AP® Physics C: Mechanics
2011 Free-Response Questions

About the College Board

TABLE OF INFORMATION FOR 2010 and 2011

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg$\cdot$s^2
Universal gas constant, $R = 8.31$ J/(mol$\cdot$K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ (T}\cdot\text{m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7} \text{ (T}\cdot\text{m)/A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5} \text{ N/m}^2 = 1.0 \times 10^{5} \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2010 and 2011

MECHANICS

$$\upsilon = \upsilon_0 + at$$

$$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$$

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$$

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \le \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$K = \frac{1}{2}m\upsilon^2$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{\upsilon^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \sum mr^2$$

$$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$$

$$\upsilon = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2}I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

- a = acceleration
- F = force
- f = frequency
- h = height
- I = rotational inertia
- J = impulse
- K = kinetic energy
- k = spring constant
- ℓ = length
- L = angular momentum
- m = mass
- N = normal force
- P = power
- p = momentum
- r = radius or distance
- $\mathbf{r}$ = position vector
- T = period
- t = time
- U = potential energy
- υ = velocity or speed
- W = work done on a system
- x = position
- μ = coefficient of friction
- θ = angle
- τ = torque
- ω = angular speed
- α = angular acceleration

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho\ell}{A}$$

$$\mathbf{E} = \rho\mathbf{J}$$

$$I = Nev_d A$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

- A = area
- B = magnetic field
- C = capacitance
- d = distance
- E = electric field
- $\mathcal{E}$ = emf
- F = force
- I = current
- J = current density
- L = inductance
- ℓ = length
- n = number of loops of wire per unit length
- N = number of charge carriers per unit volume
- P = power
- Q = charge
- q = point charge
- R = resistance
- r = distance
- t = time
- U = potential or stored energy
- V = electric potential
- υ = velocity or speed
- ρ = resistivity
- ϕ_m = magnetic flux
- κ = dielectric constant

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$

$$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 nI$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2}LI^2$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2010 and 2011

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Parallelepiped

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

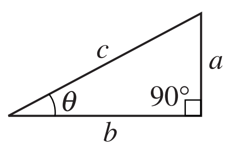
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

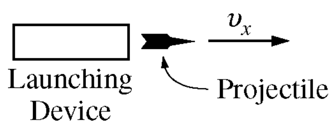
$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



Mech. 1.

A projectile is fired horizontally from a launching device, exiting with a speed v_x. While the projectile is in the launching device, the impulse imparted to it is J_p, and the average force on it is F_{avg}. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of v_x, J_p, F_{avg}, and fundamental constants, as appropriate.

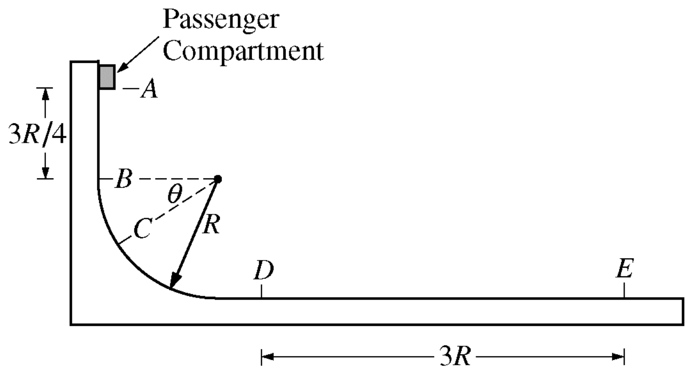
(a) Determine an expression for the time required for the projectile to travel the length of the launching device.
(b) Determine an expression for the mass of the projectile.

The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance d into the block before it stops. Express all algebraic answers to the following in terms of d and the given quantities previously indicated, as appropriate.

(c) Derive an expression for the work done in stopping the projectile.
(d) Derive an expression for the average force F_b exerted on the projectile as it comes to rest in the block.

Now a new projectile and block are used, identical to the first ones, but the block is not clamped to the table. The projectile is again fired into the block of wood and travels a new distance d_n into the block while the block slides across the table a short distance D. Assume the following: the projectile enters the block with speed v_x, the average force F_b between the projectile and the block has the same value as determined in part (d), the average force of friction between the table and the block is f_T, and the collision is instantaneous so the frictional force is negligible during the collision.

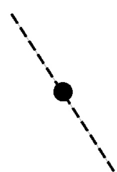
(e) Derive an expression for d_n in terms of d, D, f_T, and F_b, as appropriate.
(f) Derive an expression for d_n in terms of d, the mass m of the projectile, and the mass M of the block.



Mech. 2.

An amusement park ride features a passenger compartment of mass M that is released from rest at point A, as shown in the figure above, and moves along a track to point E. The compartment is in free fall between points A and B, which are a distance of $3R/4$ apart, then moves along the circular arc of radius R between points B and D. Assume the track is frictionless from point A to point D and the dimensions of the passenger compartment are negligible compared to R.

(a) On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point C, which is at an angle θ from point B.



(b) In terms of θ and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point C. If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Derive an expression for the speed v_D of the passenger compartment as it reaches point D in terms of M, R, and fundamental constants, as appropriate.

A force acts on the compartment between points D and E and brings it to rest at point E.

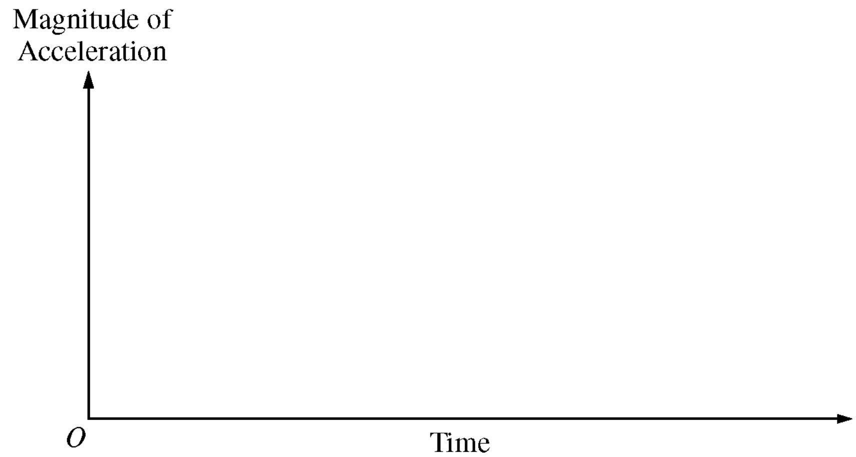
(d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track.

(e) Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where k is a constant and $\mathbf{v}$ is the velocity of the compartment. Express all algebraic answers to the following in terms of M, R, k, v_D, and fundamental constants, as appropriate.

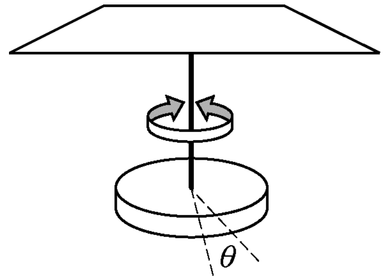
i. Write, but do NOT solve, the differential equation for $v(t)$.

ii. Solve the differential equation you wrote in part i.

iii. On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



GO ON TO THE NEXT PAGE.



Mech. 3.

The torsion pendulum shown above consists of a disk of rotational inertia I suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle θ, the twisted rod exerts a restoring torque τ that is proportional to the angular displacement: $\tau = -\beta\theta$, where β is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement θ of the torsion pendulum as a function of time t.

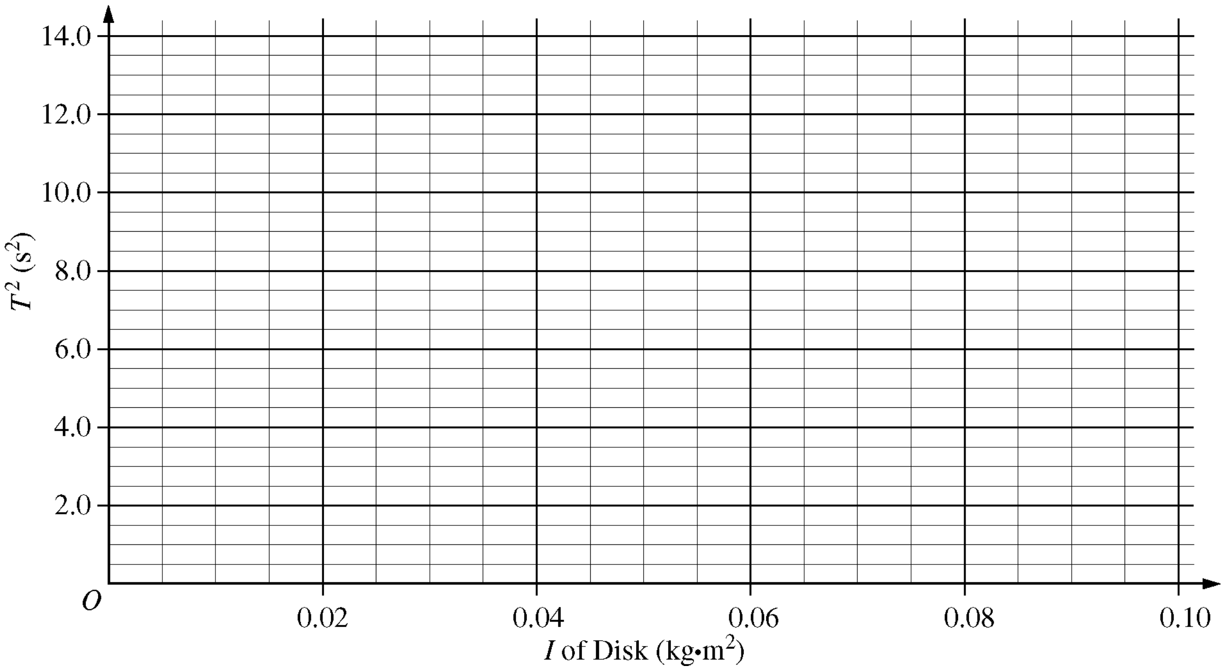
(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

To determine the torsion constant β of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

Rotational Inertia I of Disk $(\text{kg}\cdot\text{m}^2)$	Average Time for Ten Oscillations (s)	Period T (s)	T^2 (s^2)
0.025	22.4	2.24	5.0
0.036	26.8	2.68	7.2
0.049	29.5	2.95	8.7
0.064	33.3	3.33	11.1
0.081	35.9	3.59	12.9

GO ON TO THE NEXT PAGE.

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.



(d) Determine the equation for your line.

(e) Calculate the torsion constant β of the rod from your line.

(f) What is the physical significance of the intercept of your line with the vertical axis?

END OF EXAM