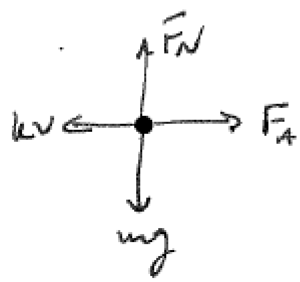
Question 2

15 points total | **Distribution of points**

(a) 4 points



For correctly showing and labeling the applied force directed to the right	1 point
For correctly showing and labeling the downward gravitational force	1 point
For correctly showing and labeling the upward normal force	1 point
For correctly showing and labeling the drag force directed to the left	1 point
One earned point was deducted for having any extraneous vectors	

(b) 2 points

$F_{net} = ma$

For the correct substitution into Newton's second law — 1 point

$F_A - kv = ma$

For a correct differential equation — 1 point

$$F_A - kv = m\frac{dv}{dt}$$

(c) 1 point

Set $\frac{dv}{dt} = 0$ in the equation from part (b)

$F_A - kv = 0$

For the correct expression for the terminal velocity — 1 point

$$v_T = \frac{F_A}{k}$$

Question 2 (continued)

Distribution of points

(d) 5 points

Use the differential equation from part (b)

$$F_A - kv = m\frac{dv}{dt}$$

For demonstrating separation of variables — 1 point

$$\frac{1}{m}dt = \frac{1}{F_A - kv}dv$$

For demonstrating that the equation must be integrated — 1 point

$$\int \frac{1}{m}dt = \int \frac{1}{F_A - kv}dv$$

For demonstrating substitution using initial and final values (or evaluating the constant of integration using the boundary conditions) — 1 point

$$\int_0^t \frac{1}{m}dt = \int_0^{v(t)} \frac{1}{F_A - kv}dv$$

$$\left[\frac{t}{m}\right]_0^t = -\frac{1}{k}\left[\ln\left(F_A - kv\right)\right]_0^{v(t)}$$

For attempting to solve for $v(t)$ — 1 point

$$-\frac{kt}{m} = \ln\left(\frac{F_A - kv(t)}{F_A}\right)$$

$$e^{-kt/m} = \frac{F_A - kv(t)}{F_A} = 1 - \frac{kv(t)}{F_A}$$

$$\frac{kv(t)}{F_A} = 1 - e^{-kt/m}$$

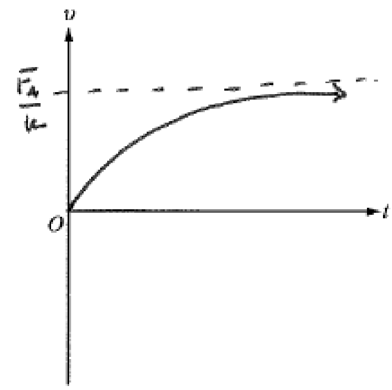
For a correct answer — 1 point

$$v(t) = \frac{F_A}{k}\left(1 - e^{-kt/m}\right)$$

Question 2 (continued)

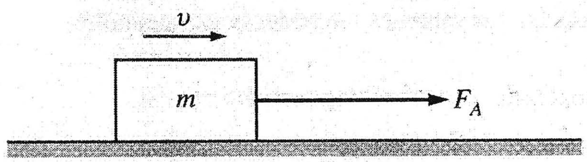
Distribution of points

(e) 3 points



For a graph that begins at the origin, with a non-negative slope everywhere, and is concave downward	1 point
For a graph with a horizontal asymptote	1 point
For the correct label of the expression for the asymptote or maximum on the vertical axis	1 point

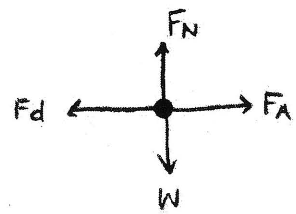
M2 A1



Mech 2.

A box of mass m initially at rest is acted upon by a constant applied force of magnitude F_A, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude kv where v is the speed of the box and k is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.

(a) The dot below represents the box. Draw and label the forces (not components) that act on the box.



(b) Write, but do not solve, a differential equation that could be used to determine the speed v of the box as a function of time t. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$\frac{dv}{dt} = a$

$F_d = kV$

$F_{net} = F_A - F_d$

$ma = F_A - kV$

$a = \frac{F_A - kV}{m}$

$$\boxed{\frac{dV}{dt} = \frac{F_A - kV}{m}}$$

(c) Determine the magnitude of the terminal velocity of the box.

$F_{net} = 0$

$F_A = F_d$

$F_A = kV$

$$\boxed{V = \frac{F_A}{k}}$$

GO ON TO THE NEXT PAGE.

(d) Use the differential equation from part (b) to derive the equation for the speed v of the box as a function of time t. Assume that $v = 0$ at time $t = 0$.

$$\frac{dV}{dt} = \frac{F_A - kV}{m} = a$$

$$\int \frac{m\,dV}{F_A - kV} = \int dt$$

$$m \int \frac{1}{F_A - kV}\, dV = t$$

$$-m \ln|F_A - kV| = t + c$$

$$-m \ln|F_A| = c$$

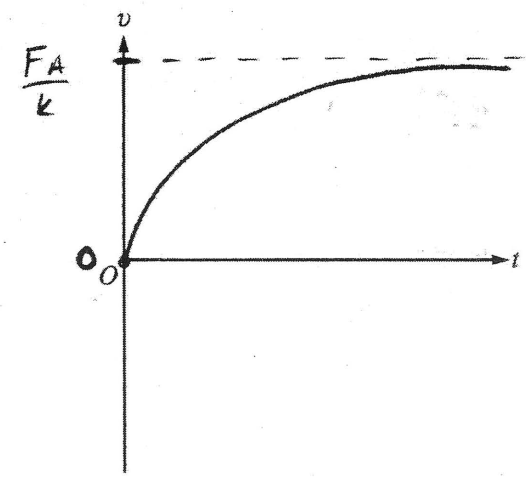
$$-m \ln|F_A - kV| = t - m\ln|F_A|$$

$$\ln|F_A - kV| = -\frac{t - m\ln|F_A|}{m}$$

$$F_A - kV = e^{-\frac{t - m\ln|F_A|}{m}}$$

$$\boxed{V = \frac{F_A - e^{-\frac{t - m\ln|F_A|}{m}}}{k}}$$

(e) On the axes below, sketch a graph of the speed v of the box as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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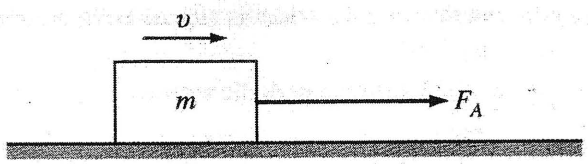
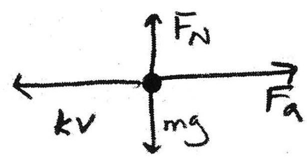


Mech 2.

A box of mass m initially at rest is acted upon by a constant applied force of magnitude F_A, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude kv where v is the speed of the box and k is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.

(a) The dot below represents the box. Draw and label the forces (not components) that act on the box.



(b) Write, but do not solve, a differential equation that could be used to determine the speed v of the box as a function of time t. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$$\Sigma F = F_A - kv = ma$$

$$\boxed{F_A - kv = m\frac{dv}{dt}}$$

(c) Determine the magnitude of the terminal velocity of the box.

At terminal velocity, $\Sigma F = 0,\ a = 0$

$$F_A - kv = 0$$

$$kv = F_a$$

$$\boxed{v_T = \frac{F_a}{k}}$$

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M2 B2

(d) Use the differential equation from part (b) to derive the equation for the speed v of the box as a function of time t. Assume that $v = 0$ at time $t = 0$.

From (b) :

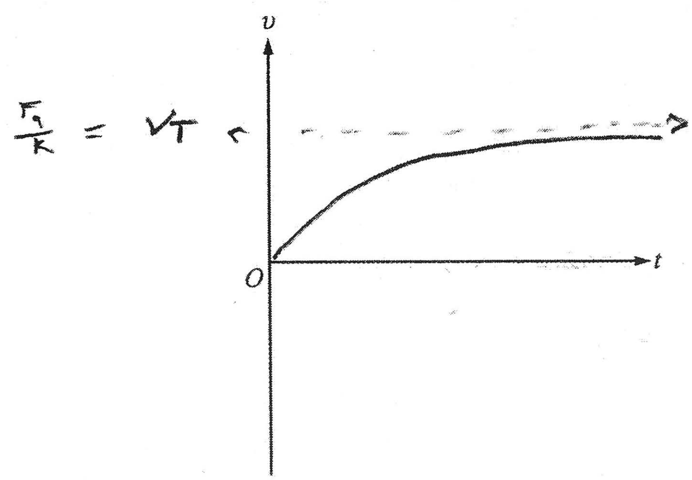
$$F_A - kv = m\frac{dv}{dt}$$

$$(F_A - kv)dt = m\,dv$$

$$\int F_A - kv\,dt = m\int dv$$

$$F_A t - \tfrac{1}{2}kv^2 = mv$$

(e) On the axes below, sketch a graph of the speed v of the box as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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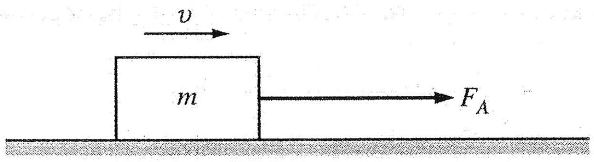

Mech 2.

A box of mass m initially at rest is acted upon by a constant applied force of magnitude F_A, as shown in the figure above. The friction between the box and the horizontal surface can be assumed to be negligible, but the box is subject to a drag force of magnitude kv where v is the speed of the box and k is a positive constant. Express all your answers in terms of the given quantities and fundamental constants, as appropriate.

(a) The dot below represents the box. Draw and label the forces (not components) that act on the box.

$KV \longleftarrow \bullet \longrightarrow Fa$

(b) Write, but do not solve, a differential equation that could be used to determine the speed v of the box as a function of time t. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

$$\frac{dx}{dt} = \frac{m\frac{dv}{dt}}{K}$$

$F = ma$

$a = \frac{dv}{dt}$

(c) Determine the magnitude of the terminal velocity of the box.

$Fa = kV$

$$V_{Terminal} = \frac{Fa}{K}$$

Terminal velocity is when the two forces are equal.

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(d) Use the differential equation from part (b) to derive the equation for the speed v of the box as a function of time t. Assume that $v = 0$ at time $t = 0$.

$$\frac{dx}{dt} = \frac{m\frac{dv}{dt}}{K}$$

$$\boxed{k\frac{dx}{dt} = m\frac{dv}{dt}}$$

$$d = \cancel{v_0 t}^{\,0} + \frac{1}{2}at^2$$

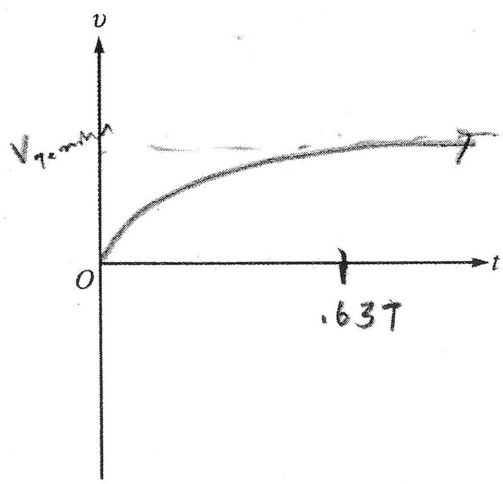
$$v = F_a - kv$$

$$a = \frac{v}{t}$$

$$\cancel{v = v_0 + 2ad}$$

$$\cancel{v = v_0(1 - e^{-t})}$$

(e) On the axes below, sketch a graph of the speed v of the box as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



The box will accelerate and then as the box gets faster, the drag force also gets stronger ultimately making it reach a terminal velocity

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Question 2

Overview

This question assessed students' ability to develop and solve a mechanical problem by using differential equations. Part (a) is a free-body diagram with four forces. Part (b) uses Newton's second law to create the appropriate differential equation for the motion of the block. Part (c) uses equilibrium conditions to determine the terminal velocity of the block. Part (d) integrates the differential equation to derive the equation for velocity of the block as a function of time. Part (e) has the graph of the motion of the block.

Sample: M2-A
Score: 14

This response is well organized and almost earned full credit. Part (a) shows a free body diagram with all four forces clearly labeled with all arrows starting on the dot and pointing outward. Part (b) uses Newton's second law to create the appropriate differential equation. Part (c) clearly demonstrates how to determine the correct terminal velocity. Part (d) does an excellent job of organizing a complicated integration to derive the equation for velocity. There is a math error, so the solution is incorrect and lost 1 point. Part (e) has the correct graph and labels the asymptote.

Sample: M2-B
Score: 11

Parts (a), (b), and (c) earned full credit. Part (d) earned 1 point for attempting to integrate the equation. The separation of variables is incorrect and there is no other work so no further credit was earned. Part (e) has the correct graph and earned full credit.

Sample: M2-C
Score: 6

Parts (a) earned 2 points for the applied and drag forces, but it is missing the weight and normal forces. No credit was earned in part (b).Full credit was earned in part (c). In part (d) there is no useful work and no credit was earned. Part (e) has the correct graph and earned full credit.