Question 2

15 points total | **Distribution of points**

(a) 2 points

For correctly drawing and labeling vectors on the block of mass $2M$ — 1 point

For correctly drawing and labeling vectors on the block of mass $3M$ using symbols for the vectors that are physically correct and different from those on the block of mass $2M$ — 1 point

Note: A maximum of one point can be earned if there are any extraneous vectors.

(b) 2 points

Using a proper expression for conservation of momentum

$$p_1 = p_2$$

For correctly substituting into the above equation — 1 point

$$3Mv_0 = (3M + 2M)v_f$$

For a correct answer — 1 point

$$v_f = \frac{3}{5}v_0$$

(c) 1 point

Using a proper expression for kinetic energy of the two-block system

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2$$

For an answer consistent with part (b) — 1 point

$$K = \frac{9}{10}Mv_0^2$$

Question 2 (continued)

Distribution of points

(d) 4 points

For a correct expression of the conservation of energy — 1 point

$\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$

$K_0 = U_{\text{final}}$

For attempting to integrate the spring force equation — 1 point

$$\frac{9}{10}Mv_0^2 = -\int_{x_1}^{x_2} -Bx^3 dx$$

$$\frac{9}{10}Mv_0^2 = \int_{x_1}^{x_2} Bx^3 dx$$

For using the correct limits of integration or an appropriate constant of integration — 1 point

$$\frac{9}{10}Mv_0^2 = \int_0^D Bx^3 dx$$

$$\frac{9}{10}Mv_0^2 = \left[\frac{Bx^4}{4}\right]_0^D$$

For an answer consistent with the speed from (b) or the kinetic energy from part (c) — 1 point

$$D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$$

(e)

i. 2 points

For selecting the correct answer "Right" with a reasonable attempt at a justification — 1 point

If the incorrect selection is made, no points are earned for the justification.

For an indication that at maximum compression the block of mass $2M$ has an acceleration to the right due to the forces acting on the block of mass $2M$ or an acceleration to the right due to the external spring force acting on the system of blocks — 1 point

Example: At maximum compression the two-block system is instantaneously at rest. The only horizontal external force acting on the system is due to the spring. This force is directed to the right. The system and therefore the block of mass $2M$ is accelerated to the right, which implies that the net force acting on the block of mass $2M$ is also to the right.

Question 2 (continued)

Distribution of points

(e)

ii. 2 points

The magnitude of the net force is greater on the block of mass $3M$.
If the incorrect selection is made, no points are earned for the justification.

For an indication that both blocks will have the same acceleration	1 point
For a correct justification for why the net force is greater on the block of mass $3M$	1 point

Example:
Because the blocks stick together, both blocks must have the same acceleration. Because the block of mass $3M$ has more mass, the net force on it must be greater than the net force on the block of mass $2M$.

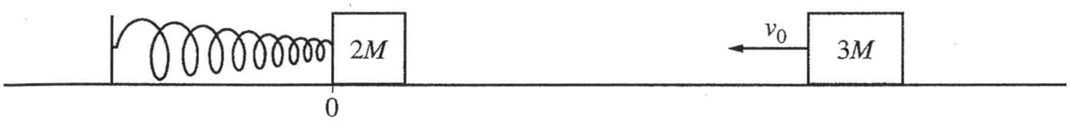
(f) 2 points

For selecting the correct answer "No," with a reasonable attempt at a justification	1 point

If the incorrect selection is made, no points are earned for the justification.
If the correct answer is selected without any justification, the point is not earned for the selection.

For an indication that the spring does not apply a linear force and that simple harmonic motion is the resulting motion of a linear restoring force $\left(a = -\frac{k}{m}\Delta x\right)$	1 point

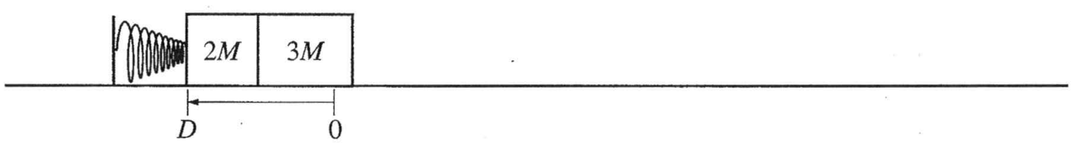
Example:
Because the blocks are sticking together and are attached to the spring, the spring will apply a restoring force to the blocks. However, because the restoring force exerted by a nonlinear spring is not proportional to the blocks' displacement from equilibrium, the blocks do not exhibit simple harmonic motion.



Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.



The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

$3MV_0 = 5MV$

$\frac{3}{5}V_0 = V$

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

$KE = \frac{1}{2}mV^2$

$= \frac{1}{2}5M\left(\frac{3}{5}V_0\right)^2$

$KE = \frac{9}{10}MV_0^2$

(d) Derive an expression for the maximum distance D that the spring is compressed.

$W = \int -Bx^3\,dx = \frac{Bx^4}{4}$

$\frac{9}{10} MV_0^2 = \frac{BD^4}{4}$

$\sqrt[4]{\frac{3.6MV_0^2}{B}} = D$

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ✓ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

The block will start accelerating to the right due to the force of the spring at the maximum distance, so the net force is to the right.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

✓ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

The acceleration is constant, but the net force on the 3M block will be greater since $3Ma > 2Ma$

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ✓ No

Justify your answer.

Since the surface is frictionless, while the 2M block will follow SMH, the 3M block will continue to the right with a constant acceleration

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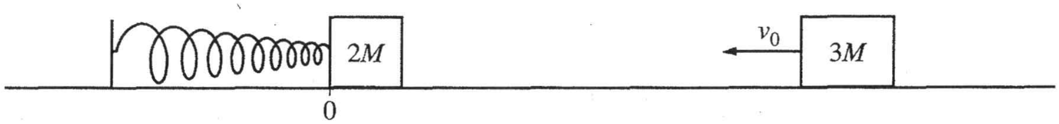
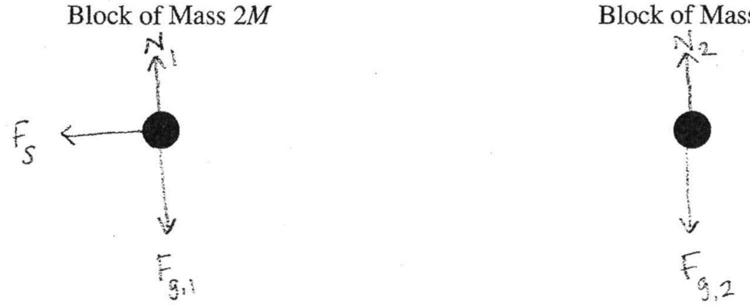


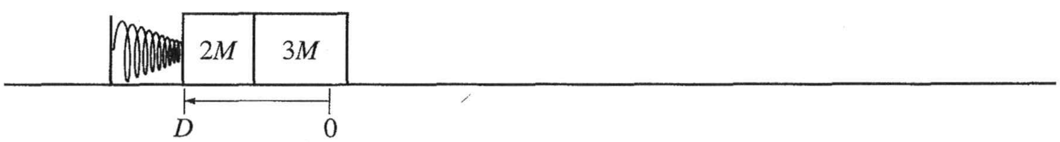
Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$ | Block of Mass $3M$





The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v = \frac{2M(0) + 3M(v_0)}{2M + 3M}$$

$$v = \frac{3v_0}{5}$$

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

$$K = \frac{1}{2} m v^2$$

$$K = \frac{1}{2}(2M + 3M)\left(\frac{3v_0}{5}\right)^2$$

$$= \frac{5M}{2}\left(\frac{9v_0^2}{25}\right)$$

$$K = \frac{9}{10} M v_0^2$$

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(d) Derive an expression for the maximum distance D that the spring is compressed.

$W = \Delta KE = F \cdot d$

$\frac{9}{10} M v_0^2 = -Bx^3 \cdot D$

$x = D$

$\frac{9}{10} M v_0^2 = -BD^4$

$D^4 = \frac{9 M v_0^2}{10 B}$

$D = \sqrt[4]{\frac{9 M v_0^2}{10 B}}$

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left __✓__ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

Since the net force is in the same direction as the acceleration, the block will accelerate to the right when the spring is at max compression and so will the net force.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

__✓__ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

The spring, gravity and the table are acting on the 2M block while gravity, the table, and the 2M block are acting on the 3M block at max compression. Since the 3M block has more mass, according to Newton's Third Law, it has a greater net force.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.

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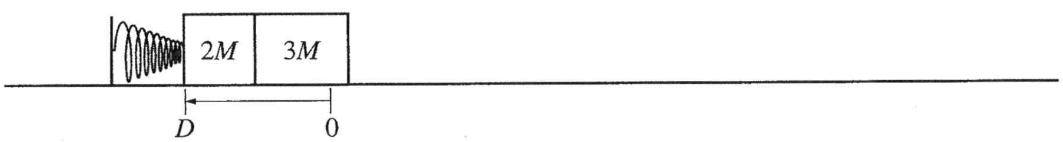
Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$

Block of Mass $3M$



The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

$K = \frac{1}{2} 5Mv^2$

$\frac{1}{2} 3mV_0^2 = \frac{1}{2} 5M v^2$ $\quad \frac{3}{5} v_0^2 = v^2$

$\sqrt{\frac{3}{5} v_0^2} = v = \sqrt{\frac{3}{5}} v_0$

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

$3mv_0$ $\quad K = \frac{1}{2} 3m V_0^2$ $\quad \sqrt{\frac{2k}{3M}} = v_0$ $\quad K = \frac{5}{2} M (\sqrt{\frac{3}{5}} v_0)^2$

$K =$ $\quad K = \frac{3}{2} M v_0^2$

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(d) Derive an expression for the maximum distance D that the spring is compressed.

$F(x) = -Bx^3 = kx \Rightarrow k = -Bx^2$

$U_s = k$

$\frac{1}{2} k(D)^2 = \frac{3}{2} M v_0^2$ $D = \sqrt[4]{3BMv_0^2}$

$\frac{1}{2} - B D^4 = \frac{3}{2} M v_0^2$

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right _✓_ The net force on the block of mass $2M$ is zero.

Justify your answer. At Maximum compression there is no acceleration on the 2M block, therefore the net force has to be zero

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

_✓_ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer. The 3M block has a greater F_g and F_N and has the same F_s as the 2M block

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

_✓_ Yes ____ No

Justify your answer. The two blocks will move back and forth with no friction and their motion would follow a sine wave.

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Question 2

Overview

The question had a block sliding on a horizontal frictionless surface and colliding with another block that was attached to a nonlinear spring. It required knowledge of free body diagrams, forces, integral calculus (determining potential energy from a force function), conservation of energy, conservation of momentum, conceptual understanding of interactions in collisions, and simple harmonic motion.

Sample: M Q2 A
Score: 12

Part (a) earned 1 point for drawing two appropriately vertical vectors on the block of mass 2M. However, the vectors are not differentiated from each other (2Mg or 3Mg or N_1 or N_2). Part (b) earned 2 points for correctly using conservation of momentum to calculate the speed of the blocks after the collision. Part (c) earned 1 point for a proper expression for the kinetic energy of the two-block system. Part (d) earned 3 points for using an appropriate approach to a correct solution and having the precise, correct expression for the "D." However, because this question was a "Derive" question, the rubric demands that the student show explicit steps along the way toward the solution and the limits of integration (from 0 to D in the integration) are not shown. Part (e)(i) earned 2 points for a correct answer and justification. Part (e)(ii) earned 2 points for indicating that both blocks have the same acceleration and justifying the greater force on the block of mass 3M. Part (f) earned 1 point for checking the correct box, but the justification does not recognize that the system has a nonlinear restoring force acting on the two-block system and, therefore, will not exhibit simple harmonic motion.

Sample: M Q2 B
Score: 8

Part (a) earned 1 point for drawing two vertical vectors on both blocks and differentiating the vectors on the two blocks; however, there is an extraneous vector on the block of mass 2M. Parts (b) and (c) earned full credit. Part (d) earned 2 points for using a correct expression for conservation of energy with previous answers but did not integrate the spring force. Part (e)(i) earned 2 points. Part (e)(ii) earned no credit because the justification is incorrect, even though the correct box is checked. Part (f) earned no credit.

Sample: M Q2 C
Score: 4

Part (a) earned 1 point for drawing two appropriately vertical vectors on the block of mass 2M. However, the vectors are not differentiated. Part (b) earned no credit as there is no use of conservation of momentum. Part (c) earned 1 point for an answer consistent with part (b). Part (d) earned 2 points for using a correct expression for conservation of energy with previous answers but did not integrate the spring force. Part (e)(i) earned no credit for an incorrect answer, and the justification is ignored. Part (e)(ii) earned no credit because the justification is incorrect, even though the correct box is checked. Part (f) earned no credit because the incorrect box is checked.