Question 3

15 points total | **Distribution of points**

(a) 2 points

For a correct expression indicating that $F_{net} = 0$ — 1 point

$2F_A - Mg = 0$

$F_A = Mg/2$

$F_A = (2.0 \text{ kg})\left(9.8 \text{ m/s}^2\right)/2$

For a correct answer — 1 point

$F_A = 9.8 \text{ N}$ (or 10 N using $g = 10 \text{ m/s}^2$)

(b) 5 points

For a correct expression of Newton's second law for translational motion — 1point

$F_{net} = Ma$

$F_A + T - Mg = Ma$ (equation 1)

For using a correct expression of torque in Newton's second law for rotational motion — 1 point

$\tau = I\alpha$

$F_A R - TR = \left(MR^2/2\right)\alpha$ or $F_A R - TR = I\alpha$

For substituting for the angular acceleration in terms of the linear acceleration $(\alpha = a/R)$ — 1 point

$F_A R - TR = \left(MR^2/2\right)(a/R)$

$F_A - T = Ma/2$ (equation 2)

For combining equations 1 and 2 to solve for the linear acceleration — 1 point

Add the two equations

$2F_A - Mg = (3/2)Ma$

$a = (2/3)((2F_A/M) - g)$

$a = (2/3)\left((2(12 \text{ N})/2.0 \text{ kg}) - 9.8 \text{ m/s}^2\right)$

For a correct answer, with units — 1 point

$a = 1.47 \text{ m/s}^2$ (1.33 m/s^2 using $g = 10 \text{ m/s}^2$)

(c) 2 points

For using the relationship between linear and angular acceleration in the equation for angular speed

$\omega = \omega_0 + \alpha t$ and $\alpha = a/R$ — 1 point

$\omega_0 = 0$, so $\omega = at/R$

$\omega = \left(1.47 \text{ m/s}^2\right)(3.0 \text{ s})/(0.10 \text{ m})$

1 point

Question 3 (continued)

Distribution of points

For an answer with units, consistent with previous work

$\omega = 44 \text{ rad/s}$ (40 rad/s using $g = 10 \text{ m/s}^2$)

(d) 4 points

Express the change in mechanical energy as the sum of the change in potential energy and the change in kinetic energy

$\Delta E = \Delta U_g + \Delta K$

For a correct expression for the change in kinetic energy including both translational and rotational kinetic energy — 1 point

$\Delta K = \frac{1}{2}M\left(v^2 - v_0^2\right) + \frac{1}{2}I\left(\omega^2 - \omega_0^2\right)$

For a correct expression for the change in potential energy, including a correct expression of the height h in terms of the time — 1 point

$\Delta U_g = Mgh = Mg\left(\frac{1}{2}at^2\right)$

v_0 and ω_0 are zero, so $\Delta E = Mg\left(\frac{1}{2}at^2\right) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

For simplifying the expression using the relationship between linear velocity and angular velocity — 1 point

$\Delta E = Mg\left(\frac{1}{2}at^2\right) + \frac{1}{2}M(R\omega)^2 + \frac{1}{2}\left(MR^2/2\right)\omega^2$

$\Delta E = \frac{1}{2}Mgat^2 + \frac{3}{4}MR\omega^2$

For correctly substituting given values and answers from previous parts into a correct expression — 1 point

$\Delta E = \frac{1}{2}(2.0 \text{ kg})\left(9.8 \text{ m/s}^2\right)\left(1.47 \text{ m/s}^2\right)(3.0 \text{ s})^2 + \frac{3}{4}(2.0 \text{ kg})(0.10 \text{ m})^2(44 \text{ rad/s})^2$

$\Delta E = 159 \text{ J}$ (144 J using $g = 10 \text{ m/s}^2$)

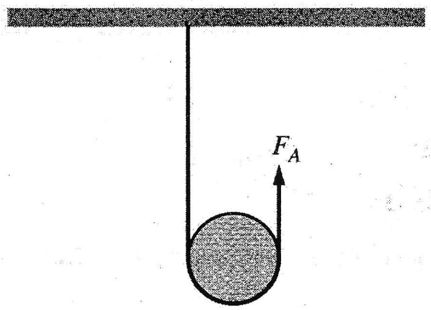
(e) 2 points

For selecting "Less than" — 1 point

For a correct justification — 1 point

<u>Example</u>

The rotational inertia of a hoop is greater than that of a solid disk of the same mass and radius, therefore the acceleration of the hoop would be less.

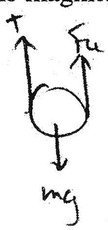


Note: Figure not drawn to scale.

Mech 3.

A disk of mass $M = 2.0$ kg and radius $R = 0.10$ m is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force F_A. The rotational inertia of the disk around its center is $MR^2/2$.

(a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.



$\tau = I\alpha$

$\tau_{net} = 0$

$TR - F_u R = 0$

$T = F_u$

$T + F_u - mg = 0$

$2F_u = mg$

$F_u = \frac{mg}{2} \rightarrow F_u = \frac{2\cdot 9.8}{2} = 9.8N$

At time $t = 0$, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

(b) Calculate the linear acceleration of the disk.

$F_a R - TR = \frac{1}{2}MR^2\alpha$

$F_u - T = \frac{1}{2}MR\frac{a}{R}$

$F_u - T = \frac{ma}{2}$

$F_u - (ma + mg - F_u) = \frac{ma}{2}$

$F_u - ma - mg + F_u = \frac{ma}{2}$

$2F_u - mg = \frac{3}{2}ma$

$2(12) - 2(9.8) = \frac{3}{2}(2)a$

$T + F_u - mg = ma$

$T = ma + mg - F_u$

$a = \frac{22}{15}\ m/s^2 = 1.466 m/s^2$

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(c) Calculate the angular speed of the disk at $t = 3.0$ s.

Constant a → Constant α

$a = R\alpha$

$\frac{a}{R} = \alpha$ $\alpha = \frac{44}{3}$ rad/sec^2

$\omega_f - \omega_i = \alpha t$

$\omega_i = 0$ $\omega_f = (\frac{44}{3}$ rad/s$^2)(3) = 44$ rad/sec

(d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0$ s.

@ t = 0, System is in static equilibrium at t = 0

No energy!

@ t = 3, gained KE_R KE_T and U_g

$v_2 = \left(\frac{22}{15}\right)(3) = \frac{22}{5}$ m/s

$KE_T = \left(\frac{22}{5}\right)^2\left(\frac{1}{2}\right)(2) = 14.36$ J

$KE_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(44)^2$

$= 9.68$ J

Mech Energy!

$14.36 + 9.68 + 129.36$

$= 158.4$ J

$\frac{v_2^{\,2}}{2a} = \Delta x$

$\Delta x = \frac{\left(\frac{22}{5}\right)^2}{2\left(\frac{22}{15}\right)} = 6.6\text{ m} \rightarrow$

$U_g = mgh$

$U_g = 2(9.8)(6.6) = 129.36$ J

(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

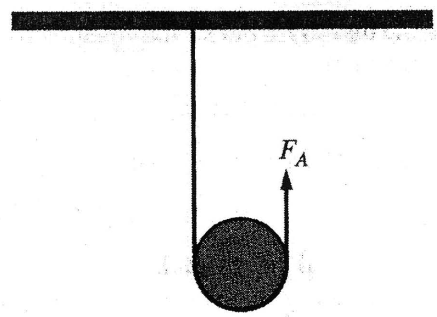
__ Greater than ✓ Less than __ The same as

Justify your answer.

$I_D = \frac{1}{2}MR^2$

$I_H = MR^2$

Because the moment of inertia of a hoop is greater than a disk The energy required to rotate the hoop will be greater, thus less energy will be available to contribute to the linear acceleration of the object. All in all, less translational energy means smaller acceleration.



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Mech 3.

A disk of mass $M = 2.0$ kg and radius $R = 0.10$ m is supported by a rope of negligible mass, as shown above. The rope is attached to the ceiling at one end and passes under the disk. The other end of the rope is pulled upward with a force F_A. The rotational inertia of the disk around its center is $MR^2/2$.

(a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.

$$F_A = \frac{W}{2} = \frac{2.0(9.8)}{2} = 9.8 \text{ N}$$

At time $t = 0$, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

(b) Calculate the linear acceleration of the disk.

$$I\frac{|a|}{R} = (F_A - mg)R$$

$$|a| = \frac{(F_A - Mg)\cancel{R^2}}{\frac{1}{2}M\cancel{R^2}} = \frac{2(F_A - Mg)}{M} = 7.6 \text{ m/s}^2$$

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(c) Calculate the angular speed of the disk at $t = 3.0$ s.

$\omega = \alpha t = \frac{a}{R} t = 228 \ \frac{rad}{s}$

(d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0$ s.

$W = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 + mgh$

$= \frac{1}{2}(2.0)(7.6 \cdot 3) + \frac{1}{2}\left[(2.0)(.10)^2/2\right](228) + (2.0)(9.80)\left(\frac{1}{2}(7.6)(3)^2\right)$

$= 694$ J

(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

__ Greater than ✓ Less than __ The same as

Justify your answer.

With an increased moment of inertia, it will take more force to accelerate the object the same.

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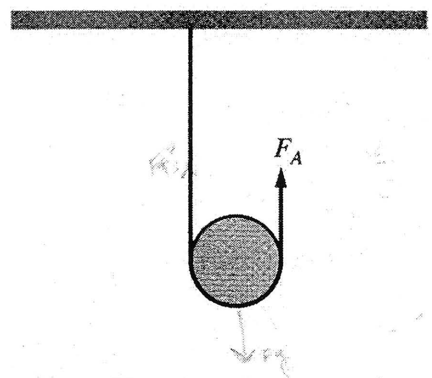

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Mech 3.

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(a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.

$\Sigma F = F_{Net} = ma$

$F_A - F_g = 0$

$F_A = mg$

$F_A = 2(10)$

$\boxed{F_A = 20\text{ N}}$

At time $t = 0$, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.

(b) Calculate the linear acceleration of the disk.

$\Sigma \tau = \tau_{Net} = I\alpha$

$F \times r = (\frac{1}{2} mR^2)(\frac{a}{r})$ $a = \alpha r$

$2F \cdot R = mr^2 a$

$a = \frac{2F \cdot \cancel{R}}{mR^2}$

$a = \frac{2F}{mr}$

$a = \frac{\cancel{2}(12)}{(\cancel{2})(.1)}$

$\boxed{a = 120\text{ m/s}^2}$

GO ON TO THE NEXT PAGE.

(c) Calculate the angular speed of the disk at $t = 3.0$ s.

$\omega_F = \cancel{\omega_0}^0 + \alpha t$

$\omega_F = \left(\frac{a}{r}\right)t$

$\omega_F = \frac{120}{.1}(3)$

$\boxed{\omega_F = 3600 \text{ rad/s}}$

(d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0$ s.

$E_T = \cancel{\Delta U}^0 + \Delta K$

$E_T = \frac{1}{2} I \alpha^2$

$E_T = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\Delta\omega}{\Delta t}\right)^2$

$E_T = \frac{1}{4}(2)(.1)^2\left(\frac{3600^2}{3^2}\right)$

$\boxed{E_T = 7200 \text{ J}}$

(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

__ Greater than ✓ Less than __ The same as

Justify your answer.

$I_{hoop} = mr^2$ $I_{disk} = \frac{1}{2}mr^2$

The linear acceleration of a hoop is less than the linear acceleration of a disk since the inertia of a hoop is greater than the inertia of a disk.

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Question 3

Overview

This question assessed students' ability to apply the laws of mechanics to a rigid body that was in equilibrium or rotating while its center of mass was also in motion. Part (a) uses Newton's second law to calculate F_A at equilibrium. Part (b) combines Newton's second law for both translational and rotational motion to calculate the linear acceleration of the disk. Part (c) uses kinematics to calculate the angular speed of the disk. Part (d) determines the increase in total mechanical energy of the disk as it rises. Part (e) compares the motion of the original disk with a hoop of the same mass and radius.

Sample: M3-A
Score: 15

This response earned full credit. In part (a) Newton's second law is used calculate F_A at equilibrium. Part (b) combines Newton's second law for both translational and rotational motion as well as the relationship $a = r\alpha$ (which is valid since the rope does not slip) to calculate the linear acceleration of the disk. Part (c) uses kinematics to get the angular speed of the disk. Part (d) determines the height gained by the disk in three seconds and uses this to calculate the increase in potential energy of the disk. It then correctly accounts for the increase in both linear and rotational kinetic energy as well as the increase in potential energy when determining the total increase in mechanical energy of the disk. "Less than" is correctly selected in part (e) and a clear and correct explanation is given.

Sample: M3-B
Score: 10

Part (a) earned full credit. Part (b) earned 1 point for using $a = r\alpha$. Part (c) is done correctly with an answer consistent with the acceleration from part (b) and earned full credit. Part (d) accounted for both rotational and translational kinetic energy, calculated the change in height and potential energy, and plugged it into the equation, but did not square both velocities and lost one point. Full credit was earned in part (e).

Sample: M3-C
Score: 6

One point was earned in part (a) for setting the net force equal to zero. Newton's second law for translational motion was not used in part (b) and is incorrect for rotational motion. One point was earned for using $a = r\alpha$. Part (c) is done correctly with an answer consistent with the acceleration from part (b) and earned full credit. There is no correct work in part (d) and credit was not earned. Part (e) earned full credit.