2017

AP CollegeBoard

AP Physics C: Mechanics

Sample Student Responses and Scoring Commentary

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General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

15 points total | **Distribution of points**

(a)

i. 2 points

For correctly using conservation of energy for the block moving down the incline — 1 point

$U_g = K$

$mgh = \frac{1}{2}mv^2$

For a correct answer — 1 point

$v = \sqrt{2gh}$

ii. 1 point

Correct answer: "Greater than"

For a correct justification — 1 point

Example: The speed is proportional to the square root of the change in height. So if the height is reduced by a factor of 2, the speed is reduced by a factor of $\sqrt{2} \approx 1.41$. Therefore, the speed halfway down the ramp is more than half the speed at the bottom of the ramp.

Note: If the incorrect selection is made, the justification cannot earn credit.

(b) 1 point

For correctly using conservation of energy, consistent with part (a), for the block compressing the spring — 1 point

$U_g = U_S$

$mgh = \frac{1}{2}kx_{max}^2$

$x_{max} = \sqrt{\frac{2mgh}{k}}$

(c) 2 points

For indicating a simple harmonic motion approach — 1 point

For a correct answer — 1 point

$t = \frac{1}{4}T = \left(\frac{1}{4}\right)\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

Question 2 (continued)

Distribution of points

(d)

i. 3 points

For correctly applying Newton's second law for the horizontally sliding block — 1 point

For correctly indicating that the direction of F_{net} is opposite the direction of motion — 1 point

$$F_{net} = ma$$

$$-\beta v^2 = ma$$

For expressing the equation as a differential equation — 1 point

$$-\beta v^2 = m\frac{dv}{dt}$$

ii. 3 points

For correctly separating variables — 1 point

$$-\frac{\beta}{m}dt = \frac{1}{v^2}dv$$

For correctly integrating the equation above — 1 point

$$\int -\frac{\beta}{m}dt = \int \frac{1}{v^2}dv$$

$$-\frac{\beta}{m}[t] = \left[-\frac{1}{v}\right]$$

For using the correct limits or constant of integration — 1 point

$$-\frac{\beta}{m}[t]_0^t = \left[-\frac{1}{v}\right]_{v_0}^{v(t)}$$

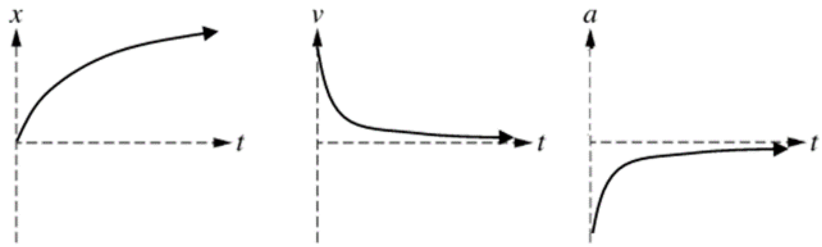
$$-\frac{\beta t}{m} = -\frac{1}{v(t)} - \left(-\frac{1}{v_0}\right)$$

$$\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$$

Question 2 (continued)

Distribution of points

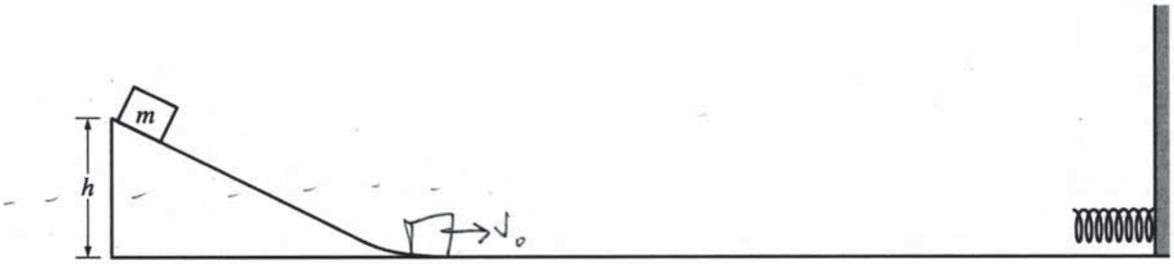
(e) 3 points



For a displacement curve that is concave down and approaching a horizontal asymptote	1 point
For a velocity curve that is concave up and has the horizontal axis as an asymptote	1 point
For an acceleration curve that is concave down and has the horizontal axis as an asymptote	1 point

Note: If an incorrect nonlinear velocity graph is generated, 1 point is earned if the position and acceleration graphs are consistent with the velocity graph.

Note: Full credit is earned if all three graphs are flipped vertically.



Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

$\frac{1}{2}mv^2 = mgh \rightarrow v = \sqrt{2gh}$

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

✓ Greater than ____ Less than ____ Equal to

Justify your answer.

$mg\left(\frac{1}{2}h\right) = \frac{1}{2}mv^2$

$v = \sqrt{gh} \rightarrow \sqrt{gh}$ is greater than $\frac{1}{2}\sqrt{2gh}$

(b) Derive an expression for the maximum compression of the spring.

$mgh = \frac{1}{2}kx^2$

$\frac{2mgh}{k} = x^2 \rightarrow x = \sqrt{\frac{2mgh}{k}}$

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

$T = 2\pi\sqrt{\frac{m}{k}}$

$\downarrow$

$\frac{1}{4}T = \frac{1}{2}\pi\sqrt{\frac{m}{k}}$

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

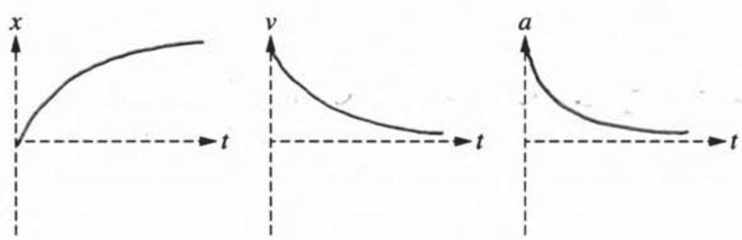
i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

$$\Sigma F = ma$$
$$-Bv^2 = m\frac{dv}{dt}$$
$$\frac{dv}{dt} = \frac{-B}{m}v^2$$

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

$$\int_{v_0}^{v} \frac{1}{v^2}\,dv = \int_0^t -\frac{B}{m}\,dt$$
$$\left[\frac{-1}{v}\right]_{v_0}^{v} = -\frac{Bt}{m}$$
$$-\frac{1}{v(t)} + \frac{1}{v_0} = -\frac{Bt}{m}$$
$$\frac{1}{v(t)} = \frac{1}{v_0} + \frac{Bt}{m}$$

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



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2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

✓ Greater than ____ Less than ____ Equal to

Justify your answer.

$v = \sqrt{2gh/2}$

$v = \sqrt{gh}$

When using the equation from part a)i.) mathematically, the equation is not divided by 2 but rather $\sqrt{2}$. So, the speed will be greater than one-half terminal speed

(b) Derive an expression for the maximum compression of the spring.

$\frac{1}{2}mv^2 = kx$

$x = \frac{2k}{mv^2} = \frac{2k}{mgh} = \Delta x$

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

$v_f = v_0 + at$

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

$F = \beta v^2$

$ma = \beta v^2$

$$\boxed{m\frac{dv}{dt} = \beta v^2}$$

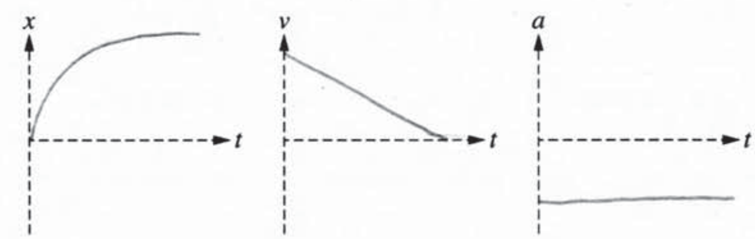
ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

$$\int_{v_0}^{v(t)} \frac{dv}{v^2} = \int_0^t \frac{\beta\, dt}{m}$$

$$v^{-1}\Big|_{v_0}^{v(t)} = \frac{\beta t}{m}\Big|_0^t$$

$$\frac{1}{v(t)} - \frac{1}{v_0} = \frac{\beta t}{m} - \frac{\beta t}{0} \rightarrow \frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$$

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



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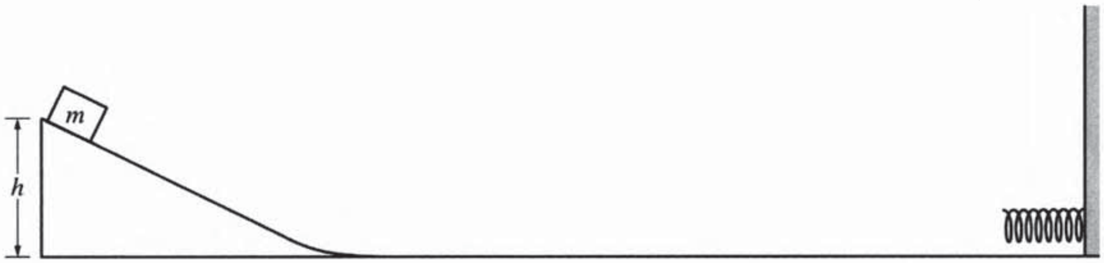

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2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

$KE = PE$ $\quad \frac{1}{2}mv^2 = mgh \quad \frac{1}{2}v^2 = gh \quad v^2 = 2gh \quad v = \sqrt{2gh}$

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ____ Less than ✓ Equal to

Justify your answer.

$\frac{1}{2}mgh = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$\frac{1}{2}mgh = \frac{1}{4}mv^2$

$2mgh = mv^2 \quad v = \sqrt{2gh}$

(b) Derive an expression for the maximum compression of the spring.

$KE = PE_{spring}$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$2ghm = kx^2$

$\frac{2ghm}{k} = x^2$

$\sqrt{\frac{2ghm}{k}} = x_{max}$

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

$F = kx$

$ma = kx$

$a = \frac{kx}{m}$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$x_{max} = \sqrt{2gh}\, t + \frac{1}{2}\frac{kx}{m} t$

$\sqrt{\frac{2ghm}{k}} = \sqrt{2gh}(t) + \frac{kx}{2m}(t)$

$\sqrt{\frac{2ghm}{k}} = t\left(\sqrt{2gh} + \frac{kx}{2m}\right)$

$t = \dfrac{\sqrt{\frac{2ghm}{k}}}{\left(\sqrt{2gh} + \frac{kx}{2m}\right)}$

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The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

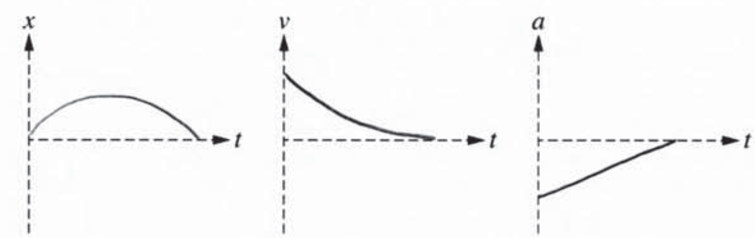
$F = ma$

$\frac{Bv^2}{m} = a$

$v(t) = \int \frac{Bv^2}{m} dt$

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



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Question 2

Overview

The responses to this question were expected to demonstrate the following:

- An understanding of the relationship between different types of mechanical energy.
- The ability to use energy considerations to calculate relevant quantities.
- An understanding of an object transitioning from linear motion to simple harmonic motion.
- An understanding of an object undergoing simple harmonic motion.
- The ability to use the equations for simple harmonic motion.
- The ability to apply Newton's second law to a system experiencing a resistive force.
- The ability to express a resistive force equation as a differential equation.
- The ability to use separation of variables to solve a differential equation.
- The ability to carry out integration, including the appropriate choice of limits or constant of integration.
- An understanding of the behavior of objects experiencing a resistive force.
- The ability to describe that behavior using position vs. time, velocity vs. time, and acceleration vs. time graphs.

Sample: M Q2 A
Score: 14

Parts (a), (b), (c), and (d) earned full credit. Part (a)(i) uses conservation of energy and has the correct answer. Part (a)(ii) makes the correct selection, correctly calculates the speed halfway down the incline, and relates it to the speed at the bottom of the incline. Part (b) relates the gravitational potential energy at the top of the hill to the elastic potential energy for the spring at maximum compression and has a correct answer. Part (c) uses the correct equation and has a correct answer. Part (d)(i) has a correct use of Newton's second law, uses the appropriate differential equation, and correctly indicates the direction of the net force. Part (d)(ii) separates the variables, uses the correct limits of integration, and integrates the equation. In part (e) the acceleration graph is flipped around the horizontal axis, so 2 points were earned.

Sample: M Q2 B
Score: 9

Part (a) earned full credit. Part (b) incorrectly sets kinetic energy equal to the spring force, so no credit was earned. Part (c) has no indication of simple harmonic motion, so no points were earned. Part (d)(i) has a correct use of Newton's second law, uses the appropriate differential equation, but is missing the negative sign for the direction of the force, so 2 points were earned. Part (d)(ii) separates the variables, uses the correct limits of integration, and integrates the equation, so full credit was earned. In part (e) the velocity and acceleration graphs are incorrectly drawn, so 1 point was earned.

Question 2 (continued)

Sample: M Q2 C
Score: 5

Part (a)(i) earned full credit. In part (a)(ii) a correct energy equation is used but the calculation does not go far enough to answer the question, so no credit was earned. Part (b) earned full credit. Part (c) has no indication of simple harmonic motion, so no points were earned. Part (d)(i) has a correct use of Newton's second law. The differential equation is not correctly written, and the negative sign for the direction is missing, so 1 point was earned. Part (d)(ii) has no response, so no credit was earned. For part (e) the position and acceleration graphs are incorrectly drawn, so 1 point was earned.