2017

AP CollegeBoard

AP Physics C: Mechanics

Sample Student Responses and Scoring Commentary

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AP® PHYSICS
2017 SCORING GUIDELINES

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 3

15 points total **Distribution of points**

(a) 2 points

For correctly applying conservation of energy to the cylinder rolling down the incline — 1 point

$U_{g\_top} = K_{table}$

$K_{table} = mgh = (0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)(1.0\ \text{m})(\sin 30)$

For a correct answer with units — 1 point

$K_{table} = 2.45\ \text{J}$ (or 2.5 J using $g = 10\ \text{m/s}^2$)

(b) 3 points

For correctly setting the kinetic energy of the cylinder equal to the sum of both the linear and rotational kinetic energy — 1 point

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For correctly substituting into the above equation for the linear velocity and moment of inertia of the cylinder — 1 point

$$K = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$$

$$\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{(4)(2.45\ \text{J})}{(3)(0.50\ \text{kg})(0.10\ \text{m})^2}}$$

For correct substitution into the equation above — 1 point

$\omega = 25.6\ \text{rad/s}$ (or $26.0\ \text{rad/s}$ using $g = 10\ \text{m/s}^2$)

(c) 2 points

For using a correct expression for the ratio of the rotational kinetic energy to the total kinetic energy of the cylinder — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh} = \frac{\frac{1}{4}M(R\omega)^2}{\frac{3}{4}M(R\omega)^2 + Mgh_{\text{table}}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{\text{table}}}$$

For substituting into the above equation — 1 point

$$\frac{K_{rot}}{K_{tot}} = \frac{[(0.10\ \text{m})(25.6\ \text{rad/s})]^2}{(3)(0.10\ \text{m})^2(25.6\ \text{rad/s})^2 + (4)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})}$$

$\frac{K_{rot}}{K_{tot}} = 0.133$ (or 0.135 using $g = 10\ \text{m/s}^2$)

Question 3 (continued)

Distribution of points

(c) (continued)

Alternate Solution — *Alternate Points*

For using a correct expression for the ratio of the rotational kinetic energy to the total potential energy of the cylinder — *1 point*

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{U_{total}} = \frac{\frac{1}{4}M(R\omega)^2}{Mg(h+y)}$$

For substituting into the above equation — *1 point*

$$\frac{K_{rot}}{U_{total}} = \frac{\frac{1}{4}(R\omega)^2}{g(h+y)} = \frac{\frac{1}{4}[(0.10\ \text{m})(25.6\ \text{rad/s})]^2}{(9.81\ \text{m/s}^2)((1.0\ \text{m})\sin(30^\circ) + (0.75\ \text{m}))}$$

$$\frac{K_{rot}}{U_{total}} = 0.13 \textit{ (or } 0.135 \textit{ using } g = 10\ \text{m/s}^2 \textit{)}$$

(d) 2 points

For correctly using motion in the vertical direction to calculate the time for the cylinder to reach the floor — 1 point

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$y - y_0 = 0 - \frac{1}{2}gt^2$$

Determine the time for the cylinder to reach the floor

$$y - y_0 = -\frac{1}{2}gt^2$$

$$0 - y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{(2)(0.75\ \text{m})}{(9.81\ \text{m/s}^2)}} = 0.39\ \text{s}$$

For correctly using the equation for constant speed in the horizontal direction — 1 point

$$x - x_0 = v_{0x}t$$

$$x - x_0 = R\omega t$$

$$x - x_0 = R\omega\sqrt{\frac{2y}{g}}$$

$$D = R\omega t = (0.10\ \text{m})(25.6\ \text{rad/s})(0.39\ \text{s})$$

$$D = 1.0\ \text{m}$$

Question 3 (continued)

Distribution of points

(e)

i. 2 points

For selecting "Equal to" and attempting a relevant justification	1 point
For a correct justification	1 point

Example: Because the sphere falls the same height as the cylinder and because they have the same mass, the sphere-Earth system has the same initial potential energy and, therefore, the same total kinetic energy when it reaches the floor.

ii. 2 points

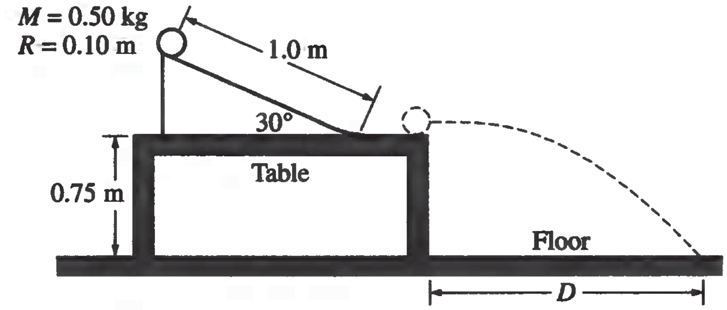
For selecting "Less than" and attempting a relevant justification	1 point
For a correct justification	1 point

Example: Because the rotational inertia of the sphere is less than the rotational inertia of the cylinder, the sphere will rotate faster and, because $v = r\omega$, will move with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere will have a greater linear kinetic energy. Because the total kinetic energies of the sphere and cylinder are the same, the sphere must have less rotational kinetic energy.

iii. 2 points

For selecting "Greater than" and attempting a relevant justification	1 point
For a correct justification	1 point

Example: Because the sphere has a greater linear speed as it leaves the table, it will travel a greater horizontal distance before it reaches the floor.



3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

$U = K_R + K_T$

$mgh = K_{Total}$

$K_{Total} = mg(1\sin\theta) = (0.5)(10)(1)\sin 30 = \boxed{2.5\text{ J}}$

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

ω

$\frac{3}{4}mv^2 = 2.5$

$v = 2.58198$

$\omega = \frac{v}{R} = \frac{v}{0.1} =$ $\boxed{25.820\text{ rad/s}}$

$K_{Total} = K_R + K_t$

$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}(\frac{1}{2}mR^2)(\frac{v^2}{R}) + \frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}mv^2 = \frac{3}{4}mv^2 = 2.5$

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

$K_{0,K}$ / K_f

$K_0 = 2.5\text{ J}$

$K_0 + U = K_f$

$2.5 + mgh =$

$2.5 + 3.75 = 6.25\text{ J}$

$K_R = \frac{1}{2}I\omega^2$

$= \frac{1}{2}(\frac{1}{2}mR^2)(25.82^2)$

$= \frac{1}{4}(0.5)(0.1)^2(25.82)^2$

$= 0.833\text{ J}$

$\frac{K_R}{K_T} = \frac{0.833}{6.25} = \boxed{0.133\text{ J}}$

GO ON TO THE NEXT PAGE.

(d) Calculate the horizontal distance D.

$x = v_0 t$ $y = \frac{1}{2} g t^2$ $t = \sqrt{\frac{2y}{g}}$

$D = (2.58198)\left(\sqrt{\frac{2(0.75)}{10}}\right)$

0.999m

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$. $\frac{1}{2}MR^2$

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than   ____ Less than   ✓ Equal to

Justify your answer.

Energy is conserved. The total KE at the floor will be equal to the total KE at the end of the ramp.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than   ✓ Less than   ____ Equal to

Justify your answer.

The moment of inertia is less than that of the sphere. This means that there is less KE_R but a greater translational KE.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

✓ Greater than   ____ Less than   ____ Equal to

Justify your answer.

There is more translational KE for the sphere at the end of the ramp because there is less rotational KE. More KE_t means more velocity. More velocity means a greater D.

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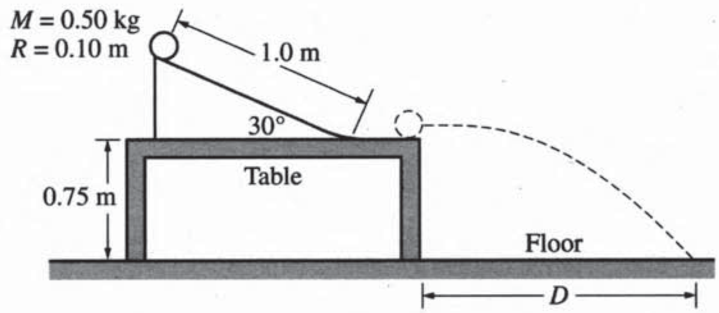

3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

$E_U = E_K$

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$.50kg(9.8\frac{m}{s^2})(1\sin 30°) = E_K$

$E_K = 2.45 J$

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

$E_K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$2.45J = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

$$\frac{E_{K_R}}{E_K} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

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(d) Calculate the horizontal distance D.

$\Delta x = \frac{1}{2} a t^2 + V_0 t$

$D = \frac{1}{2} a t^2 + V_0 t$

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than _X_ Equal to

Justify your answer.

gravitational energy is converted to kinetic energy

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than _X_ Less than ____ Equal to

Justify your answer.

The sphere has a smaller rotational inertia

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than _X_ Equal to

Justify your answer.

The sphere has the same total kinetic energy

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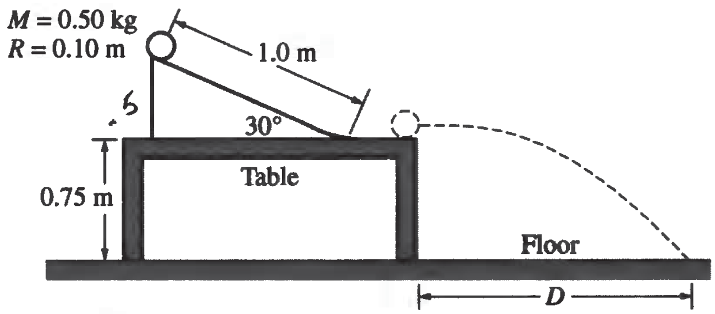

3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

$M = .5$
$R = .1$
$v_i = 0$
$d = 1$
$h = .75$
$h = 1.25$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh - \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh - \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{Mr^2}{2}\right)\frac{v^2}{r^2}$

$.5\ 9.8\ .5$

$2.45 - \frac{1}{2}(.5)v^2 + \frac{1}{2}\frac{.5v^2}{2}$

$2.45 = .25v^2 + .5v^2$

$v = 1.8$ m/s

$KE = 1.215$ J

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

$mgh - \left(\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{Mr^2}{2}\right)\frac{v^2}{r^2}\right)$

$2.45 = .25v^2 + .5v^2$

$v = 1.8$ m/s

$v = \omega r$ $\quad \frac{1.8}{.1}$ $\quad \omega = 18$ rad/s

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

$\frac{1}{2}mv^2 = \frac{1}{2}(.5)(1.8)^2 = 0.81$

$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{.5(.1)^2}{2}\right)\left(\frac{1.8^2}{(.1)^2}\right) = .405$

$.405 / .81 \rightarrow 1:2$

~~1/2~~

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(d) Calculate the horizontal distance D.

V=1.8

X	y
$V_x = 1.8$	$a = 9.8$
D = ?	$V_i = 1.8$
t = .18s	$V_f = 0$
	t =
	h = .75

$V_f = V_i + at$

$0 = 1.8 + 9.8t$

t = .18 s

$.75 = 1.8t - \frac{1}{2} 9.8t^2$

D = Vt

D = .324 m

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than __✓__ Less than ____ Equal to

Justify your answer.

KE energy is the in both interia

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

__X__ Greater than ____ Less than ____ Equal to

Justify your answer.

coffinent of inertia is less in the sphere

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

__X__ Greater than ____ Less than ____ Equal to

Justify your answer.

Sphere travels more because it has less rotaticnal inertia.

$\frac{2}{5} < \frac{1}{2}$

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Question 3

Overview

The responses to this question were expected to demonstrate the following:

- An understanding of conservation of energy, including the distinction between translational and rotational kinetic energy.
- An understanding of the difference and the relationship between linear and angular velocity of a rolling object.
- An understanding of projectile motion.
- An understanding of the effect of rotational inertia on the kinetic energy distribution for a rolling object.
- The ability to read, analyze, and correctly interpret the statement of a problem, including distinguishing between relevant and superfluous information for each part.
- The ability to apply the concept of energy conservation to a system where gravitational potential energy is converted into translational and rotational kinetic energy.
- The ability to analyze the kinematics of a rolling object.
- The ability to calculate the range of a horizontally launched projectile.
- The ability to determine how changes to the rotational inertia of an object affect an object's total kinetic energy, rotational kinetic energy, and projectile range.
- The ability to formulate concise and clear explanations of the behavior of a rolling object.

Sample: M Q3 A
Score: 14

Parts (a), (b), (c), and (d) each earned full credit, and their solutions are clearly shown. Part (a) applies conservation of energy. Part (b) uses correct expressions for both linear and rotational kinetic energy to calculate the total kinetic energy of the cylinder and substitutes the energy from part (a). Part (c) sets up the ratio of the rotational to the total kinetic energy, calculating the latter by adding the kinetic energy from part (a) to the potential energy lost due to falling from the table. Full credit was earned even though units are included. Part (d) uses a correct equation to calculate the time of the fall and uses the constant speed equation to calculate the horizontal distance. Part (e)(i) has the correct selection, but the justification is incomplete, so 1 point was earned. The justification indicates that energy is conserved but does not compare the case of the sphere to the cylinder. Parts (e)(ii) and (e)(iii) both have correct selections and justifications, so full credit was earned.

Sample: M Q3 B
Score: 7

Part (a) earned full credit. Part (b) uses both linear and rotational kinetic energy to calculate the total kinetic energy of the cylinder but does not substitute for the speed and rotational inertia, so 1 point was earned. Part (c) has a correct ratio but does not substitute values, so 1 point was earned. Part (d) does not attempt to calculate time or distance, so no credit was earned. Part (e)(i) has the correct selection, but the justification is incomplete, so 1 point was earned.
Part (e)(ii) has a correct selection and justification, so full credit was earned. Part (e)(iii) has an incorrect selection and justification, so no credit was earned.

Question 3 (continued)

Sample: M Q3 C
Score: 5

Part (a) uses conservation of energy, but the answer is incorrect, so 1 point was earned. Part (b) uses both linear and rotational kinetic energy to calculate the total kinetic energy and substitutes correctly for linear velocity and rotational inertia but does not correctly substitute values into the equation, so 2 points were earned. Part (c) has no correct ratio, so no credit was earned. In part (d) the time is implicitly used in the horizontal motion equation, but the equation for the vertical direction is not used correctly, so 1 point was earned. Parts (e)(i) and (e)(ii) have incorrect selections and justifications, so no credit was earned. Part (e)(iii) has a correct selection and attempts a justification, but the justification is incomplete, so 1 point was earned.