General Notes About 2011 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be earned. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally earn credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still earned. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics Course Description*.

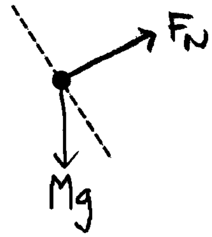
4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically earn full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

15 points total **Distribution of points**

(a) 2 points



For either a weight force or a normal force, correctly drawn and labeled — 1 point
For the second correct force and no additional forces, arrows or components — 1 point

(b) 1 point

For a correct expression for the centripetal force in terms of the forces drawn in part (a) — 1 point
For the example above:
$F_c = F_N - Mg\sin\theta$

Alternate Solution — *Alternate points*
Applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point C

$Mg\,\Delta h = Mv_C^2/2$

$v_C^2 = 2g\,\Delta h$

$\Delta h = 3R/4 + R\sin\theta$

$v_C^2 = 2g(3R/4 + R\sin\theta)$

$F_c = Mv_C^2/R$

$F_c = M(2g(3R/4 + R\sin\theta))/R$

For a correct answer — *1 point*
$F_c = 2Mg(3/4 + \sin\theta)$

(c) 2 points

For applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point D — 1 point

$Mg\,\Delta h = Mv_D^2/2$

$v_D^2 = 2g\,\Delta h$

$\Delta h = 3R/4 + R = 7R/4$

$v_D^2 = 2g(7R/4)$

For a correct answer — 1 point
$v_D = \sqrt{(7/2)gR}$

Question 2 (continued)

Distribution of points

(d) 3 points

Work-energy approach

For equating the work done by the friction force to the kinetic energy of the compartment at point D — 1 point

$W = \Delta K = 0 - \frac{1}{2}Mv_D^2$

For a correct expression for the frictional force — 1 point

$f = \mu N = \mu Mg$

$W = \mathbf{F} \bullet \Delta\mathbf{r} = fd\cos 180 = -(\mu Mg)d$

$(\mu Mg)d = \frac{1}{2}Mv_D^2$

For substituting the expression for v_D from part (c), and $d = 3R$ — 1 point

$(\mu Mg)3R = \frac{1}{2}M\left(\frac{7}{2}gR\right)$

$3\mu = \frac{1}{2}\left(\frac{7}{2}\right)$

$\mu = 7/12$

Note: Full credit is also earned for setting the initial potential energy at point A, $U_A = mg\left(\frac{7R}{4}\right)$, equal to the work done by the frictional force, and solving for μ.

Alternate solution — *Alternate points*

For using both Newton's second law and a correct kinematics equation — *1 point*

$\mathbf{F}_{net} = m\mathbf{a}$

$v_f^2 - v_i^2 = 2ad$

For a correct expression for the frictional force — *1 point*

$f = \mu N = \mu Mg$

$-\mu Mg = Ma$

$a = -\mu g$

Substituting for a, and the final and initial speeds in the kinematic equation

$-v_D^2 = 2(-\mu g)d$

For substituting the expression for v_D from part (c), and $d = 3R$ — *1 point*

$\frac{7}{2}gR = 2(\mu g)3R$

$\mu = 7/12$

Question 2 (continued)

Distribution of points

(e)

i. 2 points

$\Sigma F = ma$

For substituting the braking force into Newton's second law as the net force	1 point
For substituting the time derivative of velocity for the acceleration	1 point

$-kv = M(dv/dt)$

ii. 2 points

For separating the variables and integrating	1 point

$dv/v = -(k/M)dt$

$$\int_{v_D}^{v} dv/v = -(k/M)\int_0^t dt$$

$$\ln v\Big|_{v_D}^{v} = -(k/M)t$$

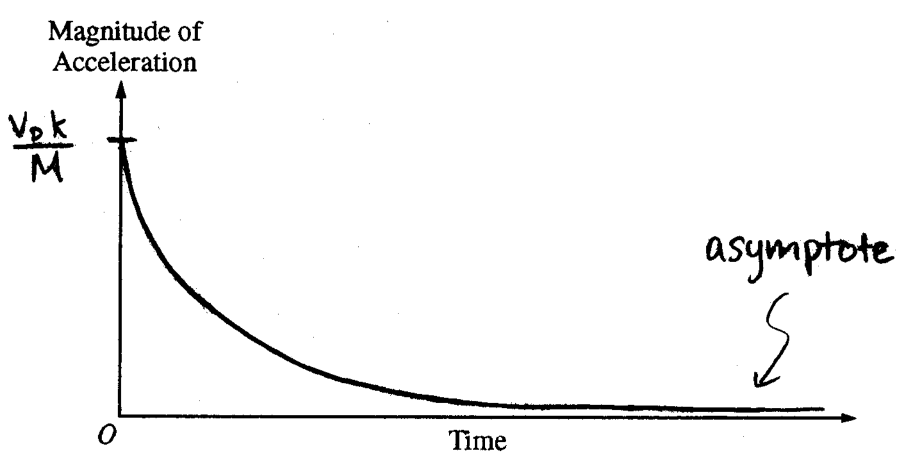
$$\ln v - \ln v_D = \ln(v/v_D) = -(k/M)t$$

$$v/v_D = e^{-kt/M}$$

For a correct expression for the velocity as a function of time	1 point

$v = v_D e^{-kt/M}$

iii. 3 points

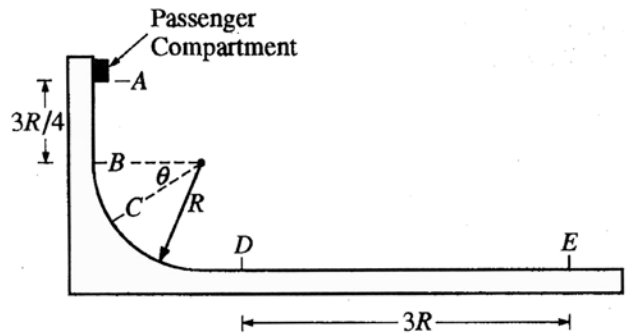


Taking the derivative of the equation for v from part (e) ii

$$a = dv/dt = d\left(v_D e^{-kt/M}\right)/dt = -(k/M)v_D e^{-kt/M}$$

At $t = 0$, $a = -kv_D/M$

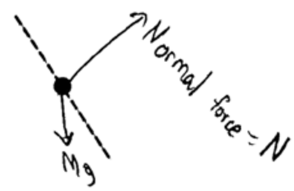
For a graph with a finite intercept on the vertical axis	1 point
For a graph that is concave upward and asymptotic to zero	1 point
For labeling the initial acceleration with the correct value	1 point



Mech. 2.

An amusement park ride features a passenger compartment of mass M that is released from rest at point A, as shown in the figure above, and moves along a track to point E. The compartment is in free fall between points A and B, which are a distance of $3R/4$ apart, then moves along the circular arc of radius R between points B and D. Assume the track is frictionless from point A to point D and the dimensions of the passenger compartment are negligible compared to R.

(a) On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point C, which is at an angle θ from point B.



(b) In terms of θ and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point C. If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

~~By Conservation of energy,~~

$\cancel{K_A + U_A = K_C + U_C}$

$\cancel{\frac{1}{2}Mv_C^2 = MgR(\frac{3}{4} + \cos\theta)}$

By Newton's 2nd Law,

$\left[F_c = N - Mg\cos\theta \right]$

(c) Derive an expression for the speed v_D of the passenger compartment as it reaches point D in terms of M, R, and fundamental constants, as appropriate.

By conservation of energy,

$K_A + U_A = K_D + U_D$

$\frac{7MgR}{4} = \frac{1}{2}Mv_D^2$

$V_D = \sqrt{\frac{7gR}{2}}$

GO ON TO THE NEXT PAGE.

A force acts on the compartment between points D and E and brings it to rest at point E.

(d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track. By Newton's 2nd Law,

$f_r = Ma \quad -\mu Mg = Ma \quad \therefore\ a = -\mu g$

$\therefore$ By linear, constant acceleration kinematics, $V^2 = V_0^2 + 2a\Delta x$

or $v_D^2 = 2\mu g(3R)$

$\frac{7gR}{2} = 6\mu gR \qquad \mu = \frac{7}{12}$

(e) Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where k is a constant and $\mathbf{v}$ is the velocity of the compartment. Express all algebraic answers to the following in terms of M, R, k, v_D, and fundamental constants, as appropriate.

i. Write, but do NOT solve, the differential equation for $v(t)$. By Newton's 2nd Law,

$$M\frac{dv}{dt} = -kv$$

ii. Solve the differential equation you wrote in part i.

$$\frac{dv}{v} = \frac{-k\,dt}{M}$$

$$\int \frac{dv}{v} = \int -\frac{k}{M}\,dt$$

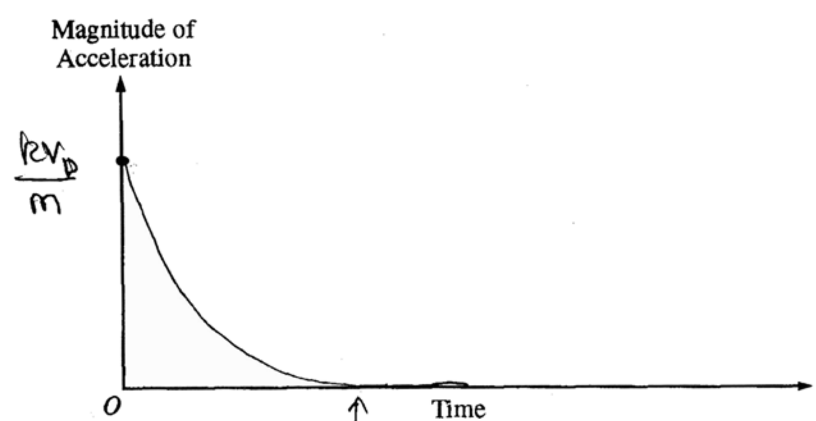
$$\ln|v| = -\frac{k}{M}t + C$$

at $t=0$, $v = v_D$

Solving w/ the intial conditions, we get

$$v(t) = v_D e^{-\frac{k}{M}t}$$

iii. On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



Asymptotically approaching 0

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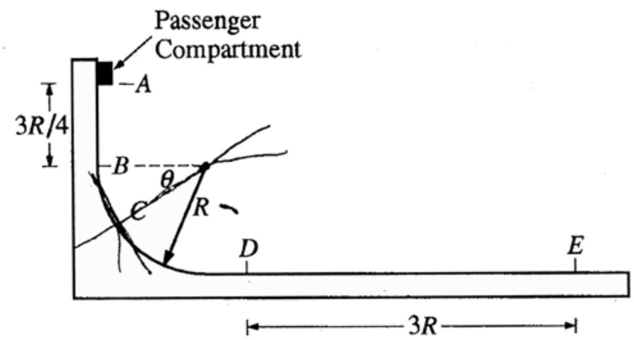
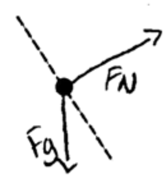


Mech. 2.
An amusement park ride features a passenger compartment of mass M that is released from rest at point A, as shown in the figure above, and moves along a track to point E. The compartment is in free fall between points A and B, which are a distance of $3R/4$ apart, then moves along the circular arc of radius R between points B and D. Assume the track is frictionless from point A to point D and the dimensions of the passenger compartment are negligible compared to R.

(a) On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point C, which is at an angle θ from point B.



(b) In terms of θ and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point C. If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$F_c = \frac{mv^2}{R}$

$F = ma$

mg

(c) Derive an expression for the speed v_D of the passenger compartment as it reaches point D in terms of M, R, and fundamental constants, as appropriate.

~~A→B~~

~~$V_f^2 = V_i^2 + 2ad$~~

~~$V_f^2 = 0 + 2(10)(\frac{3R}{4})$~~

~~$V_f = \sqrt{15R}$ point B~~

$PE = KE$

$mgh = \frac{1}{2}mv^2$

$\cancel{m}g\frac{7R}{4} = \frac{1}{2}\cancel{m}V^2$

$\boxed{\sqrt{\frac{7gR}{2}} = V_D}$

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A force acts on the compartment between points D and E and brings it to rest at point E. **M2B p2**

(d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track.

$$\Sigma F = ma$$
$$\mu \cancel{m} g = \cancel{m}\frac{7g}{12}$$
$$\boxed{\mu = \frac{7}{12}}$$

$$V_f^2 = V_i^2 + 2ad$$
$$0 = \frac{7gR}{2} + 2a3R$$
$$\frac{7gR}{2} = 6Ra$$
$$\frac{7g}{12} = a$$

(e) Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where k is a constant and $\mathbf{v}$ is the velocity of the compartment. Express all algebraic answers to the following in terms of M, R, k, v_D, and fundamental constants, as appropriate.

i. Write, but do NOT solve, the differential equation for $v(t)$.

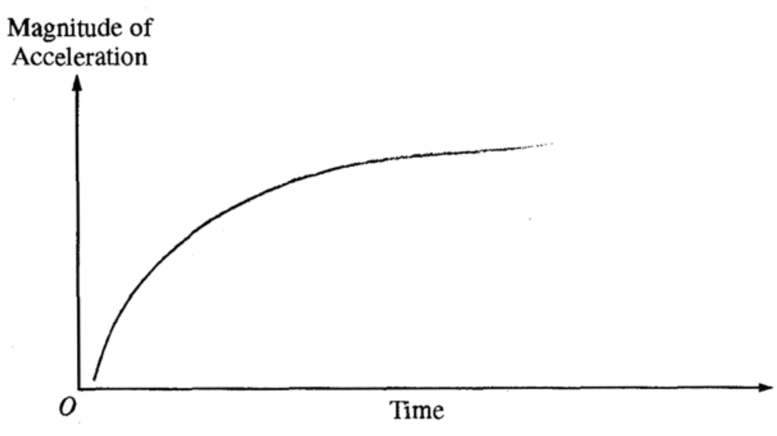
$$\Sigma F = ma$$
$$-kv = m\frac{dv}{dt}$$

ii. Solve the differential equation you wrote in part i.

$$v(0) = \sqrt{\frac{7gR}{2}}$$

$$\frac{-kv}{m} = \frac{dv}{dt}$$
$$\int dt = \int \frac{1}{-\frac{kv}{m}}\, dv$$
$$t + C = -\frac{m}{k}\ln\left(-\frac{kv}{m}\right)$$
$$-\frac{k}{m}t + C = \ln\left(-\frac{kv}{m}\right)$$
$$Ce^{-\frac{k}{m}t} = -\frac{kv}{m}$$
$$C = -\frac{k}{m}\sqrt{\frac{7gR}{2}}$$
$$\longrightarrow -\frac{\cancel{k}}{\cancel{m}}\sqrt{\frac{7gR}{2}}\, e^{-\frac{k}{m}t} = -\frac{\cancel{k}}{\cancel{m}}v$$
$$\boxed{v = \sqrt{\frac{7gR}{2}}\, e^{-\frac{k}{m}t}}$$

iii. On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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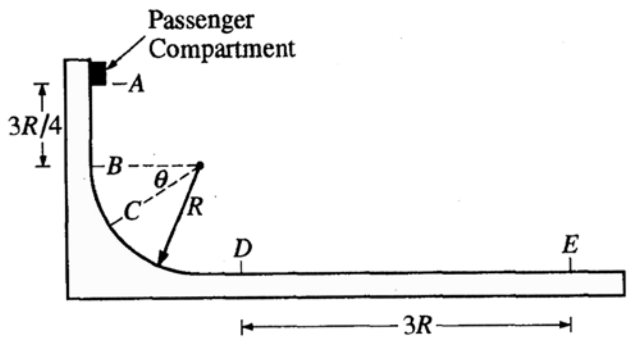

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$F_c =$

(c) Derive an expression for the speed v_D of the passenger compartment as it reaches point D in terms of M, R, and fundamental constants, as appropriate.

$E_0 = E_f$; $mgh = \frac{1}{2} m v_D^2$; $h = \frac{3R}{4} + R = \frac{7R}{4}$

$v_D^2 = \dfrac{\cancel{m} g \frac{7R}{4}}{\frac{1}{2} \cancel{m}} = \frac{7R}{2} g$

$v_D = \sqrt{\frac{7R}{2} g}$

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A force acts on the compartment between points D and E and brings it to rest at point E. **M2C p2**

(d) If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track.

$v^2 = v_0^2 + 2a\Delta x$

$0 = v_D^2 + (2a)(3R)$

$\frac{-v_D^2}{3R} = 2a \; ; \; a = \frac{-v_D^2}{6R}$

$f = m\left(\frac{-v_D^2}{6R}\right)$

(e) Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where k is a constant and $\mathbf{v}$ is the velocity of the compartment. Express all algebraic answers to the following in terms of M, R, k, v_D, and fundamental constants, as appropriate.

i. Write, but do NOT solve, the differential equation for $v(t)$.

$-kv = ma$

$a = \frac{-kv}{m}$

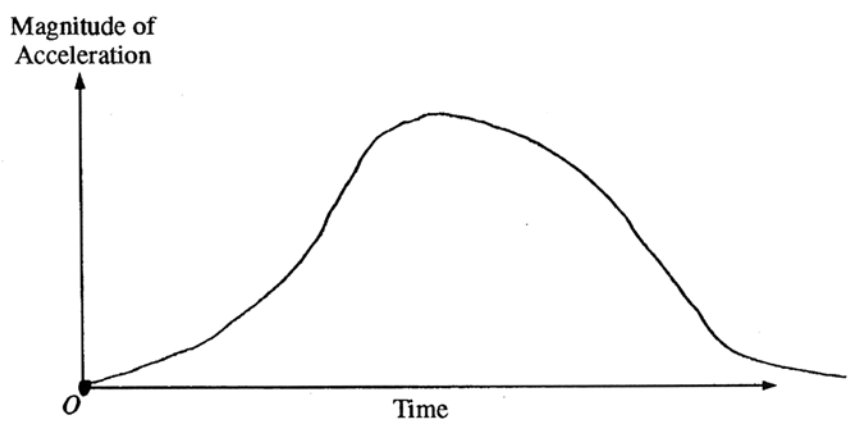
$\frac{dv}{dt} = \frac{-kv}{m}$

ii. Solve the differential equation you wrote in part i.

$dv = -\frac{kv}{m}\, dt$

$v =$

iii. On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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Question 2

Overview

This question assessed students' understanding of conservation of energy, circular motion in a vertical plane, work done by a constant retarding force, and velocity and acceleration of a particle under the influence of a variable retarding force.

Sample: M2A
Score: 14

The response earned full credit on part (a). No credit was earned in part (b) because the student uses the wrong component of gravity in the centripetal force. Full credit was earned in parts (c) through (e). The student labels the asymptote in part (e) iii. Although the question did not require this, it is good practice and should be encouraged.

Sample: M2B
Score: 11

The response earned full credit for parts (a) and (c), but no credit for part (b). Full credit was also earned for part (d); note the Newton's law and kinematics approach used. The response also earned full credit for parts (e) i and (e) ii. Note that the student uses the expression for v_D from part (c) in the $v(t)$ function. This was accepted because it is in terms of the variables listed. No credit was earned for the graph in part (e) iii.

Sample: M2C
Score: 6

Part (a) earned 1 point. Two correct forces are shown, but because the extra force F_c is included incorrectly, credit for the second correct force was not earned. No credit was earned for part (b). The response earned 2 points for part (c). Part (d) earned 1 point for combining kinematics and Newton's law. The velocity from part (c) is not substituted and the friction force is not present; therefore those points were not earned. The response earned 2 points for part (e) i but no points for parts (e) ii and (e) iii.