AP® PHYSICS C: MECHANICS
2015 SCORING GUIDELINES

Question 2

15 points total	**Distribution of points**
(a) 1 point	
Writing an equation to solve for the speed when the dart is at its maximum height $v = v_x = v_0(\cos\theta)$ $v = (10\ \text{m/s})(\cos 30°)$	
For a correct answer $v = 8.7\ \text{m/s}$	1 point
(b) 2 points	
Writing an equation to solve for time using motion in the vertical direction $v_y = v_{y0} + a_y t$ $0 = (10\ \text{m/s})(\sin 30) + \left(-9.8\ \text{m/s}^2\right)t$	
For a correct value for the time $t = 0.51\ \text{s}$ (or $t = 0.50\ \text{s}$ if using $g = 10\ \text{m/s}^2$)	1 point
For substituting into an equation for the horizontal motion consistent with the speed from part (a), or for determining the correct answer $x = v_x t$ $x = (8.7\ \text{m/s})(0.51\ \text{s})$ $x = 4.4\ \text{m}$	1 point
(c) 3 points	
For a correct expression of conservation of momentum $p_i = p_f$	1 point
For a correct expression that represents a totally inelastic collision between the dart and the block $m_1 v_{1i} = (m_1 + m_2)v_f$ $(0.020\ \text{kg})(8.66\ \text{m/s}) = (0.020\ \text{kg} + 0.10\ \text{kg})v_f$	1 point
For an answer consistent with the speed from part (a) and correct mass substitutions $v = 1.4\ \text{m/s}$	1 point

Question 2 (continued)

Distribution of points

(d) 3 points

For a correct expression of conservation of energy — 1 point

$$K_1 + U_{g1} = K_2 + U_{g2}$$

$$\frac{1}{2}mv_1^2 = mgh_2$$

For a correct expression for the height reached by the block — 1 point

$$h = L - L(\cos\theta)$$

For substituting the speed value from part (c) into a correct conservation of energy equation — 1 point

$$\frac{1}{2}mv_1^2 = mgL(1 - \cos\theta)$$

$$\cos\theta = 1 - \frac{v_1^2}{2gL}$$

$$\cos\theta = 1 - \frac{(1.44\ \text{m/s})^2}{2\left(9.8\ \text{m/s}^2\right)(1.2\ \text{m})}$$

$$\theta = 24^\circ$$

(e) 2 points

For substituting the correct length into the correct equation for the period — 1 point

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{(1.2\ \text{m})}{\left(9.8\ \text{m/s}^2\right)}} = 2.2\ \text{s}$$

For correctly dividing the period in half to solve for the time — 1 point

$$t = T/2 = (2.2\ \text{s})/2$$

$$t = 1.1\ \text{s}$$

(f)

i. 2 points

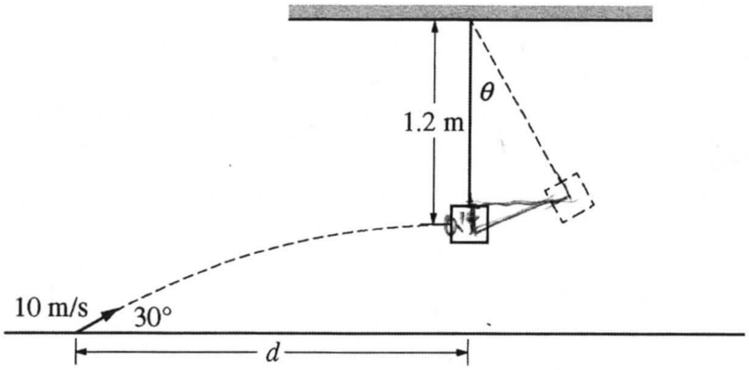
For selecting "Increase" — 1 point

For a correct justification of the larger angle for the block-dart system — 1 point

Example: A more massive dart would cause the speed after the collision with the block to increase. A greater speed after the collision would cause the block to reach a greater height and thus the angle θ would increase.

Question 2 (continued)

	Distribution of points
(f) (continued)	
ii. 2 points	
For selecting “Stay the same”	1 point
For a correct justification	1 point
Example: A more massive dart would not affect the period of the pendulum. Only changing the length of the string would change the period.	
Note: If the student correctly points out the changes to the simple pendulum could be outside the small angle approximation, then the student’s entire answer will be considered (both check box and justification are consistent and physically correct).	



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

$V_{top} = ?$

$V_{top,y} = 0\,m/s$ because top of projectory

$V_{top,x}$ only speed: $V_0 \cos\theta = 10\,m/s\,(\cos 30°) = \boxed{8.66\,m/s}$

(b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

$d = ?$ $\quad V_{top,y} = 0, \quad V_{0,y} = V_0 \sin 30°$

$d = V_x(t)$ $\quad \frac{V_{0y}}{g} = t$

$d = 8.66\,m/s \times 0.51\,s$ $\quad t = \frac{V_0 \sin 30°}{g} = 0.51\,s$

$d = \boxed{4.41\,m}$

(c) Calculate the speed of the block just after the dart strikes.

$V_{block} = ?$

cons. of angular momentum:

$L_1 = L_f$

$rmV = I\omega, \quad \omega = \frac{V}{r}, \quad M = m_{dart} + M_{block}$

$\cancel{r}mv = MR^{\cancel{2}}\left(\frac{V}{\cancel{R}}\right)$

$m_{dart}(V_0) = (M_{dart} + M_{block})(V_f)$

$10\,m/s\,(0.02) = (0.02 + 0.1)(V_f)$

$V_f = \boxed{1.67\,m/s}$

GO ON TO THE NEXT PAGE.

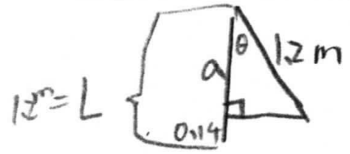
(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

$\theta = ?$

$\frac{1}{2}mv^2 = mgh$

$\frac{1}{2}(V_f)^2 = gh$ $\quad \frac{1}{2}(1.67 m/s)^2 = 9.81 m/s (h)$

$h = 0.14m$



$\cos\theta = \frac{a}{L}$

$\cos\theta = \frac{1.2m - h}{1.2m}$

$\cos\theta = \frac{1.2 - 0.14m}{1.2m}$

$\theta = \boxed{28.2°}$

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position. $Time = T/2$

$T = 2\pi\sqrt{\frac{L}{g}}$ simple pendulum

$\frac{T}{2} = \pi\sqrt{\frac{L}{g}}$

$\frac{T}{2} = \pi\sqrt{\frac{1.2}{9.81}} \Rightarrow T/2 = \boxed{1.1s}$

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

_____ Increase   __X__ Decrease   _____ Stay the same

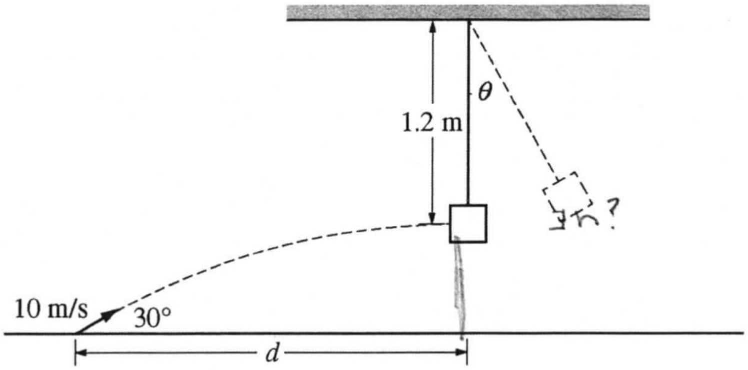
Justify your answer. because the final velocity would be less and thus a smaller angle.

ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

_____ Increase   _____ Decrease   __X__ Stay the same

Justify your answer.

$T = 2\pi\sqrt{\frac{L}{g}}$ the period would stay the same because the length and g stays the same despite change in mass



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

$y_{max} = v_y = 0$ $v_f = v_1 - at$ $t = 0.509$

$0 = 10\sin 30° - 9.81t$

$v_x = (10\cos 30°)$

$\rightarrow \boxed{8.66 \text{ m/s}}$

(b) Calculate the horizontal distance *d* between the launching point of the dart and a point on the floor directly below the block.

$t = 0.509$

$\Delta x = v_x t$

$\boxed{\Delta x = 4.414 \text{ m}}$

(c) Calculate the speed of the block just after the dart strikes.

$m_1 v_1 = (m_1 + m_2) v_f$

$(8.66 \text{ m/s})(.02 \text{ kg}) = (.1\text{kg} + .02\text{kg}) v_f$

$\boxed{v_f = 1.44 \text{ m/s}}$

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

$\boxed{v_i = 1.44\,m/s}$

$\sin^{-1}\left(\frac{1.44\,m/s}{9.81\,m/s^2}\right)$

$(m v_i) - mg\sin\theta = 0$ $\theta = \sin^{-1}(v/g)$

$\cancel{m}v_1 = \cancel{m}g\sin\theta$ 8 $\theta = 8.46°$

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position. $h = ?$ $(1.2\,m)\sin(8.46°)$

$v_{yf} = v_{yi}^{\,0} - gt$ $t = ?$ $v_{yf}^2 = v_{yi}^{2\,\to 0} - 2g\Delta h \to v_f = \sqrt{2g(1.2m)\sin 8.46}$

$v_{yf} = 1.861\,m/s$

$1.86\,m/s = (9.81\,m/s^2)(t)$

$\boxed{t = 0.189\,s}$

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

_____ Increase __✓__ Decrease _____ Stay the same

Justify your answer.

The greater mass of the new system would have to act on the same initial velocity, meaning less swing

ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

T

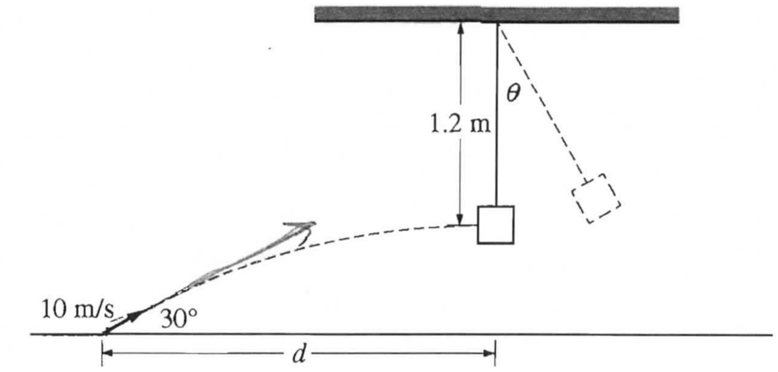
_____ Increase _____ Decrease __✓__ Stay the same $T = 2\pi\sqrt{\frac{L}{g}}$

Justify your answer.

The period of this simple pendulom does not take into account angle. only length and gravity.

GO ON TO THE NEXT PAGE.

MQ2 C1



Mech.2.

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle *θ* with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

$V_y = 0$

$V_x = 10\cos 30° = 1.54251$ m/s

(b) Calculate the horizontal distance *d* between the launching point of the dart and a point on the floor directly below the block.

x | y

$V_{f_y} = 0$ $\quad V_f = V_i + at$ $\quad (10\cos 30°)(.52)$

$0 = 10\sin 30° - 10(t)$ $\quad = 0.802108$ m

$1\cancel{0}t = 1\cancel{0}\sin 30°$

$t = 0.52$ s

(c) Calculate the speed of the block just after the dart strikes.

$\Sigma F = ma = \frac{mv^2}{r}$

$10(1.2) = V^2$

$\boxed{\sqrt{12} = V}$

$mgh + \frac{1}{2}mv^2 = mgh + \frac{1}{2}mv^2$

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

$$\frac{1}{2}mv^2 + mg\underset{0}{\cancel{h}} = \frac{1}{2}m\underset{0}{\cancel{v}}^2 + mgh$$

$$\frac{1}{2}(.02+.1)(12) = (.12)(10)(1.2 - 1.2\cos\theta)$$

$$6 = 10(1.2 - 1.2\cos\theta)$$

$$0.5 = \cos\theta$$

$$\theta = 60°$$

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

$$v_f = v_i + at$$

$$0 = \sqrt{12} + -10(t)$$

$$t = \frac{\sqrt{12}}{10} = (0.346 \text{ sec})4 = 1.39 \text{ sec}$$

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

i. Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

_____ Increase _____ Decrease [✓] Stay the same

Justify your answer.

because mass dosn't matter.

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

ii. Would the period of oscillation after the collision increase, decrease, or stay the same?

_____ Increase _____ Decrease [✓] Stay the same

Justify your answer.

because v_i and v_f would be the same.

GO ON TO THE NEXT PAGE.

AP® PHYSICS C: MECHANICS
2015 SCORING COMMENTARY

Question 2

Overview

The intent of this question was to engage the students with a classic ballistic pendulum problem. This question tested student understanding of projectile motion, inelastic collisions, conservation of energy, and simple harmonic motion.

Sample: MQ2 A
Score: 12

This paper earned 1 point for part (a) as the horizontal velocity is clearly stated. Part (b) earned both points. In part (c) the student does start incorrectly by using angular momentum, but then recovers by writing a correct statement for linear momentum. This statement earned 1 point for using conservation of momentum and 1 point for applying it to a totally inelastic collision. However, the student uses an incorrect value for the initial speed and does not get the correct final answer. Part (d) earned all 3 points, correctly using the incorrect velocity from part (c). In part (e) both points were earned. Part (f)(i) did not earn any points, as the wrong check box is marked, but part (f)(ii) earned 2 points.

Sample: MQ2 B
Score: 8

This paper earned all 3 points for the first two parts (part (a) and part (b)). Note that the student calculates the relevant time in part (a), then rewrites the number in part (b). Part (c) is completely correct and earned all 3 points. Part (d) earned no points. There is no evidence of an energy approach or recognition of the geometry involved with the pendulum arc. Part (e) also earned no points; it attempts to use kinematics, which is not useful with the pendulum since the tangential acceleration is changing with time. In part (f)(i) the student checks the incorrect checkbox and therefore earned no points. However, in part (f)(ii) the student has a correct choice and justification and earned 2 points. Note that it was not necessary to discuss the bounds of the small angle approximation for the pendulum's period to earn the justification point — only the recognition that the period is independent of mass and amplitude for a simple pendulum.

Sample: MQ2 C
Score: 6

In this paper the point in part (a) was not earned, apparently due to an error in calculation. However, in part (b) 2 points were earned. The student correctly finds the time and then correctly computes the horizontal distance by using the speed determined in part (a). Part (c) has no evidence of a conservation of momentum approach and therefore earned 0 points. Part (d) earned all 3 points. The conservation of energy and the height attained by the pendulum are correct. The value of the speed determined in part (c) (in this case $\sqrt{12}\ \text{m/s}$) is correctly substituted, so this is an example of an incorrect final answer for the angle that earned full credit. Part (e) earned 0 points as the student attempts to deal with a periodic system using kinematics. Part (f)(i) has the wrong check box marked and earned 0 points. Part (f)(ii) earned 1 point for the correct check box, but it has an incorrect justification.