Question 1

15 points total | **Distribution of points**

(a)

i. 1 points

Using Newton's second law with down the incline as the positive direction

$F_{net} = ma$

$mg\sin\theta = ma$

For a correct expression of a positive acceleration — 1 point

$a = g\sin\theta$

ii. 2 points

Using a correct kinematics equation to solve for velocity

$v_2 = v_1 + at$

For substitution into a correct kinematics equation consistent with the acceleration from part (a)i — 1 point

For having a negative sign on v_0 — 1 point

$v = -v_0 + (g\sin\theta)t$

iii. 1 points

Using a correct kinematics equation to solve for position

$$x = x_0 + v_1 t + \frac{1}{2}at^2$$

For substitution into a correct kinematics equation consistent with expressions from parts (a)i and (a)ii — 1 point

$$x = D - v_0 t + \frac{1}{2}(g\sin\theta)t^2$$

(b) 2 points

Using an equation that can be solved for the closest position to the sensor

$v_2^2 = v_1^2 + 2ad$

For substitution into a correct kinematic equation consistent with part (a) — 1 point

For setting v_2 to zero and using D for the initial position — 1 point

$$0 = v_0^2 + 2(g\sin\theta)(x - D)$$

$$x = D - \frac{v_0^2}{2g\sin\theta}$$

Question 1 (continued)

Distribution of points

(b) (continued)

Alternate solution: *Alternate points*

Using a conservation of energy approach to find the highest point the cart moves along the incline

$$K_1 + U_{g1} = K_2 + U_{g2}$$

$$K_1 = U_{g2}$$

$$\frac{1}{2}mv_0^2 = mgh_2$$

For using the correct energy statement with the correct initial velocity *1 point*

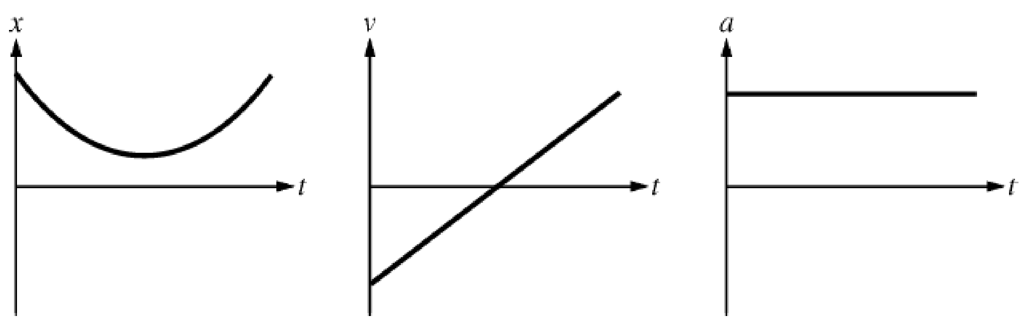
For a correct statement of the height of the cart along the incline *1 point*

$$h = (D - x)\sin\theta$$

$$\frac{1}{2}v_0^2 = g(D - x)(\sin\theta)$$

$$x = D - \frac{v_0^2}{2g\sin\theta}$$

(c) 4 points



For a position graph that is a parabola that does not cross the t-axis and has a vertex that does not touch the t-axis — 1 point

For a velocity graph that is a straight line and crosses the t-axis — 1 point

For an acceleration graph that is a horizontal line — 1 point

For a set of graphs that are consistent with each other — 1 point

Question 1 (continued)

Distribution of points

(d) 2 points

Using an equation that can be solved for the distance

$v_2^2 = v_1^2 + 2ad$

For a correct expression of the frictional force — 1 point

$f = -\mu_k mg = ma$

$a = -\mu_k g$

$0 = v_0^2 - 2\mu_k gd$

For a correct answer — 1 point

$d = \dfrac{v_0^2}{2\mu_k g}$

Alternate solution: — *Alternate points*

Using an equation that can be solved for the distance

$Fd = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$

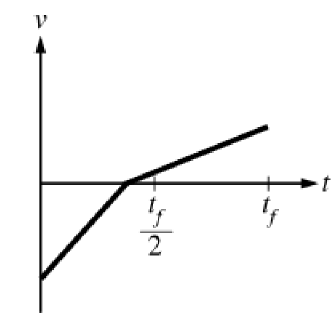
For a correct expression of the frictional force — *1 point*

$-\mu_k mgd = \frac{1}{2}m\left(0 - v_0^2\right)$

For a correct answer — *1 point*

$d = \dfrac{v_0^2}{2\mu_k g}$

(e) 3 points



The graph has two straight line portions.

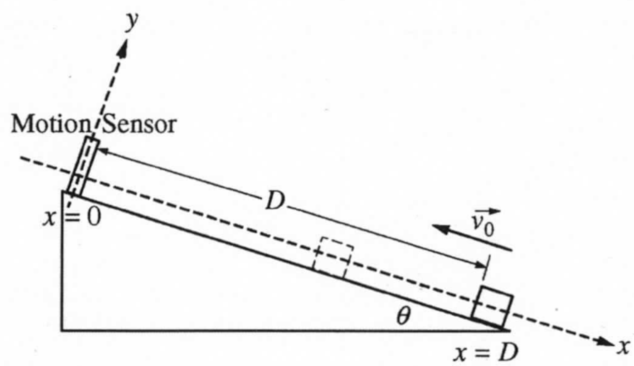
For having a change in slope at $v = 0$ — 1 point

For having slope values of each segment that have the same sign and the correct relative magnitudes (segment I slope magnitude greater than segment II slope magnitude, as shown in the graph above) — 1 point

For having a graph that crosses the t-axis earlier than $t_f/2$ and extends to t_f — 1 point

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech.1.

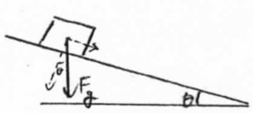
A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

i. Determine the acceleration a of the block.

Acceleration is only in the x-direction

$F_x = F_g \sin\theta$



$$a = \frac{F_x}{m} = \frac{\cancel{m} g \sin\theta}{m} = g \sin\theta$$

ii. Determine an expression for the velocity v of the block.

Note that initial velocity is $-v_0$

$$v = -v_0 + \int_0^t a\,dt$$

$$= -v_0 + at$$ (constant acceleration)

$$= -v_0 + gt\sin\theta$$

(x-direction)

GO ON TO THE NEXT PAGE.

iii. Determine an expression for the position x of the block.

$$x = x_0 + \int_0^t -V_0 + gt\sin\theta \, dt \qquad x_0 = D$$

$$= x_0 - V_0 t + \frac{1}{2} g t^2 \sin\theta \qquad \text{(constant acceleration)}$$

$$= D - V_0 t + \frac{1}{2} g t^2 \sin\theta$$

(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

At x_{min}, the block's velocity is zero

$$-V_0 + gt\sin\theta = 0$$

$$t = \frac{V_0}{g\sin\theta}$$

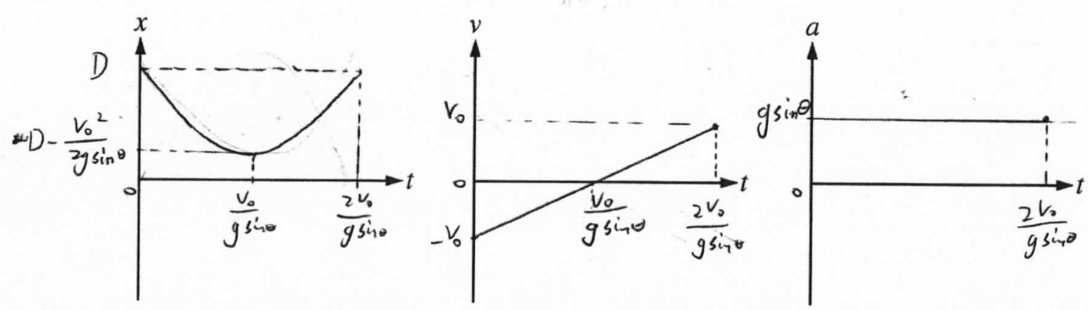
substitute this in the expression for x

$$x = D - V_0 \cdot \frac{V_0}{g\sin\theta} + \frac{1}{2} g \left(\frac{V_0}{g\sin\theta}\right)^2 \sin\theta$$

$$= D - \frac{V_0^2}{g\sin\theta} + \frac{1}{2}\frac{V_0^2}{g\sin\theta}$$

$$= D - \frac{V_0^2}{2g\sin\theta}$$

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



Question 1 continues on next page.

GO ON TO THE NEXT PAGE.

(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

$$F_N = mg$$

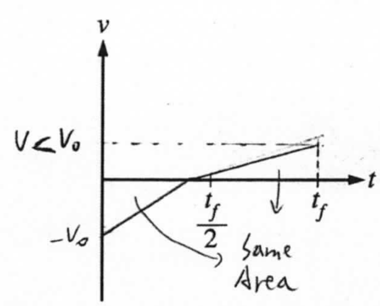
Friction $F_{f} = -\mu_k mg$

$$a = \frac{\mu_k mg}{m} = \mu_k g$$

$$\cancel{V_f^2} = V_i^2 - 2a\Delta d$$

$$\Delta d = \frac{V_0^2}{2\mu_k g}$$

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

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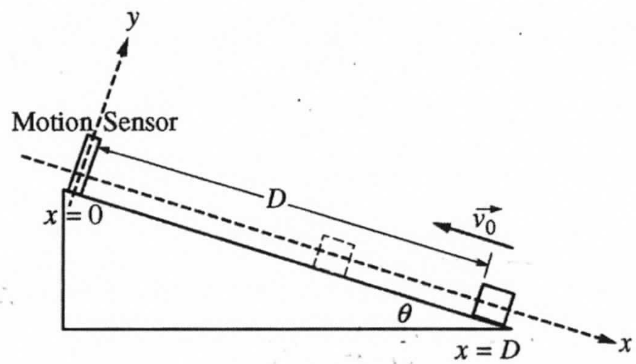


Mech.1.

A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ and physical constants, as appropriate.

i. Determine the acceleration a of the block.

$$\Sigma F = mg\sin\theta = ma$$

$$a \boxed{= g\sin\theta}$$

ii. Determine an expression for the velocity v of the block.

$$V = V_0 + at \quad \boxed{V = -V_0 + (g\sin\theta)(t)}$$

GO ON TO THE NEXT PAGE.

iii. Determine an expression for the position x of the block.

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$\boxed{x = D - v_0 t + \left(\tfrac{1}{2}g\sin\theta\right)\left(t^2\right)}$$

(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

$$v^2 = v_0^2 + 2a\Delta x$$

$$0 = v_0^2 + 2a\Delta x$$

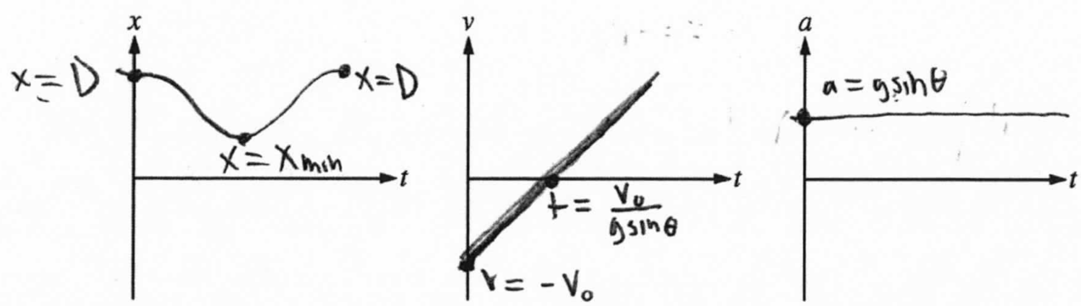
$$v_0^2 = 2g\sin\theta\,\Delta x$$

$$\Delta x = \frac{v_0^2}{2g\sin\theta}$$

$$x_{min} = D - \Delta x$$

$$\boxed{x_{min} = D - \frac{v_0^2}{2g\sin\theta}}$$

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



Question 1 continues on next page.

GO ON TO THE NEXT PAGE.

(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

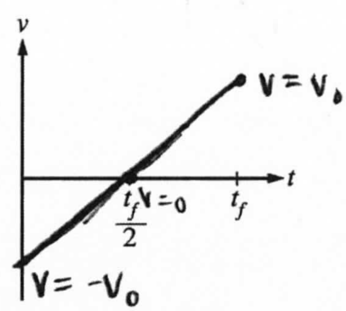
initial $v = v_0$

$$\frac{1}{2}mv_0^2 = \mu_k F_N (d_s)$$

$$\frac{1}{2}\cancel{m}v_0^2 = \mu_k \cancel{m}g(d_s)$$

$$\boxed{d_s = \frac{\frac{1}{2}v_0^2}{\mu_k g}}$$

(e) Suppose the ramp now has friction. The same block is projected up with the same initial speed v_0 and comes back down the ramp. On the axes provided below, sketch a graph of the velocity v as a function of time t for the motion of the block while it goes up and back down the ramp, arriving at the bottom of the ramp at time t_f. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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MQ1 C1

PHYSICS C: MECHANICS
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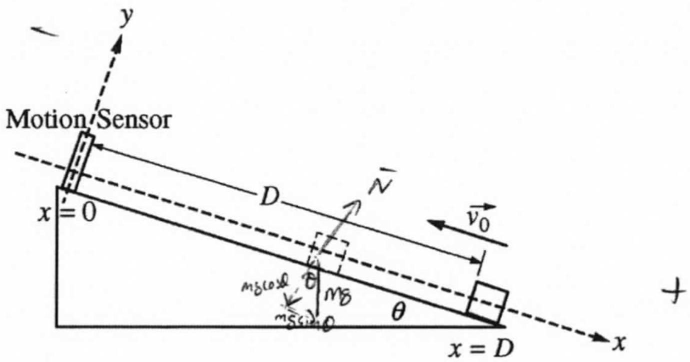
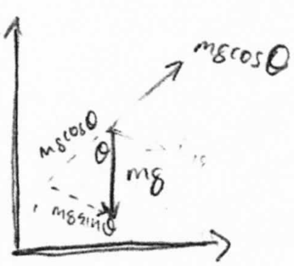


Mech.1.

A block of mass *m* is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance *D* from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time *t* after it has been projected up the ramp. Express your answers in terms of *m*, *D*, v_0, *t*, θ and physical constants, as appropriate.

i. Determine the acceleration *a* of the block.



$$a = \frac{\Sigma F}{m}$$

$$a = \frac{\cancel{m} g \sin\theta}{m}$$

$$a = g \sin\theta$$

ii. Determine an expression for the velocity *v* of the block.

$$v = at + v_0$$

$$v = g\sin\theta\, t + v_0$$

GO ON TO THE NEXT PAGE.

iii. Determine an expression for the position x of the block.

$$\int v(t) = \int -(g \sin\theta\, t + v_0)$$

$$x(t) = -\int g \sin\theta\, t + v_0$$

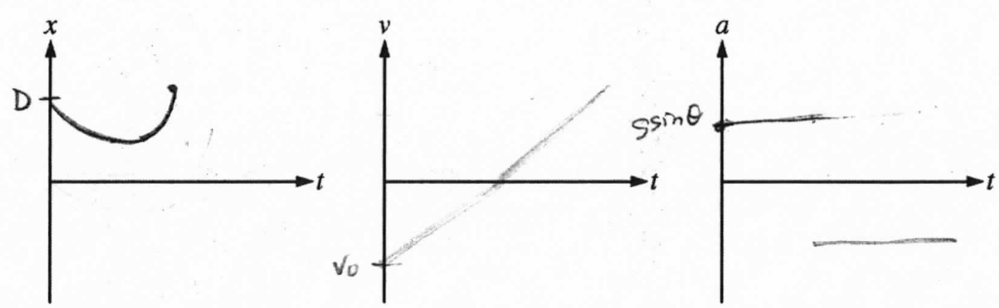
$$x(t) = -\left(\frac{g \sin\theta}{2} t^2 + v_0 t\right) + C$$

$x_0 = D$

(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

X_{min}

(c) On the axes provided below, sketch graphs of position x, velocity v, and acceleration a as functions of time t for the motion of the block while it goes up and back down the ramp. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

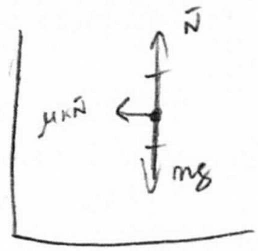


Question 1 continues on next page.

GO ON TO THE NEXT PAGE.

(d) After the block slides back down and leaves the bottom of the ramp, it slides on a horizontal surface with a coefficient of friction given by μ_k. Derive an expression for the distance the block slides before stopping. Express your answer in terms of m, D, v_0, θ, μ_k, and physical constants, as appropriate.

$\mu_k \bar{N} = F_{KF}$



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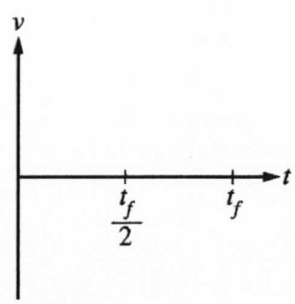

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Question 1

Overview

The question required kinematics, Newton's laws, and energy considerations to describe the motion of a block on an inclined plane with friction, using equations and a graph.

Sample: MQ1 A
Score: 15

The solutions in this full-credit response are clear and well organized. In part (a) the student correctly uses calculus derivations instead of beginning with standard kinematics equations. In part (b) the equations derived in the previous two parts are correctly used to arrive at the answer.

Sample: MQ1 B
Score: 10

Parts (a)(i), (a)(ii), and (a)(iii) of this response earned the full 4 points. All of the responses are completely correct. Part (b) of this response also earned the full 2 points credit. Part (c) only earned 2 points. The position graph appears to be more of a trigonometric function (sine or cosine) at the edges of the curve, which resulted in losing the point for that graph. Since this graph cannot lead to linear graphs for velocity and acceleration, the point for consistency between all three graphs was also not earned. Part (d) of this response earned both points. Part (e) did not earn any points, since the sketch does not have two linear segments of different slopes, and it crosses the time axis at $t_f/2$.

Sample: MQ1 C
Score: 5

Part (a)(i) of this response earned 1 point. Part (a)(ii) also earned 1 point. The correct kinematic expression is used and the acceleration value from part (a)(i) is substituted into the expression, however, the sign on the initial velocity is not correct. Part (a)(iii) of this response did earn 1 point. The expression from (a)(ii) is correctly used, and the initial position D is noted. Part (b) earned no points. Part (c) earned 2 points. The position and velocity graphs earned credit. However, the points for the acceleration graph and consistency were not earned. Part (d) of this response earned no points. The response begins by analyzing using friction, but the response does not have enough evidence of writing a correct expression for frictional force with the symbols required by the prompt. Part (e) of this response is blank and earned no points.