ⁹	giga	G
10 ⁶	mega	M
10 ³	kilo	k
10 ⁻²	centi	c
10 ⁻³	milli	m
10 ⁻⁶	micro	μ
10 ⁻⁹	nano	n
10 ⁻¹²	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan \theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following conventions are used in this exam.

- I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.
- II. The direction of any electric current is the direction of flow of positive charge (conventional current).
- III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2010 and 2011

MECHANICS

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\Sigma \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F} dt = \Delta \mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \leq \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$K = \frac{1}{2} mv^2$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\Sigma \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \Sigma mr^2$$

$$\mathbf{r}_{cm} = \Sigma m\mathbf{r} / \Sigma m$$

$$v = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2} I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

a = acceleration
 F = force
 f = frequency
 h = height
 I = rotational inertia
 J = impulse
 K = kinetic energy
 k = spring constant
 ℓ = length
 L = angular momentum
 m = mass
 N = normal force
 P = power
 p = momentum
 r = radius or distance
 \mathbf{r} = position vector
 T = period
 t = time
 U = potential energy
 v = velocity or speed
 W = work done on a system
 x = position
 μ = coefficient of friction
 θ = angle
 τ = torque
 ω = angular speed
 α = angular acceleration

$$\mathbf{F}_s = -kx$$

$$U_s = \frac{1}{2} kx^2$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1 m_2}{r^2} \hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa \epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2} QV = \frac{1}{2} CV^2$$

$$R = \frac{\rho \ell}{A}$$

$$\mathbf{E} = \rho \mathbf{J}$$

$$I = Nev_d A$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

A = area
 B = magnetic field
 C = capacitance
 d = distance
 E = electric field
 \mathcal{E} = emf
 F = force
 I = current
 J = current density
 L = inductance
 ℓ = length
 n = number of loops of wire per unit length
 N = number of charge carriers per unit volume
 P = power
 Q = charge
 q = point charge
 R = resistance
 r = distance
 t = time
 U = potential or stored energy
 V = electric potential
 v = velocity or speed
 ρ = resistivity
 ϕ_m = magnetic flux
 κ = dielectric constant

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$d\mathbf{B} = \frac{\mu_0 I d\boldsymbol{\ell} \times \mathbf{r}}{4\pi r^3}$$

$$\mathbf{F} = \int I d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 nI$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$U_L = \frac{1}{2} LI^2$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2010 and 2011

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

Parallelepiped

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

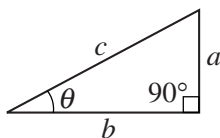
Right Triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$



A = area
 C = circumference
 V = volume
 S = surface area
 b = base
 h = height
 ℓ = length
 w = width
 r = radius

CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$$

$$\int e^x dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x dx = \sin x$$

$$\int \sin x dx = -\cos x$$

2010 AP[®] PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS

PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

Students are to conduct an experiment to investigate the relationship between the terminal speed of a stack of falling paper coffee filters and its mass. Their procedure involves stacking a number of coffee filters, like the one shown in the figure above, and dropping the stack from rest. The students change the number of filters in the stack to vary the mass m while keeping the shape of the stack the same. As a stack of coffee filters falls, there is an air resistance (drag) force acting on the filters.

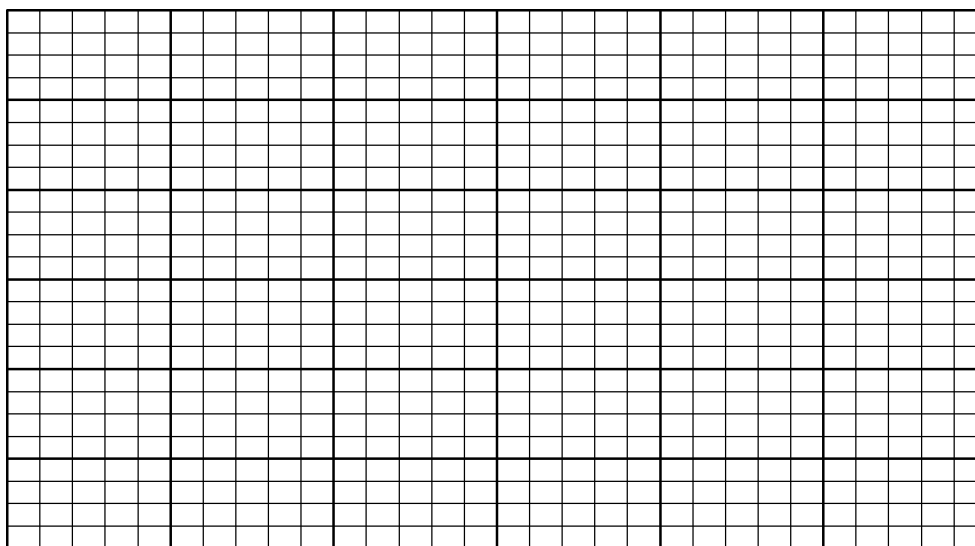
- (a) The students suspect that the drag force F_D is proportional to the square of the speed v : $F_D = Cv^2$, where C is a constant. Using this relationship, derive an expression relating the terminal speed v_T to the mass m .

The students conduct the experiment and obtain the following data.

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	0.51	0.62	0.82	0.92	1.06

(b)

- (i) Assuming the functional relationship for the drag force above, use the grid below to plot a linear graph as a function of m to verify the relationship. Use the empty boxes in the data table, as appropriate, to record any calculated values you are graphing. Label the vertical axis as appropriate, and place numbers on both axes.



m (kg)

2010 AP[®] PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS

(ii) Use your graph to calculate C .

A particular stack of filters with mass m is dropped from rest and reaches a speed very close to terminal speed by the time it has fallen a vertical distance Y .

(c)

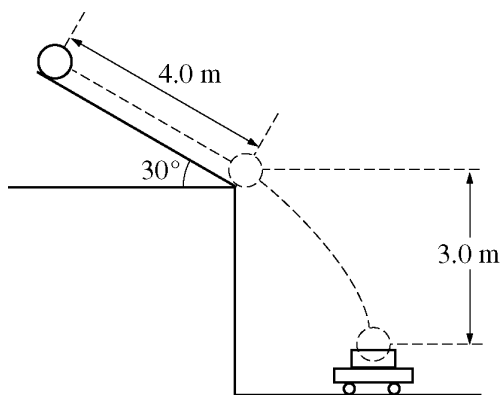
(i) Sketch an approximate graph of speed versus time from the time the filters are released up to the time $t = T$ that the filters have fallen the distance Y . Indicate time $t = T$ and terminal speed $v = v_T$ on the graph.



(ii) Suppose you had a graph like the one sketched in (c)(i) that had a numerical scale on each axis. Describe how you could use the graph to approximate the distance Y .

(d) Determine an expression for the approximate amount of mechanical energy dissipated, ΔE , due to air resistance during the time the stack falls a distance y , where $y > Y$. Express your answer in terms of y , m , v_T , and fundamental constants.

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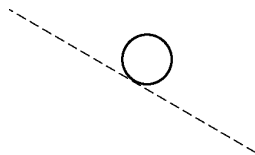


Note: Figure not drawn to scale.

Mech. 2.

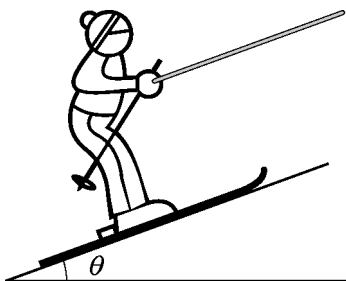
A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $\frac{2}{5}MR^2$.

- (a) On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



- (b) Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).
- (c) Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.
- (d) A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

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Mech. 3.

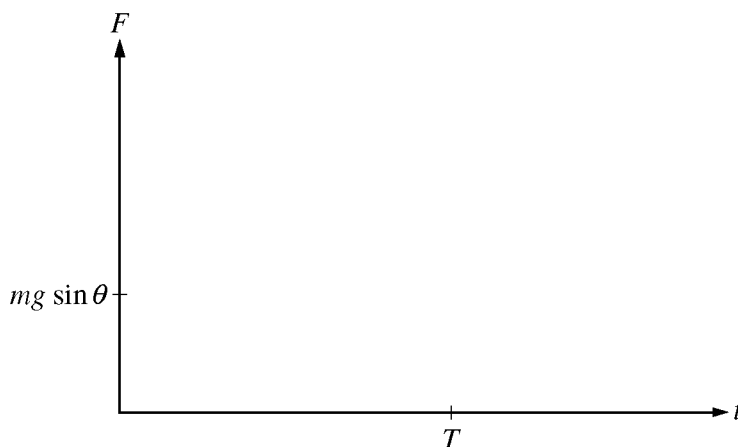
A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{\max} \sin \frac{\pi t}{T} \quad (0 < t < T)$$

$$= 0 \quad (t \geq T),$$

where a_{\max} and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

- Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.
- Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.
- Determine the magnitude of the force exerted by the rope on the skier at terminal speed.
- Derive an expression for the total impulse imparted to the skier during the acceleration.
- Suppose that the magnitude of the acceleration is instead modeled as $a = a_{\max} e^{-\pi t/2T}$ for all $t > 0$, where a_{\max} and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2 .



END OF EXAM