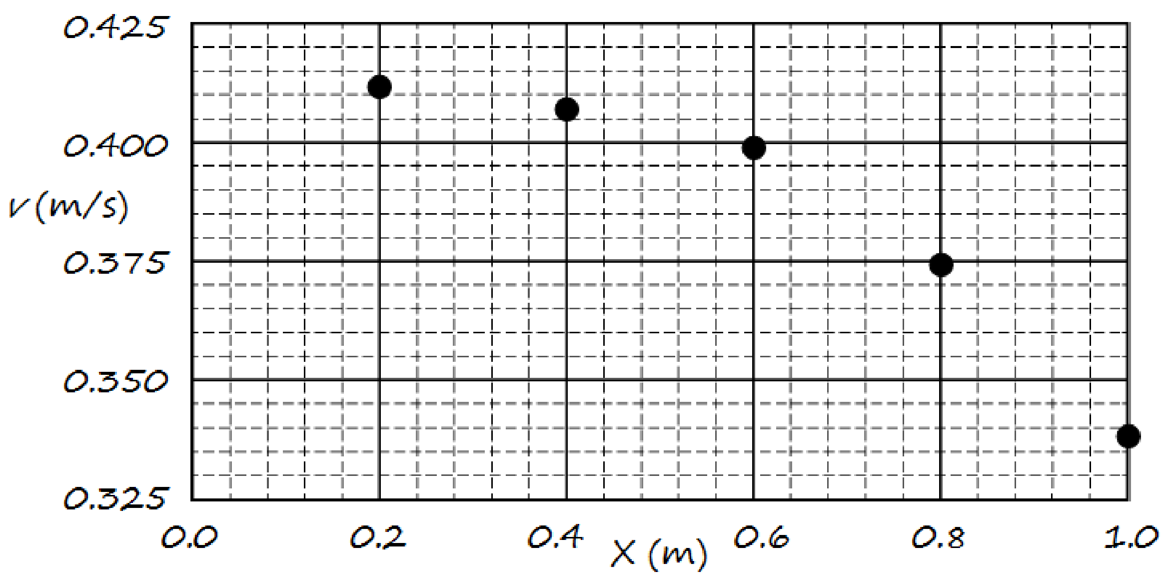
Question 1

15 points total	**Distribution of points**
(a) 2 points	
For using a correct expression for the potential energy of a spring expressed as the integral of the force	1 point
$U_S = \int F(x)dx$	
$U_S = \int_0^s \left(Ax^2 + Bx\right)dx$	
Evaluate the definite integral to get an answer with the correct magnitude.	
$U_S = \left[\frac{1}{3}Ax^3 + \frac{1}{2}Bx^2\right]_0^s$	
For any correct answer with a local minimum at $x = 0$	1 point
$U_S = \frac{1}{3}As^3 + \frac{1}{2}Bs^2$	
Note: Full credit is given for any consistent use of sign, since the guidelines are concerned with the magnitude. Full credit is also given if there is a constant term added to the correct expression.	
(b) i. 3 points	
For any statement of conservation of mechanical energy	1 point
$U_S = K$	
For using the expression for potential energy from part (a) and a correct expression for kinetic energy	1 point
$\frac{1}{3}As^3 + \frac{1}{2}Bs^2 = \frac{1}{2}mv^2$	
Solve for v^2	
$v^2 = 2\left(\frac{1}{3}As^3 + \frac{1}{2}Bs^2\right)\Big/m$	
For correct substitution	1 point
$v^2 = 2\left(\left(\frac{1}{3}\right)\left(200\,\text{N/m}^2\right)(0.040\text{ m})^3 + \left(\frac{1}{2}\right)(150\,\text{N/m})(0.040\text{ m})^2\right)\Big/(0.30\text{ kg})$	
$v^2 = 0.828\ \text{m}^2/\text{s}^2$	
$v = 0.91\ \text{m/s}$	

Question 1 (continued)

	Distribution of points
(b) continued	
ii. 3 points	
For using a correct expression for impulse in terms of change in velocity	1 point
$J = \Delta p = m(v_2 - v_1)$	
For substitution of values consistent with the answer in part (b)i	1 point
$J = (0.30\ \text{kg})(0.91\ \text{m/s} - 0)$	
$J = 0.27\ \text{kg}\bullet\text{m/s}$ or $\text{N}\bullet\text{s}$	
Units 1 point	
For correct units in both answers in part (b)	1 point

(c) 3 points



For correctly labeling both axes with variables and units	1 point
For correctly scaling both axes with scales that are linear and such that the curvature in the data is apparent	1 point
For correctly plotting the data points	1 point

Question 1 (continued)

	Distribution of points
(d) i. 2 points	
For stating that the measured initial speed of the cart is greater than the predicted value	1 point
For correctly identifying a source of error regarding the initial speed of the cart	1 point

Examples:

The student compressed the spring more than was determined. This would lead to more potential energy in the spring and greater kinetic energy for the cart. The cart would therefore move faster than predicted.

The table is not level, sloping downward would result in a greater measured speed.

The constants A and B for the spring are not accurate. The true values are larger than what is given. This would lead to smaller predicted potential energy of the spring and a smaller predicted value for the kinetic energy of the cart. Therefore, the cart would move faster than predicted

ii. 2 points

For correctly identifying the trend	1 point
For a correct physical explanation for the cart slowing down	1 point

Examples:

Friction in the axles and air resistance against the cart are slowing it down.

The track is not perfectly level and the cart is going uphill. This is slowing down the cart.

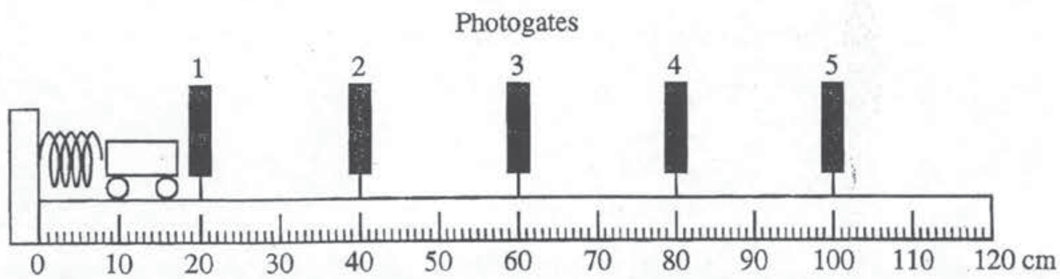
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

$$U = -W_s$$

$$U = -\int_0^s (As^2 + Bs)\,ds$$

$$U = -\left[\frac{As^3}{3} + \frac{Bs^2}{2}\right]$$

GO ON TO THE NEXT PAGE.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \text{N/m}^2$ and $B = 150\ \text{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

i. The speed of the cart immediately after it loses contact with the spring

$$\frac{(200)(.040)^3}{3} + \frac{(150)(.040)^2}{2} = \frac{1}{2}(.3)V^2$$

$$V = .91 \tfrac{m}{s}$$

ii. The impulse given to the cart by the spring

$$J = m\Delta V$$

$$= .30(.91)$$

$$J = .273$$

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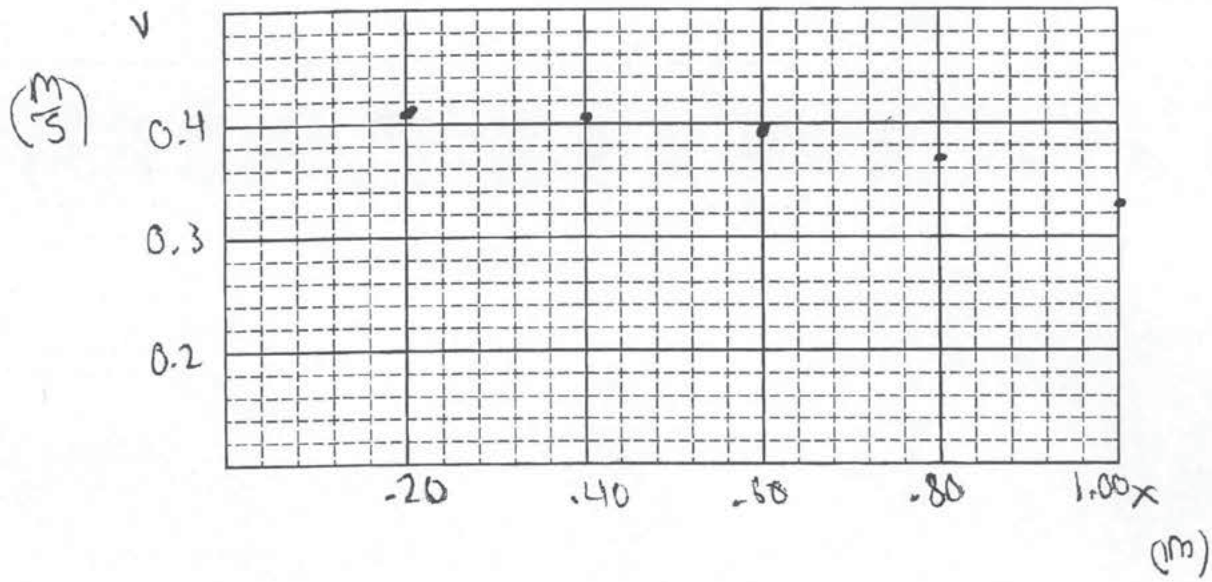
In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \text{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \text{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

(c) On the axes below, plot the data points for the <u>speed</u> v of the cart as a function of <u>position</u> x. Clearly scale and label all axes, as appropriate.



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(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

$.320\frac{m}{s}$ $.412\frac{m}{s}$

The student pushes the cart back too far causing the resulting impulse from the spring to be greater.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

a frictional force causes an acceleration that opposes the carts vel. causing it to slow down.

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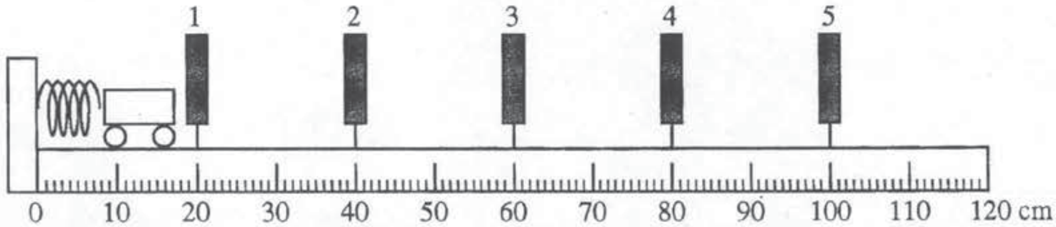

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(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

$F_s = kx$

$U = \frac{1}{2}kx^2 = \frac{1}{2}F_s \cdot x$

$\therefore U = \frac{1}{2}(As^2 + Bs) \cdot s$

$\boxed{U = \frac{As^3 + Bs}{2}}$

GO ON TO THE NEXT PAGE.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \text{N/m}^2$ and $B = 150\ \text{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible <u>only</u> during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

i. The speed of the cart immediately after it loses contact with the spring

$U = K$

$\frac{As^3 + Bs}{2} = \frac{1}{2}mv^2$

$\frac{200(0.04)^3 + 150(0.04)}{2} = \frac{1}{2}(0.3)v^2$

$\boxed{v = 4.477\ \text{m/s}}$

ii. The impulse given to the cart by the spring

$J = \Delta p = m\Delta v = (0.3)(4.477 - 0) = \boxed{1.343\ \text{kg}\cdot\text{m/s}}$

GO ON TO THE NEXT PAGE.

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of ± 0.002 m/s. The positions are measured with a precision of ± 0.005 m.

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Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
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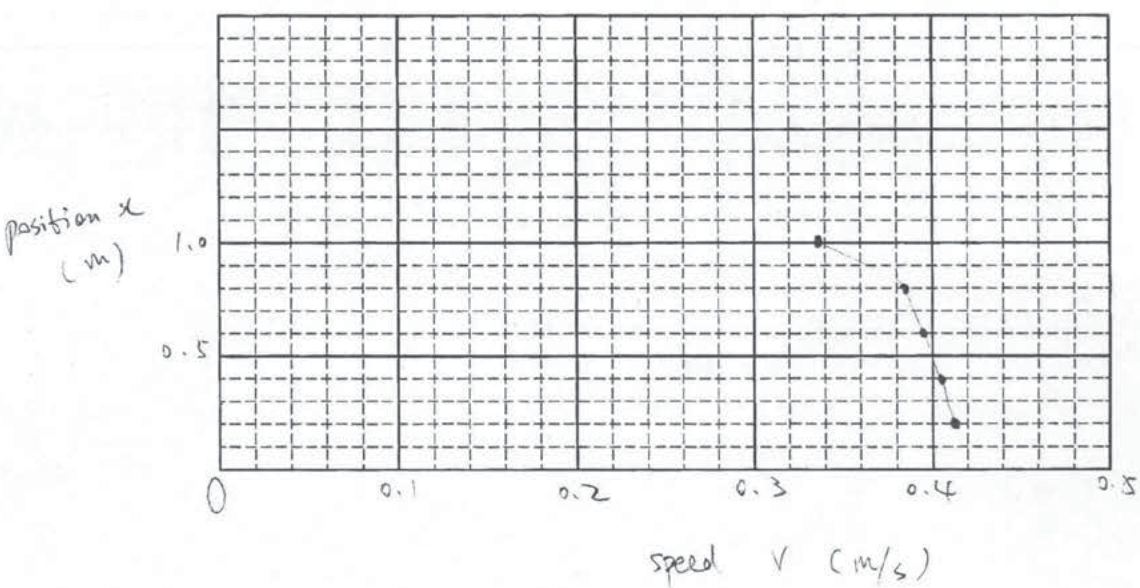
(c) On the axes below, plot the data points for the speed v of the cart as a function of position x. Clearly scale and label all axes, as appropriate.



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(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

$0.412 - 0.320 = 0.092$ m/s

① Human measurement errors

② Air resistance

③ friction.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

When the cart is more closer to lose the contact with the spring, the speed of the cart decreases.

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PHYSICS C: MECHANICS
SECTION II
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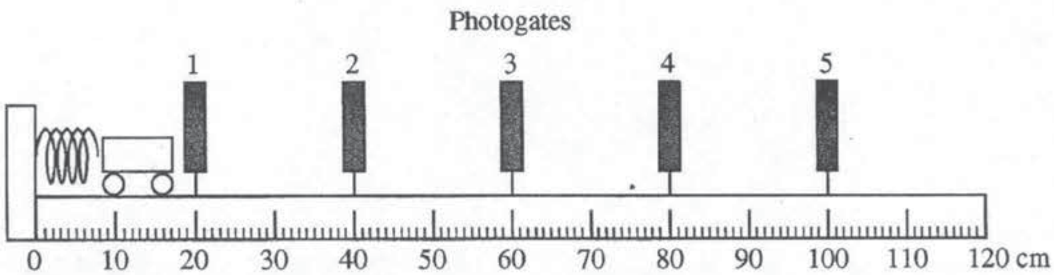

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(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

$\frac{1}{2} k x^2$

$\frac{1}{2} k s^2$

$\boxed{\frac{1}{2}(A+B) s^2 = U}$

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In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \text{N/m}^2$ and $B = 150\ \text{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

i. The speed of the cart immediately after it loses contact with the spring

$\frac{1}{2} 350 (.04)^2 = .28$

$.28 = \frac{1}{2}(.3)v^2$

$\boxed{V = 1.3663\ m/s}$

ii. The impulse given to the cart by the spring

ΔKE

$\frac{1}{2}(.3)(1.36)^2 = \boxed{0.279\ J}$

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In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \text{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \text{m/s}$. The positions are measured with a precision of ± 0.005 m.

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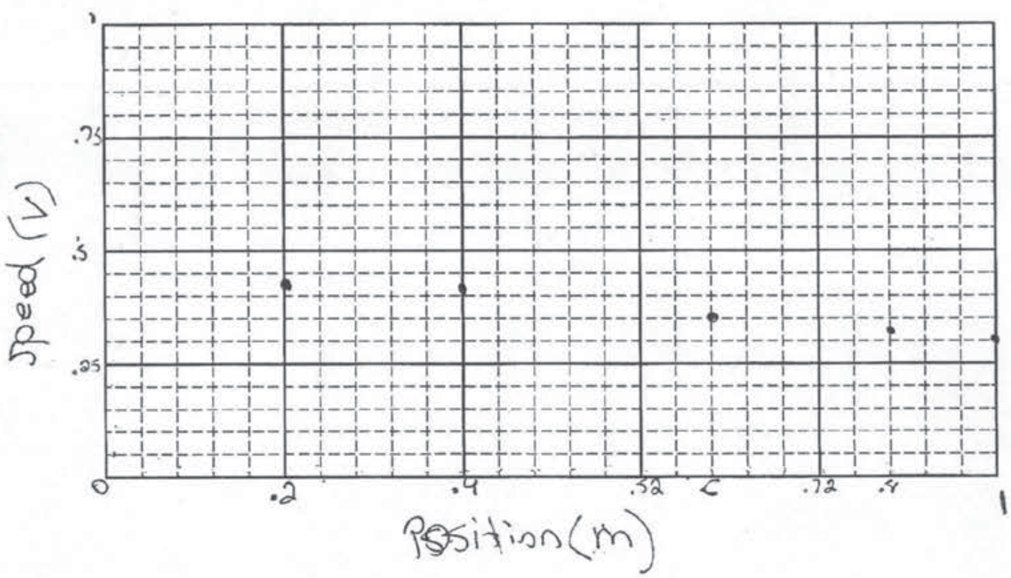

(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference. friction

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.

there is a nonlinear negative acceleration meaning friction is not equal throught

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Question 1

Overview

This question assessed understanding of the relationships between force, energy, and impulse, as well as application of conservation of energy. In addition, the analysis of data was used to evaluate the students' understanding of the system and ability to provide physical explanations for the results.

Sample: M1 A
Score: 14

The only point lost by this response was the units point in part (b), since there are no units on the answer for impulse. Writing the two values of speed in part (d)(i) was minimal but does acknowledge their difference.

Sample: M1 B
Score: 9

The work in part (a) has no attempt to evaluate an integral and earned no credit. Part (b) earned full credit, including the units point, since the incorrect answer from (a) was used correctly. One point was lost in part (c) for having the variables on the wrong axes. Part (d)(i) earned 1 point for the numerical comparison, but the explanation was not correct for an actual speed that is greater than the predicted one. Part (d)(ii) earned no credit, since the speed of the cart decreases as it moves farther from the point of losing contact.

Sample: M1 C
Score: 5

The work in part (a) has no attempt to evaluate an integral and earned no credit. Part (b)(i) earned full credit. The expression from (a) was correctly used to calculate a potential energy, and then set equal to a correct expression for kinetic energy. While the final answer does not follow from the work, all the substitutions are correct. Part (b)(ii) has nothing related to impulse and earned no credit. The units for impulse are wrong, so the units point was not earned. Parts (c) and (d)(i) also earned no credit. In part (c), "v" does not designate a unit, the horizontal scale is not linear, and the last three points are clearly not plotted at the correct speeds. Part (d)(ii) earned full credit.