2018

AP® CollegeBoard

AP Physics C: Mechanics

Free-Response Questions

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\ \text{eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg} = 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^3\ \text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9\ (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1\ \text{atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5\ \text{Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}{}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu\left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum mr^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2}k(\Delta x)^2$

$x = x_{max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{Gm_1 m_2}{r^2}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa\varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$R = \frac{\rho\ell}{A}$

$\vec{E} = \rho\vec{J}$

$I = Nev_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\,d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 nI$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

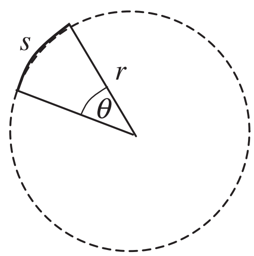
$a^2 + b^2 = c^2$

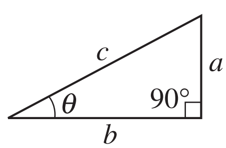
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

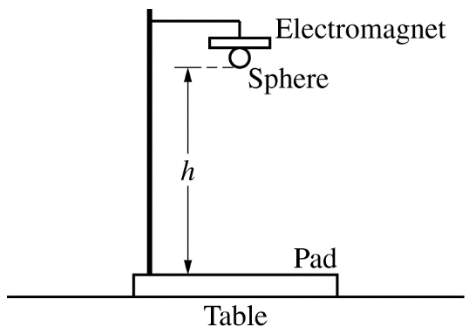
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

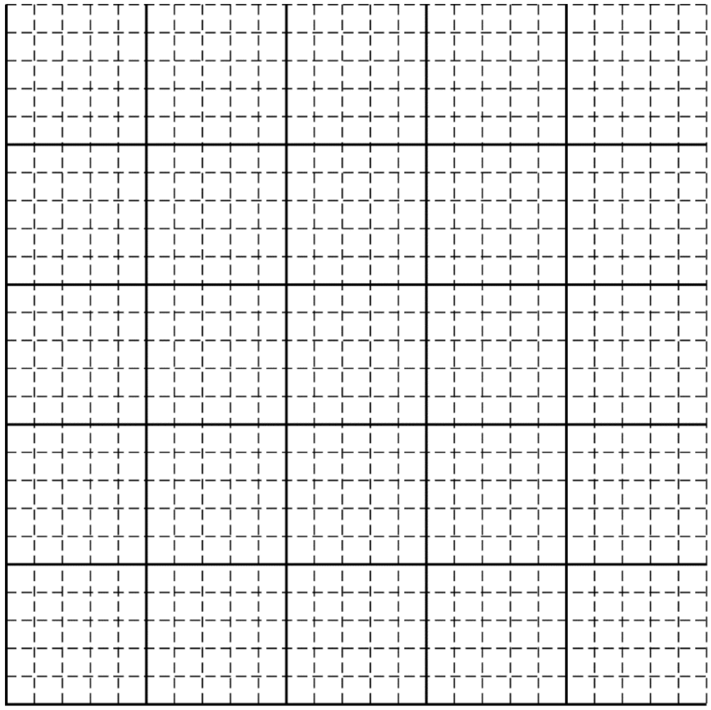
(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

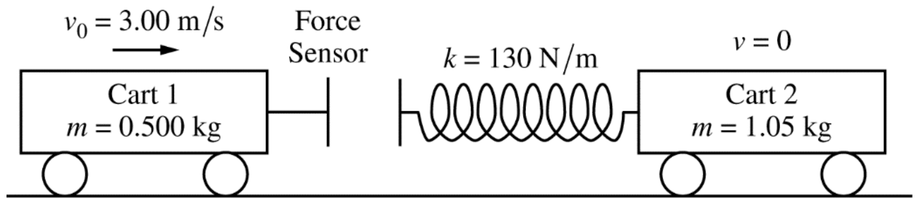
(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

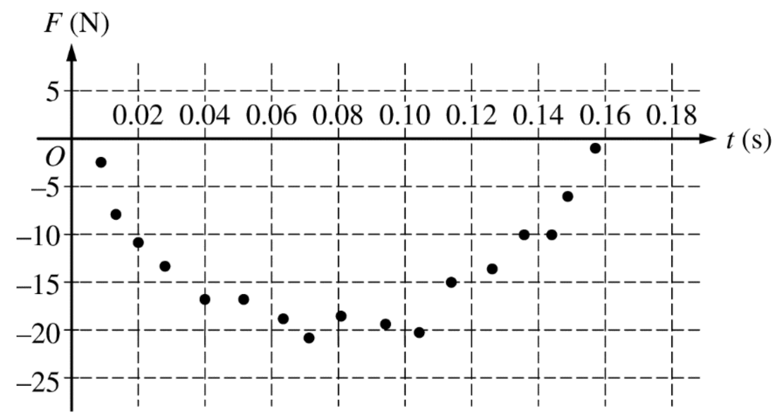
ii. Calculate the new experimental value for g.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00 \text{ m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130 \text{ N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right)t^2 - (500 \text{ N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

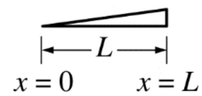
ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

$$\lambda = \left(\frac{2M}{L^2}\right)x$$



3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

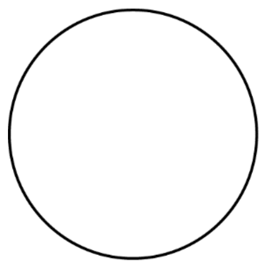


Figure 1

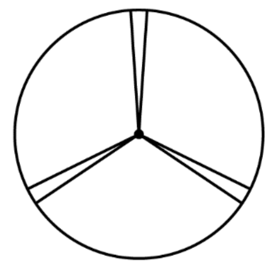


Figure 2

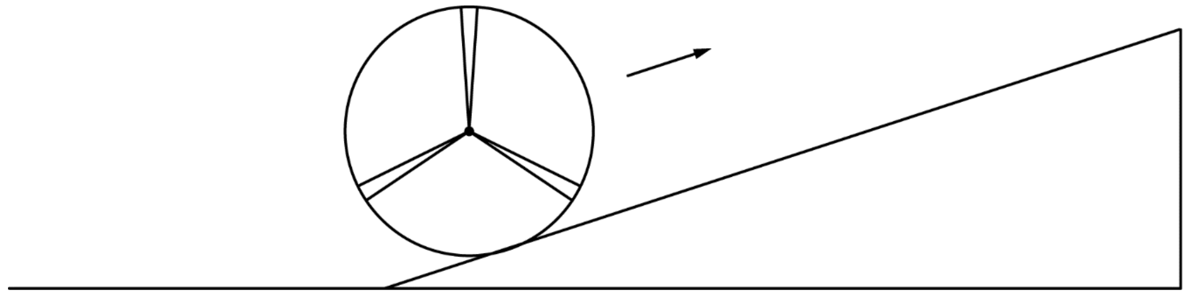
The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

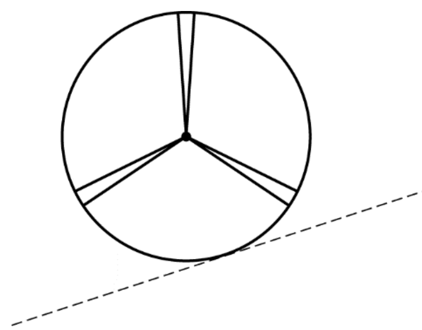
GO ON TO THE NEXT PAGE.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

STOP

END OF EXAM