Question 3

15 points total	**Distribution of points**
(a) 2 points	
For using a correct kinematics expression for the vertical motion	1 point
$y - y_0 = v_1 t + \frac{1}{2}at^2$	
$h = 0 + \frac{1}{2}gt^2$	
For a correct answer	1 point
$t = \sqrt{2h/g}$	
(b) 3 points	
For a statement of conservation of momentum or Newton's third law	1 point
$p_i = p_f$	
For substituting the momentum of the stone into a correct expression for conservation of momentum	1 point
For substituting the momentum of the person-disk system into a correct expression for conservation of momentum	1 point
$0 = m_1 v_1 + m_2 v_2$	
$0 = \left(\frac{m}{20}\right)(v_0) + \left(m + \frac{m}{2}\right)v$	
$\frac{3}{2}mv = -\frac{1}{20}mv_0$	
$v = -\frac{1}{30}v_0$	
Note: Since the question asks for speed, the negative sign is not needed. There is no penalty for including it.	
(c) 3 points	
For using a correct expression of Newton's second law with friction as the net force	1 point
$f = ma$	
$\mu mg = ma$	
$a = \mu g$	
For correctly substituting the velocity from part (b) and the acceleration into an appropriate kinematics equation	1 point
$v_2 = v_1 + at$	
$0 = -\frac{1}{30}v_0 + \mu gt$	
For an answer consistent with part (b)	1 point
$t = \frac{v_0}{30\mu g}$	

Question 3 (continued)

	Distribution of points
(c) (continued)	
Alternate Solution	*Alternate points*
For using a correct expression for impulse and change in momentum, with friction as the force	*1 point*
$Ft = \Delta p = m(v_2 - v_1)$	
$-\mu mgt = m(v_2 - v_1)$	
For correct substitutions	*1 point*
$-\mu mgt = m\left(0 - \frac{1}{30}v_0\right)$	
For an answer consistent with part (b)	*1 point*
$t = \frac{v_0}{30\mu g}$	
(d) 4 points	
For a statement of conservation of total angular momentum	1 point
$L_i = L_f$	
$L = mrv$ for linear motion	
$L = I\omega$ for rotation	
For substituting the angular momentum of the stone into a correct expression of conservation of angular momentum	1 point
For substituting the angular momentum of the person into a correct expression of conservation of angular momentum	1 point
For substituting the angular momentum of the disk into a correct expression of conservation of angular momentum	1 point

$$0 = m_s r_s v_s + I_D\omega_D + I_P\omega_P$$

$$0 = \left(\frac{m}{20}Rv_0\right) - \left(\frac{mR^2}{2}\omega + \frac{m}{2}R^2\omega\right)$$

$$\left(\frac{m}{20}Rv_0\right) = \left(\frac{mR^2}{2}\omega + \frac{m}{2}R^2\omega\right) = mR^2\omega$$

$$\omega = \frac{\frac{m}{20}Rv_0}{mR^2}$$

$$\omega = \frac{v_0}{20R}$$

Question 3 (continued)

Distribution of points

(e) 3 points

For selecting "Less than"	1 point
For correctly identifying a decrease in the stone's angular momentum	1 point
For correctly identifying a decrease in the person's rotational inertia	1 point

Example: If the stone is thrown from a point closer to the center of the disk, its angular momentum around the center of the disk decreases, resulting in a decrease of the angular momentum gained by the disk/person system. In addition, the person's rotational inertia is decreased, which decreases the rotational inertia of the disk-person system. The effect of this decrease in the rotational inertia of the person is less than the effect of the decrease in the angular momentum. Therefore, the disk/person system must have a decrease in its angular speed.

Example: From angular momentum conservation, we have

$$L_{stone} = L_{disk+person}$$

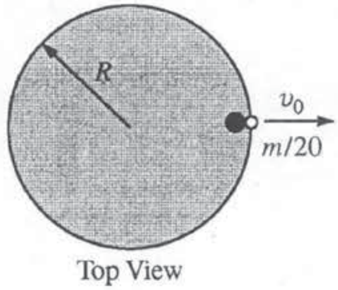
$$L_{stone} = \left(I_{disk} + I_{person}\right)\omega$$

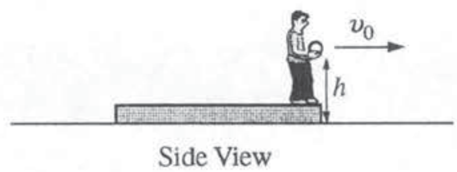
$$\boxed{m_{stone}v_0 r = \left(\frac{mR^2}{2} + \frac{m}{2}r^2\right)\omega}$$

where r is the distance of the person from the center of the disk. If r is decreased from R to $R/2$, then the stone's rotational inertia (left-hand-side of the boxed equation) is reduced to half of its previous value. The person's rotational inertia is reduced to one quarter of its previous value, but the combined disk+person rotational inertia (included in right-hand-side of boxed equation) is still greater than half of its previous value. Solving for ω therefore yields a value that is less than it was previously.

Note: If neither the stone nor the person are explicitly mentioned, one point may still be earned for the justification. If just one of either the stone or person are explicitly mentioned, both points for the justification may be earned.

Note: Indicating a decrease in the torque is equivalent to indicating a decrease in the change in angular momentum.





Mech. 3.

A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

$$\Delta x = V_0 t + \tfrac{1}{2}at^2$$
$$h = 0 + \tfrac{1}{2}gt^2$$
$$\frac{2h}{g} = t^2$$
$$t = \sqrt{\frac{2h}{g}}$$

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown. $P_i = P_f$ (conservation of momentum)

$$0 = \frac{m}{20}(V_0) + \left(m + \frac{m}{2}\right)s$$
$$0 = \frac{\cancel{m}}{20}(V_0) + \frac{3\cancel{m}}{2}(s)$$
$$\frac{1}{20}V_0 = \frac{3}{2}(s)$$
$$\frac{1}{30}V_0 = s$$

speed of disk and person $= \frac{1}{30}V_0$

(c) Derive an expression for the time it will take the disk to stop sliding.

$$F_{net} = F_f$$
$$\cancel{\left(m + \frac{m}{2}\right)}a = \mu \cancel{\left(m + \frac{m}{2}\right)}g$$
$$a = \mu g$$
$$V_f = V_i - at$$

$$0 = \frac{1}{30}V_0 - \mu g t$$
$$\mu g t = \frac{1}{30}V_0$$
$$\boxed{t = \frac{V_0}{30\mu g}}$$

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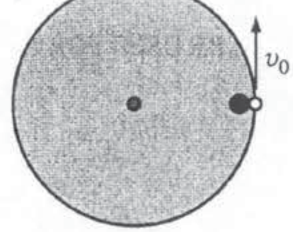

Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

$Li = Lf \qquad V = WR \quad W = \frac{V_0}{R}$

$0 = \left(\frac{M}{20}\right)R^2 W_1 + \frac{1}{2}MR^2 W_2 + \left(\frac{M}{2}\right)R^2 W_2$

$0 = \left(\frac{M}{20}\right)R^{\cancel{2}}\left(\frac{V_0}{\cancel{R}}\right) + mR^2 W_2$

$0 = \frac{\cancel{m} V_0 R}{20} + \cancel{m}R^2 W$

$\frac{V_0 \cancel{R}}{20} = R^{\cancel{2}} W$

$\boxed{W = \frac{V_0}{20R}}$

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

____ Greater than ✓ Less than ____ Equal to

Justify your answer.

$0 = \left(\frac{M}{20}\right)\left(\frac{R}{2}\right)^2\left(\frac{V_0}{R}\right) + \frac{1}{2}mR^2 W_1 + \left(\frac{M}{2}\right)\left(\frac{R}{2}\right)^2 W_2$

$0 = \left(\frac{M}{20}\right)\left(\frac{R^{\cancel{2}}}{4}\right)\left(\frac{V_0}{\cancel{R}}\right) + \frac{1}{2}mR^2 W_2 + \frac{1}{4}mR^2 W_2$

$0 = \frac{\cancel{m} R V_0}{80} + \frac{3}{4}\cancel{m}R^2 W_2$

$\frac{3}{4}R^{\cancel{2}} W_2 = \frac{\cancel{R} V_0}{80}$

$RW_2 = \frac{V_0}{60}$

$\boxed{W = \frac{V_0}{60R}}$

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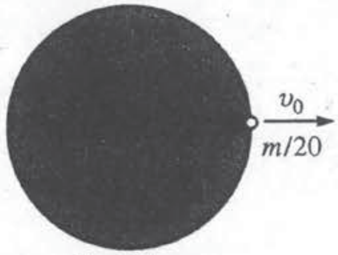

Top View

Side View

Mech. 3.

A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

$$\Delta y = V_0 t + \frac{1}{2} a t^2$$

$$h = 0 + \frac{1}{2} g t^2$$

$$\boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

$$\left(m + \frac{m}{2} + \frac{m}{20}\right) 0 = \frac{m}{20} V_0 + \left(m + \frac{m}{2}\right) V_{dp}$$

$$V_{dp} = -\frac{m V_0}{20\left(m + \frac{m}{2}\right)}$$

$$= \frac{-m V_0}{20m + 10m} = \boxed{\frac{-V_0}{30}}$$

(c) Derive an expression for the time it will take the disk to stop sliding.

$$-\left(m + \frac{m}{2}\right) g \mu = \left(m + \frac{m}{2}\right) a$$

$$a = -g\mu$$

$$0 = V_{dp} + at$$

$$\frac{V_0}{30} = at$$

$$\boxed{\frac{V_0}{30 g \mu} = t}$$

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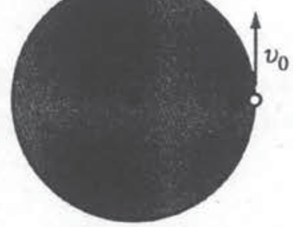

Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

$$\frac{1}{2}\left(\frac{m}{20}\right)v_0^2 = \frac{1}{2}I\omega^2$$

$$\frac{\cancel{m}v_0^2}{40} = \frac{\cancel{m}R^2}{4}\omega^2$$

$$\frac{v_0^2}{10R^2} = \omega^2$$

$$\boxed{\omega = \frac{v_0}{R\cdot\sqrt{10}}}$$

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d)?

____ Greater than ✓ Less than ____ Equal to

Justify your answer.

Torque given is now less. Torque is greater when the radius is greater ($\Sigma\tau = r\times F$). Since R is cut in half, the torque, and hence, the ω will be less.

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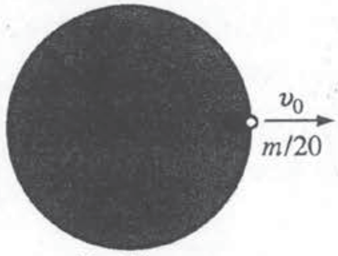

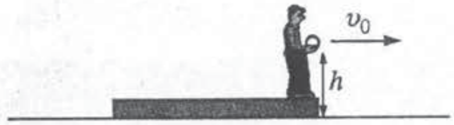

Mech. 3.

A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

$V_{0x} = v_0$ $\quad V_{0y} = 0$

$V_x = ?$ $\quad V_y = 0$

$a_x = 0$ $\quad a_y = 9.81$

$\Delta x = ?$ $\quad \Delta y = h$

$t = ?$

$\Delta y = \frac{1}{2}at^2$

$\boxed{t = \sqrt{2hg}}$

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

$\Delta p = \Delta p$

$\left(\frac{m}{2}\right)(v_0) = \left(\frac{m}{2} + m\right)V$

$\boxed{\dfrac{\left(\frac{m}{2}v_0\right)}{\frac{m}{2} + m} = V}$

(c) Derive an expression for the time it will take the disk to stop sliding.

$v = v_0 + at$

$\frac{v}{t} = a$

$F = ma$

$\mu N = \frac{3}{2}ma$

$\mu \frac{3}{2}mg = \frac{v}{t}\frac{3}{2}m$

$\frac{\mu g}{v} = \frac{1}{t}$

~~$\frac{1}{2}mv^2 = \mu N \Delta x = 0$~~

$\frac{v}{\mu g} = t$

$\boxed{\dfrac{\frac{\frac{m}{2}v_0}{\frac{m}{2}+m}}{\mu g} = t}$

$\Delta x =$

$v = 0$

$v_0 = V$

$a =$

$\Delta x = ?$

$t = ?$

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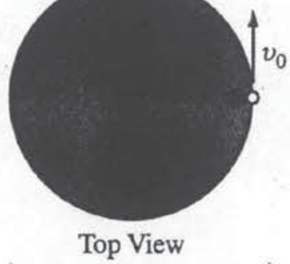

Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

~~p=p~~

$$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$$

$$\frac{mR^2}{2}\omega^2 = mv^2$$

$$\omega = \sqrt{\frac{2v^2}{R^2}}$$

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

____ Greater than   ____ Less than   X Equal to

Justify your answer.

Angular speed does not matter how far away from the center you are.

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Question 3

Overview

This question dealt with interactions between two objects and the related conservation of linear and angular momentum. It also included aspects of projectile motion, friction, and moment of inertia.

Sample: M3 A
Score: 15

This response earned full credit. Part (e) uses a mathematical justification.

Sample: M3 B
Score: 10

The first three parts earned full credit. Part (d) uses energy instead of angular momentum and earned no credit. Part (e) does not mention the rotational inertia of the person, so it earned only 2 points.

Sample: M3 C
Score: 6

Part (a) earned 1 point for a correct expression for the vertical motion but made a mistake in solving for the time. Part (b) lost 1 point because the momentum of the stone was not correct. Part (c) earned full credit, since the previous incorrect answer was used correctly. Part (d) uses energy instead of angular momentum and earned no credit. Part (e) also earned no credit.