Question 3

15 points total | **Distribution of points**

(a) 3 points

For an indication that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

For a correct substitution for x in the equation — 1 point

For a correct substitution for r in the equation that can be used to solve for k — 1 point

$k\left(\frac{d}{2}\right) = m(2d)\omega^2$

Correct answer:

$k = 4m\omega^2$

(b)

i. 1 point

Substitute values for the block into the equation for rotational inertia

$I = \sum mr^2 = m(2d)^2$

For a correct answer or an answer consistent with the radius used in part (a) — 1 point

$I = 4md^2$

ii. 2 points

For an indication that the rotational inertia of the system must include the platform, the rod, and the object — 1 point

$I = I_P + I_R + I_O$

$I = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$

For correctly substituting into the equation the rotational inertia of the platform, the rod, and the object consistent with (b) i. — 1 point

$I = \frac{5m(2d)^2}{2} + \frac{3m\,d^2}{3} + 4md^2$

$I = (10 + 1 + 4)md^2$

Correct answer:

$I = 15md^2$

(c) 1 point

Using a correct expression of angular momentum

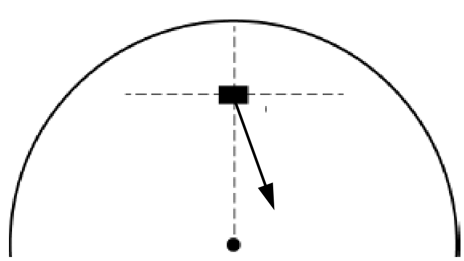
$L = I\omega$

For an answer consistent with the answer from part (b) ii. — 1 point

$L = 15md^2\omega$

Question 3 (continued)

	Distribution of points
(d) 3 points	
For indicating that the spring force equals the mass times the centripetal acceleration	1 point
$F_S = F_C = ma_C$ $kx = mr\omega^2$	
For a correct substitution for k, or a substitution consistent with part (a), in the equation	1 point
For a correct substitution for r in the equation that can be used to solve for x	1 point
$\left(4m\omega^2\right)x = m(d + x)\omega^2$ $4x = d + x$ $3x = d$ Correct answer: $x = d/3$	
(e) 2 points	
For selecting "Decreasing"	1 point
If the wrong selection is made, no points are earned for the justification.	
For a correct justification	1 point
Example: The rotational inertia I of the system decreases while the angular velocity w stays the same. Because the angular momentum is Iw, it decreases.	
(f) 2 points	
For selecting "Negative"	1 point
If the wrong selection is made, no points are earned for the justification.	
For a correct justification	1 point
Example: Because the angular velocity is constant and the rotational inertia has decreased, the rotational kinetic energy has decreased. Therefore, work must be negative.	
(g) 1 point	



For an arrow starting on the box and pointing down and to the right	1 point

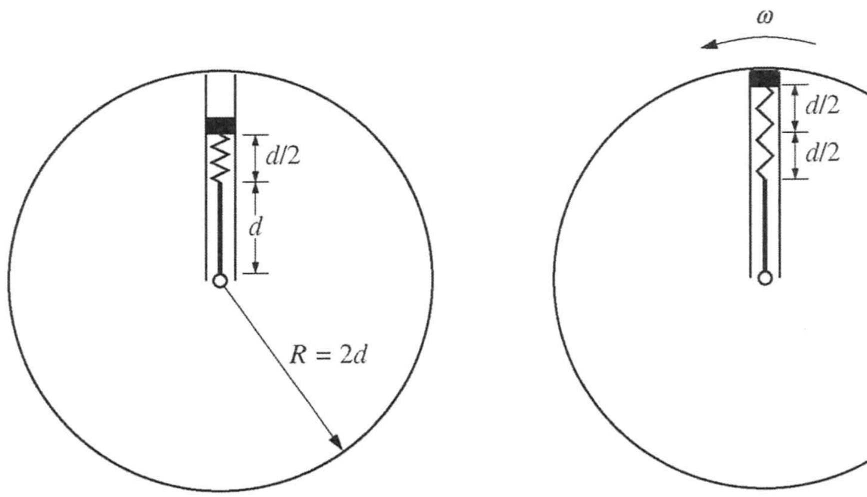


Figure 1 Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

Since $F_{spring} = F_{centripetal}$,

$$\frac{kd}{2} = mw^2\left(d+\frac{d}{2}+\frac{d}{2}\right)$$

So $\frac{kd}{2} = 2mdw^2$

So $k = 4mw^2$

GO ON TO THE NEXT PAGE.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

$$I = mr^2 = m\left(d + \frac{d}{2} + \frac{d}{2}\right)^2 = 4md^2$$

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

$$I_{sys} = I_{block} + I_{rod} + I_{platform}$$
$$= 4md^2 + \frac{3md^2}{3} + \frac{5m(2d)^2}{2}$$
$$= 4md^2 + md^2 + 10md^2$$
$$= 15md^2$$

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

$$L = I\omega = 15md^2\omega$$

Question 3 continues on next 2 pages.

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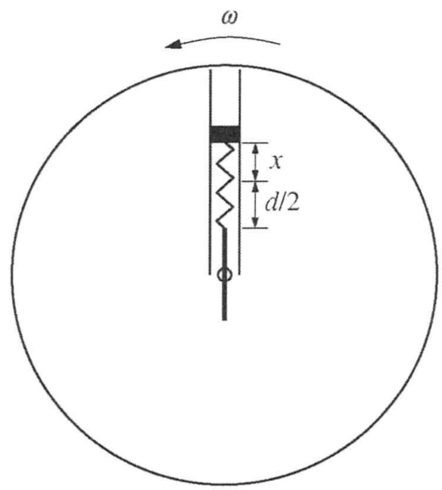

Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

Since $F_{spring} = F_{centripetal}$,

$$kx = mw^2\left(\frac{d}{2}+\frac{d}{2}+x\right)$$

So $4mw^2x = mw^2(d+x)$

So $d+x = 4x$

So $3x = d$

So $x = \frac{d}{3}$

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

____ Increasing ✓ Decreasing ____ Staying the same

Justify your answer.

By parallel-axis theorem, we know that the closer the center of mass of an object is to the rotational axis, the lower the rotational inertia is. Since the rod takes the rod and the block closer to the center, their inertia decreases. Since w stay the same, $L = Iw$ will decrease.

GO ON TO THE NEXT PAGE.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive   __✓__ Negative   _____ No work

Justify your answer.

If there were no motor, by conservation of angular momentum, since I decreased, w will increase. So in order to maintain constant w, the motor have to do negative work to slow down the angular speed.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

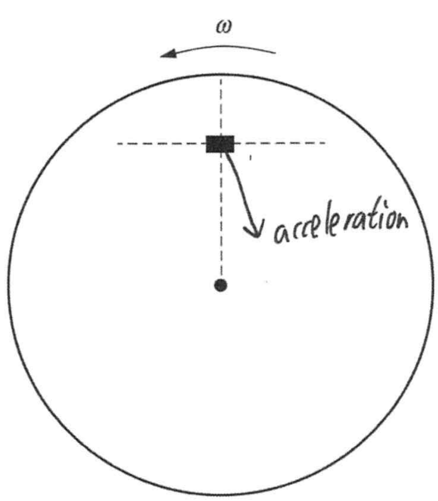

Figure 4

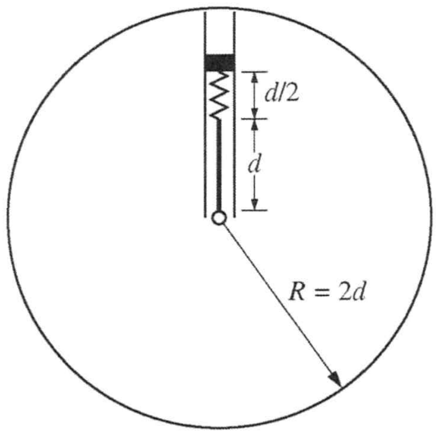

Figure 1

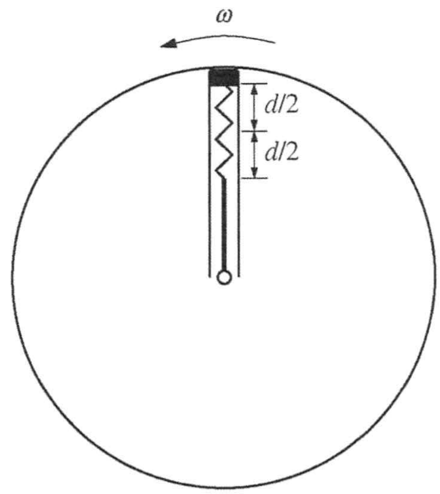

Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

$k = 4m\omega^2$

$F_c = F_s$

$-ma_c = -k\Delta x$

$-m\omega^2 r = -k\Delta x$

$-m\omega^2(2d) = -k\left(\frac{d}{2}\right)$

$-4m\omega^2 = -k$

GO ON TO THE NEXT PAGE.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

$I = 4md^2$

$I = \Sigma m r^2$

$I = m(2d)^2$

$I = 4md^2$

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

$I_{Total} = 15md^2$

$\Sigma I = I_B + I_R + I_P$

$= 4md^2 + md^2 + \frac{5m(2d)^2}{2}$

$= 5md^2 + 10md^2$

$= 15md^2$

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

$L = 15md^2\omega$

$\vec{L} = I\vec{\omega}$

$L = 15md^2\omega$

Question 3 continues on next 2 pages.

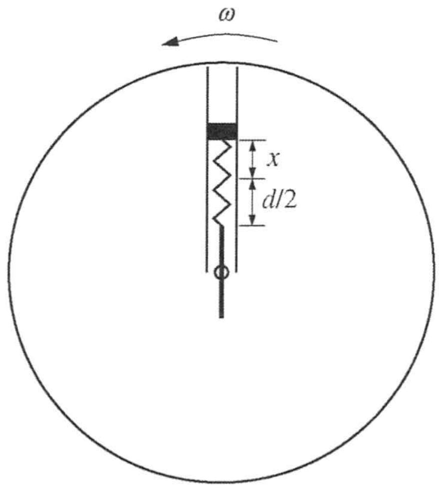

Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

$F = -k\Delta x$

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing   ✓ Decreasing   _____ Staying the same

Justify your answer.

$\vec{L} = I\vec{\omega}$

ω is constant and I is decreasing so L is decreasing.

$I = \int r^2 dm$ and r is decreasing for the rod so I is decreasing.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive   __✓__ Negative   _____ No work

Justify your answer.

$W = \Delta E$

$K = \frac{1}{2} I \omega^2$

Since I is decreasing, K is also decreasing if ω is constant. Thus ΔE is negative and work is negative.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

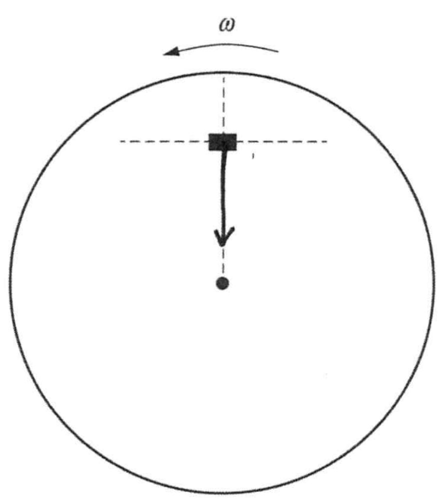

Figure 4

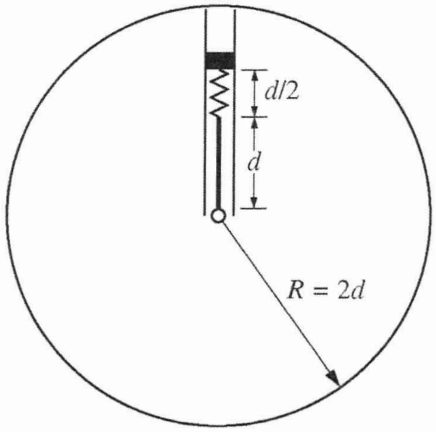

Figure 1

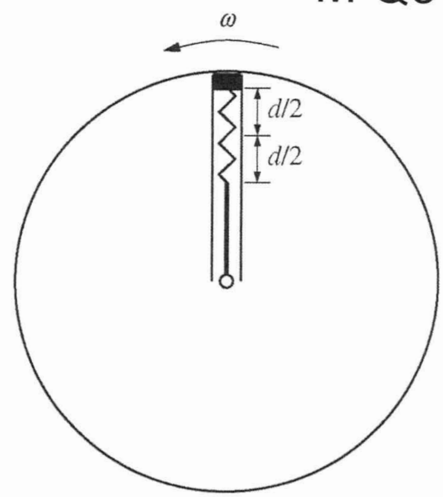

Figure 2

Mech.3.

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	Mass	Rotational Inertia
Block	m	$4md^2$
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Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

$T = \frac{2\pi}{\omega}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$\frac{\cancel{2\pi}\sqrt{\frac{m}{k}}}{\cancel{2\pi}} = \frac{\frac{\cancel{2\pi}}{\omega}}{\cancel{2\pi}}$

$\left(\sqrt{\frac{m}{k}}\right)^2 = \left(\frac{1}{\omega}\right)^2$

$\frac{m}{k} = \frac{1}{\omega^2}$

$\boxed{m\omega^2 = k}$

GO ON TO THE NEXT PAGE.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

$\cancel{I = I + mx^2}$

$\cancel{I = mr^2}$

$I = mr^2$

$I = m(2d)^2$

$\boxed{I = 4md^2}$

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

$I = \cancel{4md^2}\ I_{block} + I_{Rod} + I_{platform}$

$I = 4md^2 + \frac{\cancel{3}md^2}{3} + \frac{5m(2d)^2}{2}$

$I = \frac{24md^2}{6} + \frac{6md^2}{6} + \frac{60md^2}{6} = \frac{90md^2}{6} = \boxed{15md^2}$

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

$L = I\omega$

$\boxed{L = 15md^2\omega}$

Question 3 continues on next 2 pages.

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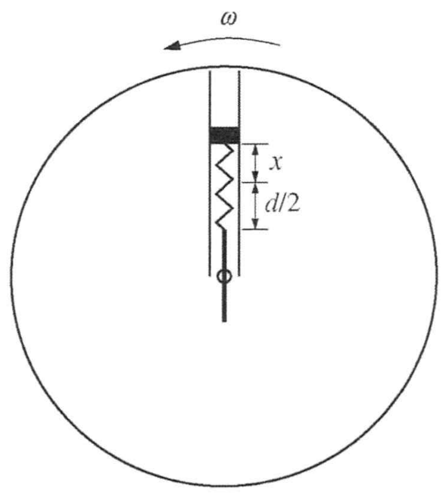

Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

$F = -kx$ $L = 15md^2\omega$

$k = m\omega^2$

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

____ Increasing ✓ Decreasing ____ Staying the same

Justify your answer.

It is decreasing because as it moves to the center the radius is becoming smaller making the r^2 part of the equation smaller.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative __✓__ No work

Justify your answer.

No work is being done because since it is moving in a circle, the displacement will always be zero and since $W = F \cdot d$, work will equal zero

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

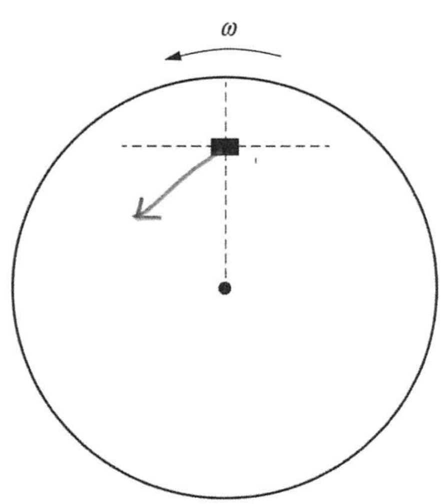

Figure 4

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2016 SCORING COMMENTARY

Question 3

Overview

The question had a rotating platform with a mass on a spring. This question assessed student comprehension and ability to make connections in the topics of rotational motion, rotational inertia, angular momentum, and behavior of springs.

Sample: M Q3 A
Score: 15

Part (a) earned 3 points for using a correct equation and correctly substituting for the stretched length of the spring and radius. Part (b)(i) earned 1 point for stating a value that was correct for the rotational inertia of the block. Part (b)(ii) earned 2 points for indicating that the rotational inertia of the system is the sum of the block's, rod's, and platform's rotational inertias and correctly substituting the values for each object. Part (c) earned 1 point for stating a value that is correct for the angular momentum of the entire system. Part (d) earned 3 points for using the correct equation and substituting from part (a) for the spring constant and the radius. Part (e) earned 2 points for selecting "Decreasing" and a correct justification relating the decreasing rotational inertia to the angular momentum. Part (f) earned 2 points for selecting "Negative" and a correct justification explaining why the motor has to do negative work to keep the angular velocity constant as the rotational inertia is decreasing. Part (g) earned 1 point for an arrow drawn in the correct direction.

Sample: M Q3 B
Score: 11

Parts (a), (b), and (c) earned full credit. Part (d) earned no credit because there is no indication that a correct equation was used. Part (e) earned full credit. Part (f) earned 2 points for selecting "Negative" and a correct justification relating the decreasing rotational inertia to a decrease in rotational kinetic energy that results in negative work. Part (g) earned no credit for an arrow drawn toward the center instead of down and to the right.

Sample: M Q3 C
Score: 6

Part (a) earned no credit because there is no indication that a correct equation was used. Part (b) and (c) earned full credit. Part (d) earned no credit because there is no indication that a correct equation was used. Part (e) earned 2 points for selecting "Decreasing" and a correct justification that a smaller distance from the center of rotation is related to a smaller angular momentum. Part (f) earned no credit for selecting an incorrect choice. Part (g) earned no credit for an arrow drawn down and to the left instead of down and to the right.