2018

AP® CollegeBoard

AP Physics C: Mechanics

Sample Student Responses and Scoring Commentary

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AP® PHYSICS
2018 SCORING GUIDELINES

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

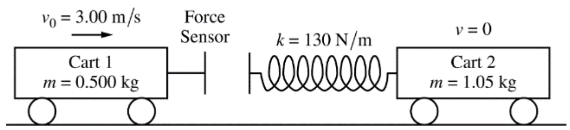
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

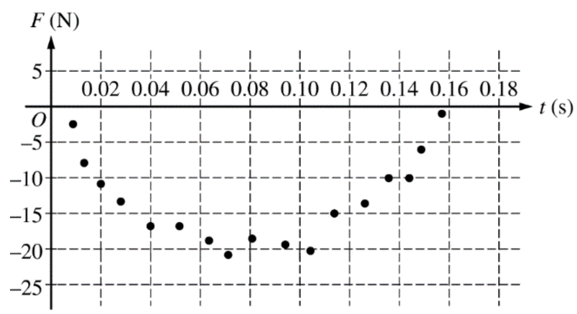
6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

15 points total **Distribution of points**



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - \left(500\ \text{N/s}\right)t$.



(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \cdot dt = \int \left(3200t^2 - 500t\right) dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\text{ s}} \left(3200t^2 - 500t\right)dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03\ \text{N}\cdot\text{s}$		

Question 2 (continued)

Distribution of points

(b)

i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1(v_{1f} - v_{1i}) \therefore v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{(-2.03\ \text{N•s})}{(0.500\ \text{kg})} + (3.00\ \text{m/s})$		
$v_{1f} = -1.06\ \text{m/s}$		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting "Left"		1 point

(c)

i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f} \therefore v_{2f} = \dfrac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \dfrac{(2.03\ \text{N•s})}{(1.05\ \text{kg})}$		
$v_{2f} = 1.93\ \text{m/s}$		

Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \;\therefore\; v_{2f} = \dfrac{m_1 \left(v_{1i} - v_{1f}\right)}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500\ \text{kg})((3.00\ \text{m/s}) - (-1.06\ \text{m/s}))}{(1.05\ \text{kg})} = 1.93\ \text{m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\frac{1}{2}\right)(0.500\ \text{kg})(3.00\ \text{m/s})^2 = \left(\frac{1}{2}\right)(0.500\ \text{kg})(-1.06\ \text{m/s})^2 + \left(\frac{1}{2}\right)(1.05\ \text{kg})(1.93\ \text{m/s})^2$		
$2.25\ \text{J} \approx 2.24\ \text{J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1 v_{1i} = (m_1 + m_2) v_{cm} \;\therefore\; v_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500\ \text{kg})(3.00\ \text{m/s})}{(0.500\ \text{kg} + 1.05\ \text{kg})} = 0.97\ \text{m/s}$		

Question 2 (continued)

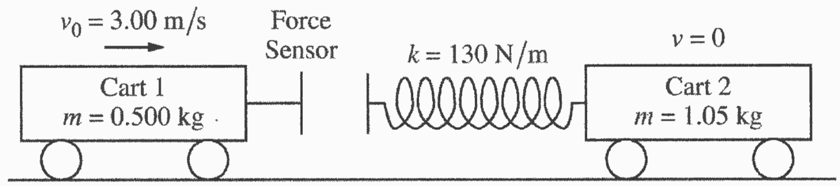
Distribution of points

(d) (continued)
ii. 3 points

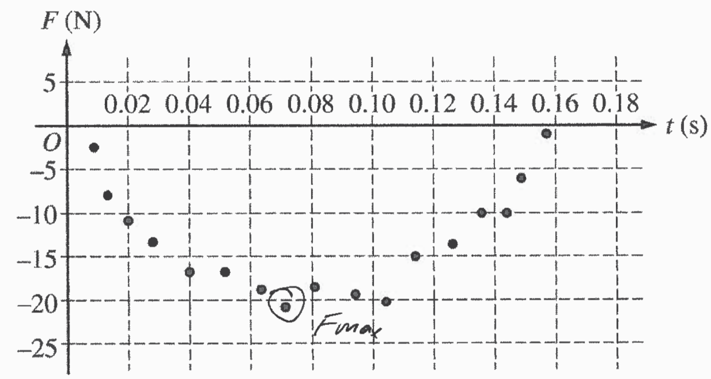
Calculate the maximum elastic potential energy stored in the spring.

For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}(m_1 + m_2)v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 - \frac{1}{2}(0.500\text{ kg} + 1.05\text{ kg})(0.97\text{ m/s})^2$		
$U_S = 1.52\text{ J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\frac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\text{ s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX}\rvert = 19.5\text{ N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\frac{F_{MAX}}{k} = -\frac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2}kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$		
$U_S = 1.46\text{ J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

Units point: 1 point for correct units on all calculated answers



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\text{ m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\text{ N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\text{ N/s}^2\right)t^2 - (500\text{ N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

$F = \frac{dp}{dt}$

$dp = F\,dt$

$\Delta P = \int_{t_0}^{t_f} F\,dt$

$= \int_0^{0.16} (3200t^2 - 500t)\,dt$

$= \frac{3200t^3}{3} - \frac{500t^2}{2} \Big|_{t=0}^{t=0.16}$

$= \frac{3200(.16)^3}{3} - \frac{500(.16)^2}{2}$

$= \boxed{-2.03 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}}$

(b)

i. Calculate the speed of cart 1 after the collision.

$\Delta P = m\Delta V = -2.03$

$\Delta V = \frac{-2.03}{0.500} = -4.06 \text{ m/s}$

$V_f = V_0 + \Delta V = 3.00 - 4.06 = -1.06 \rightarrow \boxed{1.06 \text{ m/s}}$

ii. In which direction does cart 1 move after the collision?

X Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

By Newton's 3rd law + conservation of momentum:

$\Delta P_1 = -\Delta P_2$

$\Delta P_2 = +2.03 = m_2 \Delta V_2$

$\Delta V_2 = \frac{+2.03}{1.05} = 1.933 \text{ m/s}$

$V_{f2} = V_{o2} + \Delta V_2 = 0 + 1.933 = \boxed{1.933 \text{ m/s}}$

ii. Show that the collision between the two carts is elastic.

$KE_o = KE_f$ (elastic condition)

$KE_o: \frac{1}{2}(0.500)(3)^2 = 2.25$

$KE_f: \frac{1}{2}(0.500)(1.06)^2 + \frac{1}{2}(1.05)(1.933)^2 = 2.24$

$2.25 \approx 2.24$, thus collision is elastic

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

$\Sigma P = \Sigma m (V_{com})$

$V_{com} = \frac{\Sigma P}{\Sigma m} = \frac{m_1 V_1}{m_1 + m_2} = \frac{0.500(3)}{0.500 + 1.05} = \boxed{0.968 \text{ m/s}}$

ii. Calculate the maximum elastic potential energy stored in the spring.

$U_{max} = \frac{1}{2} k x_{max}$

$F_{max} = k x_{max}$, $x_{max} = \frac{F_{max}}{k} \approx \frac{21}{130} = .161$

$U_{max} = \frac{1}{2}(130)(0.161) = \boxed{10.465 \text{ J}}$

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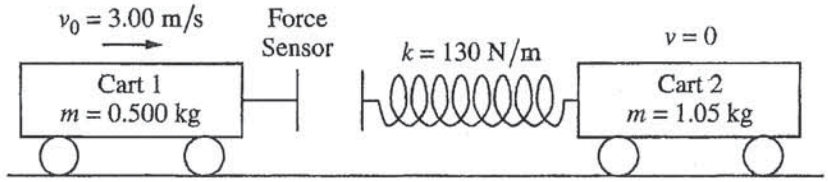
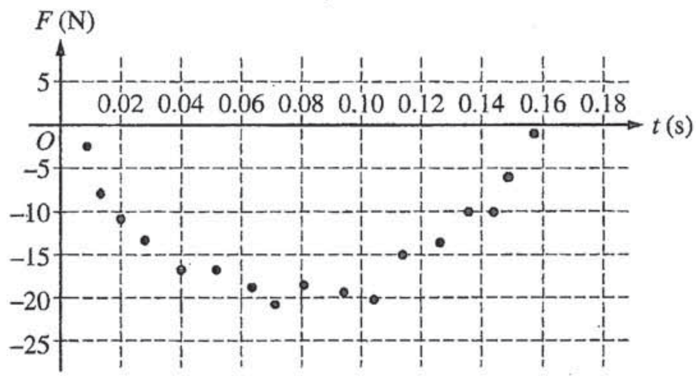


2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200\ \text{N/s}^2)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

$J = \int F\,dt = \Delta P$ $\quad 4.9 - 6.4$

$J = \int_0^{0.16} 3200t^2 - 500t\ dt = \boxed{-2.03\ N\cdot s}$

$\frac{3200t^3}{3} - \frac{500t^2}{2} \Big|_0^{0.16}$

(b)

i. Calculate the speed of cart 1 after the collision.

$J = \Delta P = mv_f - mv_i \quad -2.03 = mv_f - (0.5)(3)$

$p_i = p_f$

$mv_{f_1} = -0.53$ $\quad \boxed{V_1 = -1.061}$

$(0.5) = v_{f_1} = -0.53$

ii. In which direction does cart 1 move after the collision?

__✓__ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

GO ON TO THE NEXT PAGE.

(c)

i. Calculate the speed of cart 2 after the collision.

$P_f = P_i$

$m_{cart1} v_0 + \cancel{M_{cart2}}(0) = m_{cart_1} v_1 + m_{cart2} V_2$

$(0.5)(3) = (0.5)(-1.061) + (1.05) v_f$

$\boxed{V_2 = 2.44 \text{ m/s}}$

ii. Show that the collision between the two carts is elastic.

elastic colisions means KE Conserved

$\frac{1}{2} m v_0^2 + \frac{1}{2} m v_{20}^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$

$\frac{1}{2}(0.5)(9) + 0 = \frac{1}{2}(0.5)(-1.061) + \frac{1}{2}(1.05)(2.44)^2$

$2.25 = 0.2814 + 3.12564$

$2.25 \approx 1.8$

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

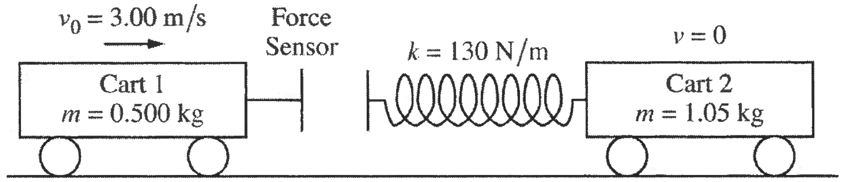
$\frac{1}{2} k x^2 = \frac{1}{2} m v^2$

$\frac{1}{2}(130)(x^2) = \frac{1}{2}(1.55) v^2$

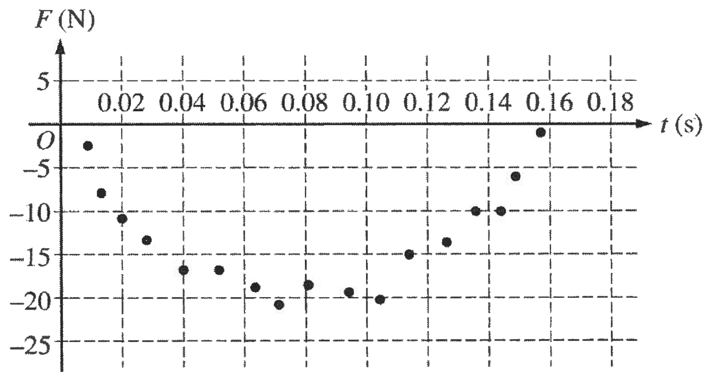
ii. Calculate the maximum elastic potential energy stored in the spring.

$U_s = \frac{1}{2} k x^2 = \frac{1}{2} m v^2$

M Q2 C p1



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

$\vec{J} = \int F\,dt$

$= \int_{.02}^{.16} (3200)t^2 - 500t\,dt$

$= \frac{1}{3}3200t^3 - \frac{1}{2}500t^2 \Big|_{.02}^{.16}$

$= 1.94$ m m/s

(b)

i. Calculate the speed of cart 1 after the collision.

$\vec{J}$ ~~Impulse~~ = ~~Δp~~

~~$\int F\,dt = mv$~~

1.~~94 = mv~~

Ke f ~~tot~~ = Ke i ~~tot~~

$v_f = \sqrt{\dfrac{\frac{1}{2}.5\,3^2}{\frac{1}{2}m}}$

$v_f = 0$

ii. In which direction does cart 1 move after the collision?

~~X~~ Left ____ Right

X The direction is undefined, because the speed of cart 1 is zero after the collision.

GO ON TO THE NEXT PAGE.

(c) $\vec{p}_0 = \vec{p}_f$

i. Calculate the speed of cart 2 after the collision.

$m_1 v_1 + \cancel{m_2 v_2} = m_1 v_1 + m_2 v_2$

$v_2 = \frac{(m_1 v)_0 - (m_1 v_1)_f}{m_2} = \frac{(.5 \cdot 3) - 0}{1.05}$ $\boxed{= 1.43 \text{ m/s}}$

ii. Show that the collision between the two carts is elastic.

$\vec{p} = \vec{p}_f$

$m_1 v_1 + \cancel{m_2 v_2} = \cancel{m_1 v_1} + m_2 v_2$

$(.5)(3) = (1.05)(1.43)$

$\boxed{1.5 = 1.5}$

(d)

i. Calculate the ~~speed of the center of mass~~ of the two-cart–spring system.

Center of mass $= \frac{(m_1 + m_2)}{(m_1 - m_2)} = \frac{(1.05 + .5)}{(1.05 - .5)}$ $\boxed{= 2.82 \text{ m}}$

ii. Calculate the maximum elastic potential energy stored in the spring.

$U = \frac{1}{2} k x^2$

$= (\frac{1}{2})(130)(20)^2$

$\boxed{= 26{,}000 \text{ J}}$

Question 2

Overview

The responses to this question were expected to demonstrate the following:

- An understanding of elastic collisions, impulse-momentum, and elastic potential energy.
- The ability to apply integral calculus to solve for impulse applied to an object.
- Overall, the knowledge of how to determine if a collision is elastic or not. In order to properly do this, students needed to understand the relationship between impulse, force, and the change in momentum.
- The ability to integrate the function and use that result to determine the velocities of each cart after the collision occurred. In addition, once students determine the velocities, they were asked to demonstrate, through the conservation of energy, that the collision is elastic.
- Finally, students were asked to find the maximum energy stored in the spring during the collision.
  - A comprehension of conservation of momentum and the conservation of energy.
  - Graphical analysis skills.

Sample: M Q2 A
Score: 14

Parts (a), (b), and (c) earned full credit. In part (a) the force equation is integrated with appropriate limits, and a correct answer is indicated, which earned 3 points. In part (b)(i) 1 point was earned for a correct substitution into an equation for impulse. In part (b)(ii) 1 point was earned for a correct selection. In part (c)(i) a correct substitution into an appropriate equation is shown, which earned 2 points. In part (c)(ii) the initial and final kinetic energy are set equal, and a correct substitution of answers from parts (b) and (c) is shown, which earned 2 points. In part (d)(i) 2 points were earned for a correct substitution into a correct equation. In part (d)(ii) a correct estimation of the maximum force is indicated, a correct calculation of maximum compression is shown, but the x_{MAX} is not squared in the elastic potential energy equation, so 2 points were earned. Finally, 1 point was earned for corrects units on all calculated values.

Sample: M Q2 B
Score: 10

Parts (a), (b), and (c) earned full credit, 3 points, 2 points, and 4 points, respectively. In part (c)(i) the answer is incorrect, but the substitution is correct, so full credit was still earned. In part (c)(ii) the final values of kinetic energy are not equal, but the substitution is consistent with previous answers, so full credit was earned. Part (d)(i) has no valid equation for speed of the center of mass, so no points were earned. In part (d)(ii) conservation of energy is shown, but there is no calculation of the maximum compression, so 1 point was earned. Units are not shown on the answer for (b)(i), so that point was not earned.

Sample: M Q2 C
Score: 3

In part (a) the force equation is integrated, but the limits are not correct, and the answer is incorrect, so 1 point was earned. In part (b)(i) an incorrect equation is used, so no points were earned. In part (b)(ii) an incorrect selection is indicated, so no points were earned. Part (c)(i) substituted into conservation of momentum consistent with the answer from part (a), so full credit of 2 points was earned. In part (c)(ii) the equation is not specific to the situation, so no points were earned. In part (d)(i) the equation shown is for center of mass, not speed of center of mass, so no points were earned. In part (d)(ii) the substitution into the elastic potential

Question 2 (continued)

energy is not related to kinetic energy or maximum compression, so no points were earned. The units for part (a) are incorrect, so that point was not earned.