Question 3

15 points total | **Distribution of points**

(a) 3 points

Writing an integral to derive the rotational inertia of the rod

$I = \int r^2 dm$

For a correct expression for *dm* — 1 point

$\lambda = M/L$, $M = \lambda L$, $dm = \lambda dr$

For using the correct limits of integration or a correct constant of integration — 1 point

$$I = \int_{r=0}^{r=L} \lambda r^2 dr$$

For correctly evaluating the integral above, leading to the answer $ML^2/3$ — 1 point

$$I = \left[\frac{\lambda r^3}{3}\right]_{r=0}^{r=L} = \frac{1}{3}\lambda\left(L^3 - 0\right) = \frac{1}{3}\left(\frac{M}{L}\right)\left(L^3\right) = \frac{1}{3}ML^2$$

(b) 4 points

For using any expression of conservation of energy — 1 point

$K_1 + U_{g1} = K_2 + U_{g2}$

For a correct energy expression relating gravitational potential energy to rotational kinetic energy — 1 point

$$mgh_1 = \frac{1}{2}I\omega_2^2$$

For correctly substituting $L/2$ for the change in height — 1 point

$$Mg(L/2) = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$$

For using $v = r\omega$ with $r = L$ to solve for the velocity of the end of the rod — 1 point

$$\frac{MgL}{2} = \frac{1}{6}ML^2\left(\frac{v}{L}\right)^2$$

$$v = \sqrt{3gL}$$

(c) 1 point

For correctly identifying a relationship between length and velocity that will result in a straight line — 1 point

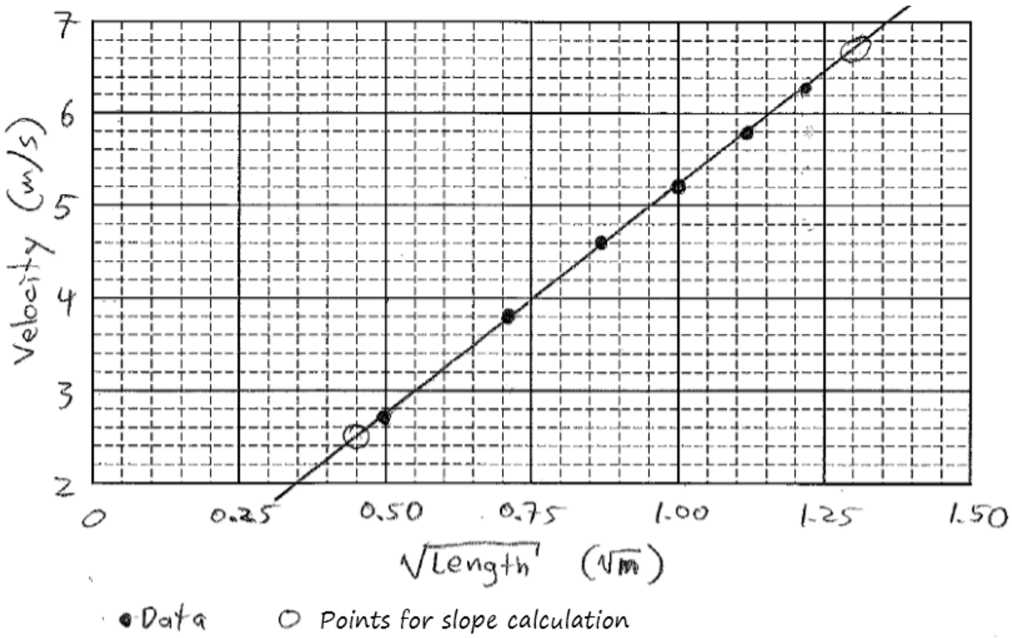
Example 1: Horizontal axis: velocity
Vertical axis: $\sqrt{\text{length}}$

Example 2: Horizontal axis: $(\text{velocity})^2$
Vertical axis: length

Note: Each of the above axis choices can also be switched to yield a straight line.

Question 3 (continued)

		Distribution of points
(d)	3 points	



For a correct scale that uses at least half the grid and for correctly labeling the axes, including units	1 point
For plotting data consistent with quantities in the data table in part (c)	1 point
For drawing a straight line consistent with the data in part (c)	1 point

(e)

i. 2 points

For correctly calculating the slope using the straight line drawn in part (d), and not using data points unless the points lie on the line — 1 point

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(6.70 - 2.50)}{(1.30 - 0.45)} = 4.94\ \sqrt{\text{m}}/\text{s}$$

For correctly calculating g using the slope — 1 point

$$m = \sqrt{3g}$$

$$g = m^2/3 = \left(4.94\ \sqrt{\text{m}}/\text{s}\right)^2/3 = 8.1\ \text{m/s}^2$$

Alternate Solution — *Alternate points*

For stating that linear regression was used and getting one of the results noted below — *1 point*

For correctly calculating g using the slope — *1 point*

When plotting velocity as a function of $\sqrt{\text{length}}$ *, the slope is* $4.94\ \sqrt{\text{m}}/\text{s}$ *and* $g = 8.14\ \text{m/s}^2$.

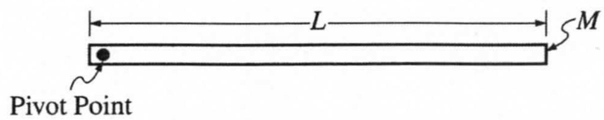
When plotting the square of velocity as a function of length, the slope is $25.77\ \text{m/s}^2$ *and* $g = 8.59\ \text{m/s}^2$.

Question 3 (continued)

	Distribution of points
(e) (continued)	
ii. 2 points	
For one example that directly decreases the effect of air resistance	1 point
For another example that directly decreases the effect of air resistance	1 point

Some examples include:

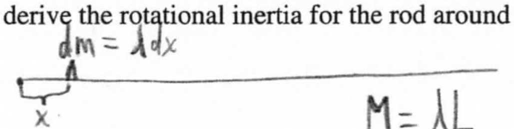
- Do the experiment in a vacuum
- Use shorter rod lengths
- Use more massive (or denser) rods
- Use a more aerodynamic shape for the rods



Mech.3.

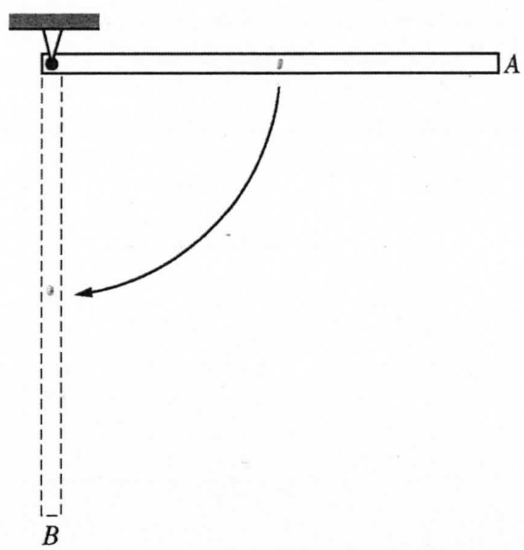
A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.



$$M = \lambda L \qquad dI = dm\,R^2$$

$$I = \int_0^L dI = \int_0^L dm\,R^2 = \int_0^L \lambda x^2\,dx = \lambda\left[\frac{x^3}{3}\right]_0^L = \frac{\lambda L^3}{3} = \frac{ML^2}{3}$$



The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.

(b) Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

$$\text{CoM} = \frac{L}{2}$$

$$\Delta PE = Mg\frac{L}{2} = \frac{1}{2}I\omega^2 \qquad Mg\Delta = \frac{ML^2}{3} \qquad \frac{3g}{L} = \frac{V^2}{L^2}$$

$$\omega = \frac{V}{L} \qquad V = \sqrt{3gL}$$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position *A* and uses photogates to measure the velocity of the free end at position *B*. The data are recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3
Square Velocity (m^2/s^2)	7.3	14	21	27	34	40.

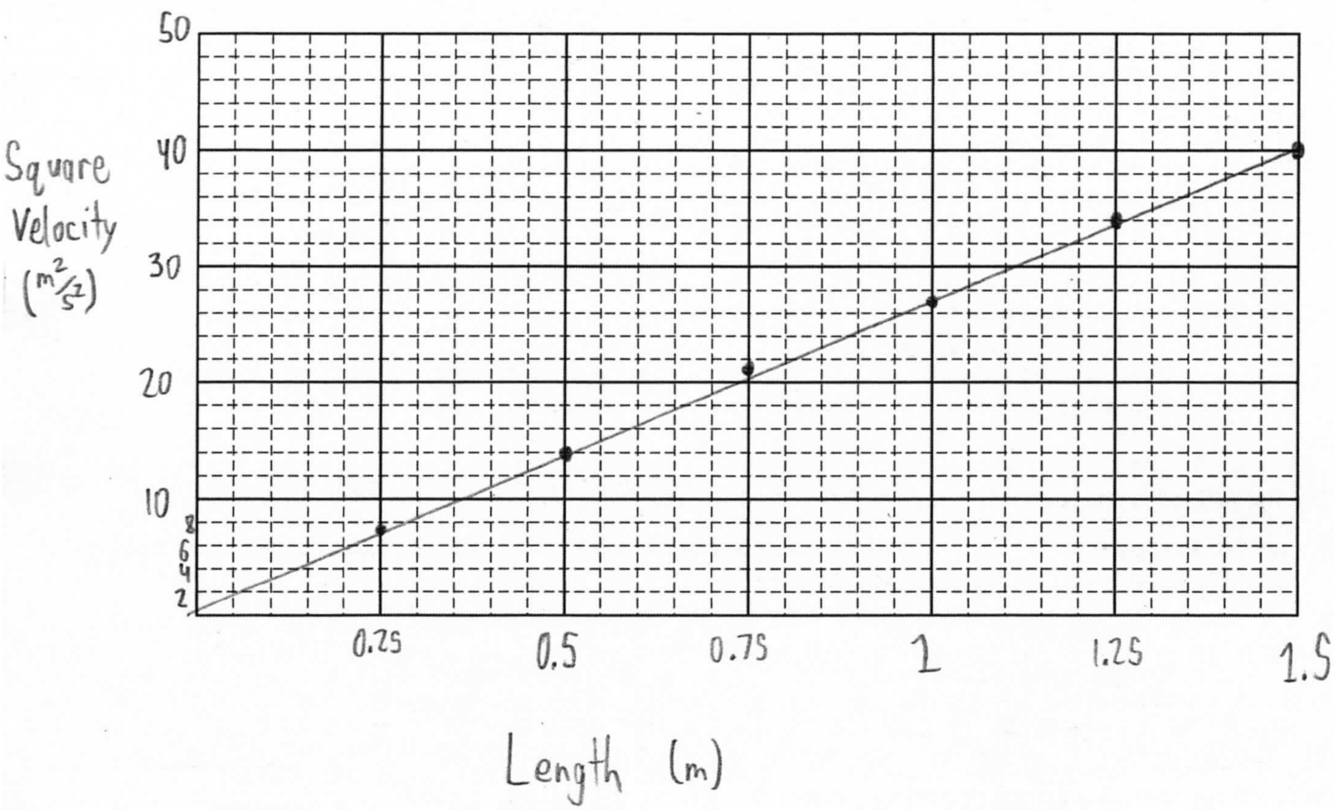
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity *g*.

Horizontal axis: Length

Vertical axis: Square Velocity

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



GO ON TO THE NEXT PAGE.

(e)

i. Using your straight line, determine an experimental value for g.

(0.75, 20)
(1.5, 40)

$v^2 = 3gL$

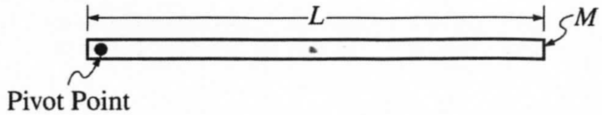
$3g = \left(\frac{20}{0.75}\right) = 26.7$

$g = 8.9$

ii. Describe two ways in which the effects of air resistance could be reduced.

Air resistance could be reduced by using a heavier rod or a thinner rod.

GO ON TO THE NEXT PAGE.



Mech.3.

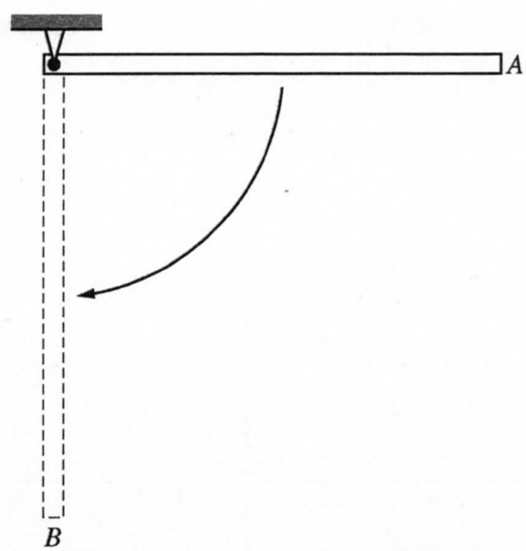
A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.

$$I = \int r^2\,dm$$

$$I = \int (L)^2\,dm$$

$$I = ML^2/3$$



The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.

(b) Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

$$K_f = U_i$$

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gL$$

$$v^2 = 2gL$$

$$V = \sqrt{2gL}$$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position *A* and uses photogates to measure the velocity of the free end at position *B*. The data are recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3
Velocity^2 (m^2/s^2)	7.29	14.44	21.16	27.04	33.64	39.69
2L (m)	.50	1.0	1.5	2.00	2.5	3.0

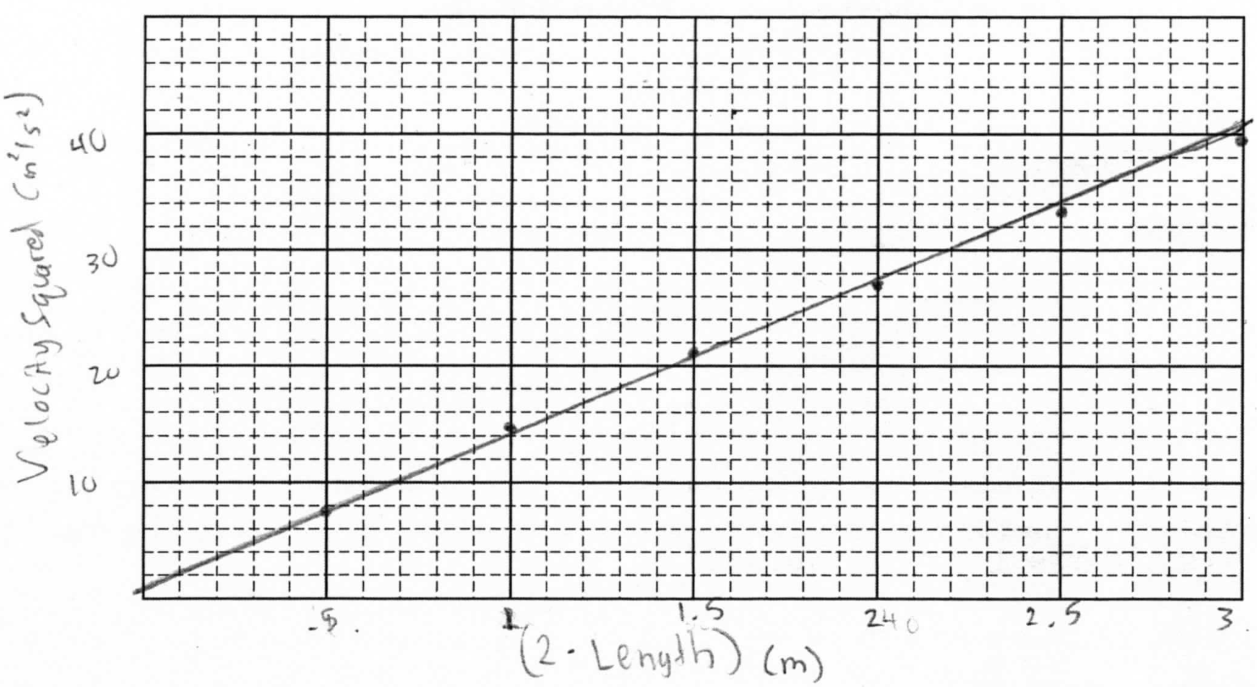
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Horizontal axis: 2 · Length

Vertical axis: Velocity Squared

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



GO ON TO THE NEXT PAGE.

(e)

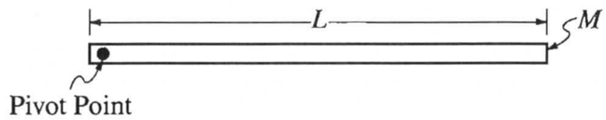
i. Using your straight line, determine an experimental value for g.

slope of line = g

$\frac{34-14}{2.5-1} = 13.333 = g$

ii. Describe two ways in which the effects of air resistance could be reduced.

Air resistance could be reduced by performing the experiment in a vacuum or



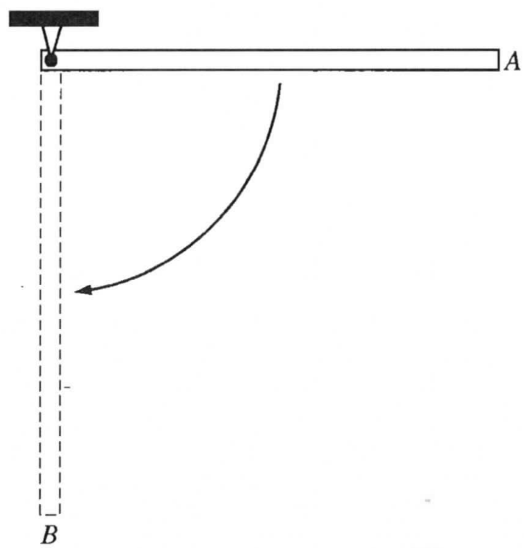
Mech.3.

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.

$$I = \int r^2 = \sum m r^2$$

$$I = \sum \frac{mL^3}{3} / L = \boxed{\frac{mL^3}{3}}$$



The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.

(b) Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

$$V = \frac{d}{t} \qquad V = \frac{\frac{2\pi r}{4}}{\frac{2\pi\sqrt{L/G}}{2}} \qquad \frac{2\pi r}{4} \cdot \frac{2}{2\pi\sqrt{L/G}} = V$$

$$\boxed{V = \frac{1}{2\sqrt{L/G}}}$$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position A and uses photogates to measure the velocity of the free end at position B. The data are recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

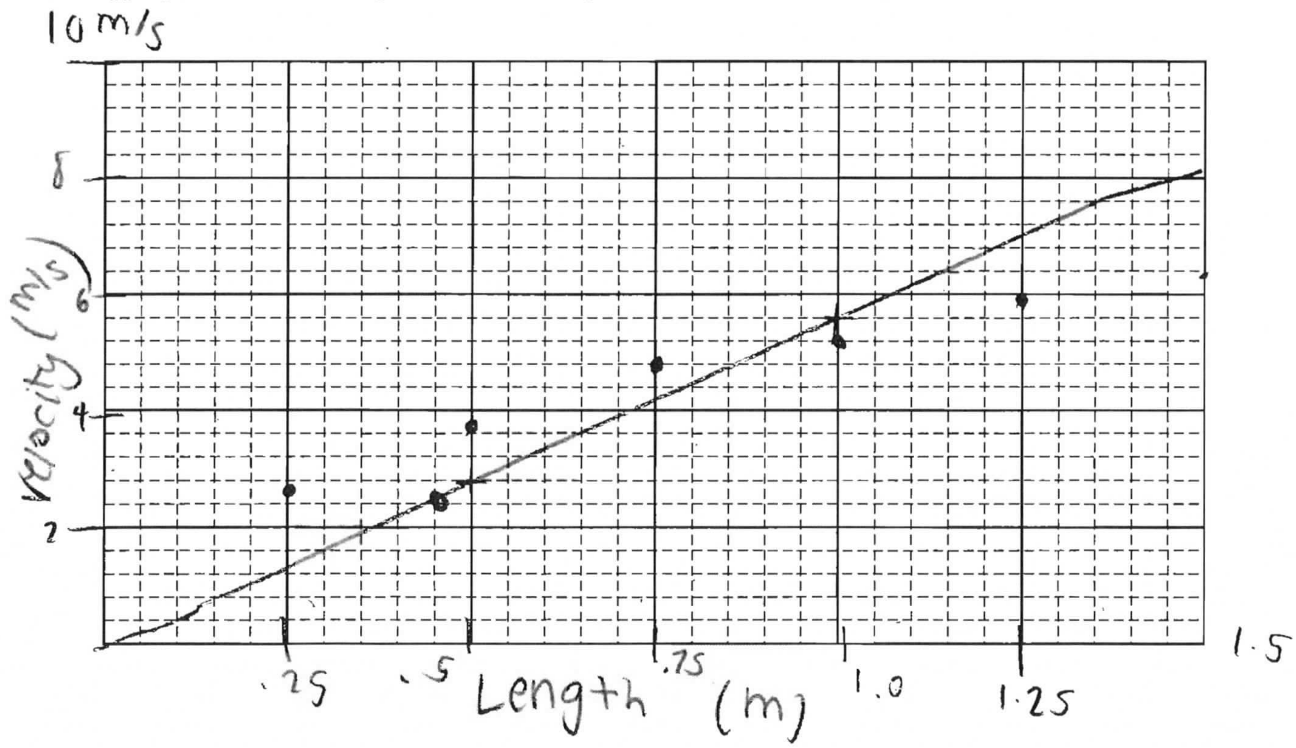
(c) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Horizontal axis: Length

Vertical axis: velocity

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data.



GO ON TO THE NEXT PAGE.

(e)

i. Using your straight line, determine an experimental value for g.

(.5, 2.8)
(1, 5.4)

$$\frac{2.8 - 5.4}{.5 - 1} = \frac{-2.6}{-.5} = 5.2\ m/s^2$$

ii. Describe two ways in which the effects of air resistance could be reduced.

① - If the road was in a vaccuum

② - If the rod had less mass

GO ON TO THE NEXT PAGE.

Question 3

Overview

This is a rotational dynamics question that requires integration and graphical analysis of data (identifying sources of error) to determine *g*.

Sample: MQ3 A
Score: 15

Part (a) of this response earned all 3 points, since all substitutions, bounds, and the evaluated integral were stated correctly. Part (b) earned all 4 points for use of the correct relationship between gravitational potential energy and rotational kinetic energy, the correct substitution for the change in height of the center of mass, and the correct substitution for angular velocity. In the line of three equations, there appears to be an erasure in the middle equation resulting in a missing factor of ω^2, but it correctly reappears in the right-hand equation. Part (c) earned 1 point for a correct relationship between velocity and length that will result in a straight line. Part (d) earned all 3 points for a graph that includes all the required elements and is consistent with the data table in part (c). Part (e)(i) earned 2 points for use of data points from the straight line to calculate slope and for using this slope to determine a value for *g*. Part (e)(ii) earned 2 points for two different methods to decrease the effect of air resistance.

Sample: MQ3 B
Score: 8

Part (a) of this response earned 0 points, since there is no indication that integration is used correctly. Part (b) earned only 1 point for use of conservation of energy equation. Part (c) earned 1 point for a correct relationship between velocity and length that will result in a straight line. Part (d) earned all 3 points for a graph that includes all the required elements and is consistent with the data table in part (c). Part (e)(i) earned 2 points for use of data points from the straight line to calculate slope and the use of this slope to determine a value for g that is consistent with part (b). Part (e)(ii) earned 1 point for one method to decrease the effect of air resistance.

Sample: MQ3 C
Score: 4

Part (a) of this response earned 0 points, since there is no indication that integration is used correctly. Part (b) earned 0 points since the method is incorrect. Part (c) earned 0 points for an incorrect relationship between velocity and length, which is not even consistent with the incorrect answer in (b). Part (d) earned 2 points for correct labels and scale and data plotted correctly; however, the line drawn does not represent a best-fit line to show the trend. Part (e)(i) earned 1 point for using points on the line but neglected to use the relationship between slope and g from part (b). Part (e)(ii) earned 1 point for only one valid method to decrease the effect of air resistance.