2018

AP CollegeBoard

AP Physics C: Mechanics

Sample Student Responses and Scoring Commentary

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Free Response Question 3

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General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

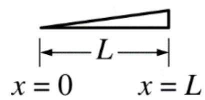
5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 3

15 points total | **Distribution of points**

$$\lambda = \left(\frac{2M}{L^2}\right)x$$



A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) 3 points

Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

For relating x to r properly in an integral to calculate the moment of inertia		1 point
$I = \int r^2 dm = \int x^2 dm$		
For correctly using the linear mass density to substitute into the equation above		1 point
$m = \int \lambda dx = \int \left(2M/L^2\right) x dx \therefore dm = \left(2M/L^2\right) x dx$		
$I = \int \left(2M/L^2\right) x^3 dx$		
For integrating using the correct limits or constant of integration		1 point
$I = \int_{x=0}^{x=L} \left(2M/L^2\right) x^3 dx = \left[\frac{\left(2M/L^2\right) x^4}{4}\right]_{x=0}^{x=L} = \frac{2M}{L^2}\left(\frac{L^4}{4} - 0\right) = ML^2/2$		

Question 3 (continued)

Distribution of points

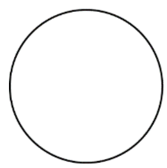


Figure 1

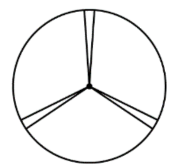


Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) 2 points

Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

For setting the total rotational inertia for the hoop-rod system equal to the sum of the rotational inertias of the hoop and the three rods		1 point
$I = 3I_{rod} + I_{hoop}$		
$I = 3\left(ML^2/2\right) + ML^2$		
For a correct answer with work		1 point
$I = \frac{5}{2}ML^2$		

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

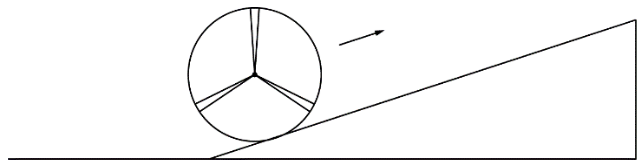
(c) 3 points

Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

For using an appropriate equation to determine the final angular speed of the hoop		1 point
$\tau\Delta t = I(\omega_2 - \omega_1)$		
$Fr\Delta t = I\omega$		
$\omega = \frac{Fr\Delta t}{I}$		
For relating the lever arm to the length of the rod		1 point
$\omega = \frac{FL\Delta t}{I}$		
For correct substitution from part (b)		1 point
$\omega = \frac{FL\Delta t}{\left(\frac{5}{2}\right)ML^2} = \frac{2F\Delta t}{5ML}$		

Question 3 (continued)

Distribution of points

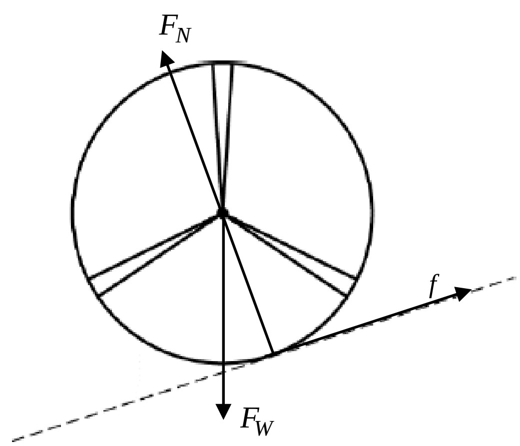


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. 3 points

On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

Question 3 (continued)

Distribution of points

ii. 1 point

Justify your choice for the direction of each of the forces drawn in part (d)(i).

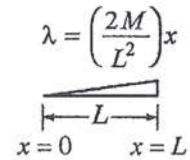
For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

(e) 3 points

Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

For including both linear and rotational kinetic energy in an equation for the conservation of energy to determine the final height of the hoop		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$		
$H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$		
For correct substitution of $v = L\omega$		1 point
$H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$		
$H = \dfrac{\frac{1}{2}\left(m_{tot}L^2 + I_{tot}\right)\omega^2}{m_{tot}g}$		
For correct substitution of inertias into energy equation		1 point
$H = \dfrac{\frac{1}{2}\left((3M + M)L^2 + I_{tot}\right)\omega^2}{(3M + M)g} = \dfrac{\left(4ML^2 + I_{tot}\right)\omega^2}{8Mg}$		

M Q3 A p1



3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

$I = \int R^2 dm = \int_0^L x^2 \frac{2mx}{L^2} dx = \frac{2m}{L^2} \cdot \frac{x^4}{4}\Big|_0^L = \frac{2mL^4}{4L^2} = \frac{mL^2}{2}$

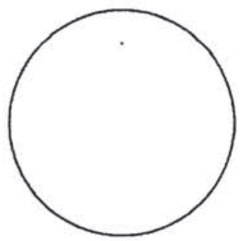


Figure 1

Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

$I = I_1 + 3I_2 = ML^2 + \frac{3}{2}mL^2 = \frac{5}{2}mL^2$

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

$\tau \Delta T = I \Delta w$

$\Delta w = w_0^{\,0} + \frac{\tau \Delta T}{I} = \frac{(FL)(\Delta T)}{5/2\, mL^2} = \frac{2}{5}\frac{F\Delta T}{mL}$

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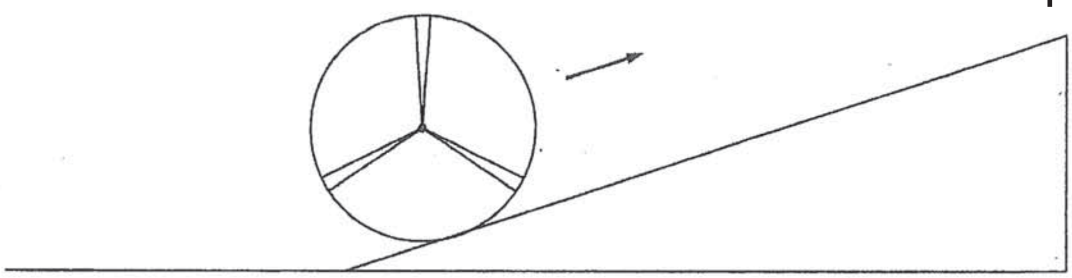
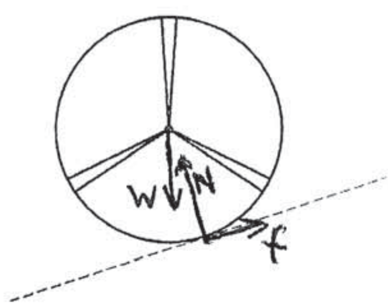


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

Weight is always toward the center of Earth or down in this case

Normal is directed orthognally from the surface

friction must point counterclockwise because the clockwise rotation of the hoop is slowing down, so net torque must be ccw, and friction provides the only torque

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

$E_{before} = \frac{1}{2} I_{tot} W^2 + \frac{1}{2}(4M)(L^2 W^2)$

$E_{After} = 4mgH \qquad 4mgH = \frac{1}{2} I_{tot} W^2 + 2mL^2 W^2$

$$H = \frac{I_{tot} W^2}{8mg} + \frac{L^2 W^2}{2g}$$

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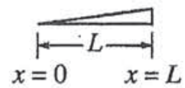
$$\lambda = \left(\frac{2M}{L^2}\right)x$$



3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

$I = \int r^2 dm = \int r^2 \lambda dr = \int r^2 \frac{2m}{L^2} x\, dr = \frac{2m}{L^2} \int_0^L x^3 dx$

$= \frac{2m}{L^2} \Big|_0^L \frac{x^4}{4} = \frac{2m}{L^2}\left(\frac{L^4}{4}\right) = \boxed{\frac{mL^2}{2}}$

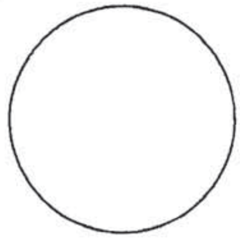


Figure 1

Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

$I_{total} = I_1 + I_2 + I_3 + I_4 = \frac{2mL^2}{2} + \frac{mL^2}{2} + \frac{mL^2}{2} + \frac{mL^2}{2}$

$= \boxed{\frac{5}{2} mL^2}$

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

$rF\sin 90 = LF$

$\Sigma \tau = I\alpha$

$LF = \frac{5mL^2}{2}\alpha$

$\frac{2LF}{5mL^2} = \alpha = \frac{2F}{5mL}$

$\omega_f = \omega_0 + \alpha t$

$\omega_f = \alpha \Delta t$

$\omega_f = \boxed{\frac{2F\Delta t}{5mL}}$

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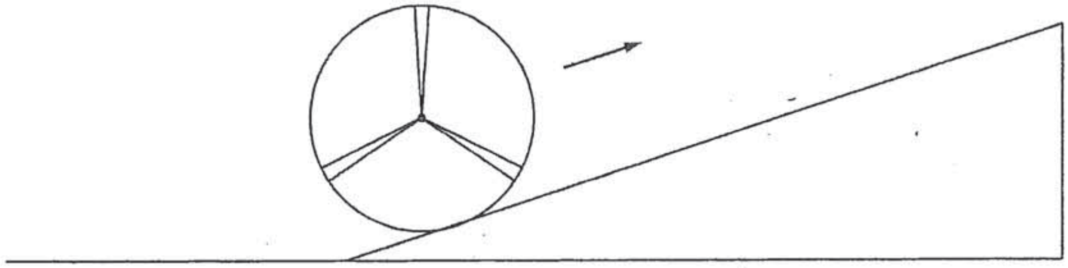
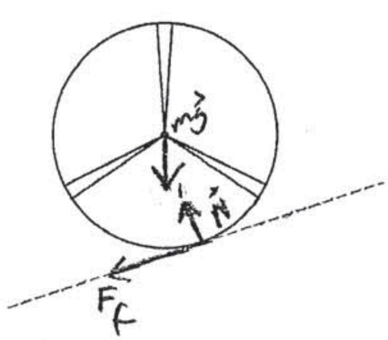


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

Friction has to be opposite to the direction moving, rolling without slipping, so Friction direction needed for torque
$m\vec{g}$ always straight down from center.
$\vec{N}$ always perpendicular to surface, from pt. of contact.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

$$KE_0 + KE_{0R} = PE_f$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}\frac{5}{2}ML^2\left(\frac{2}{5}\frac{F\Delta t}{m}\right)^2$$

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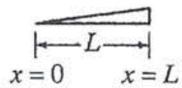
$$\lambda = \left(\frac{2M}{L^2}\right)x$$



3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

$I = \int_0^L L^2\, dm \quad dm = \lambda \frac{dx}{2}$

$= \int_0^L L^2 \cdot \frac{2Mx}{L^2} \frac{dx}{2} = \frac{2M}{0} \int_0^L \frac{x\, dx}{2} = \frac{2ML^2}{4} = \frac{1}{2}ML^2$

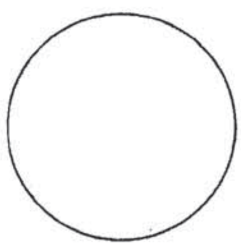


Figure 1

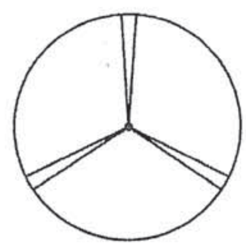


Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

$I_{tot} = I_{tot} + mr^2$

$= ML^2 + ML^2$

$= 2ML^2$

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

$\Sigma I_{net} = I\alpha$

$F = I\alpha$

$F = 2ML^2\alpha$

$\alpha = \frac{F}{2ML^2}$

$w = w_0 + \alpha t \qquad w_0 = 0$

$w = \frac{F}{2ML^2}t$

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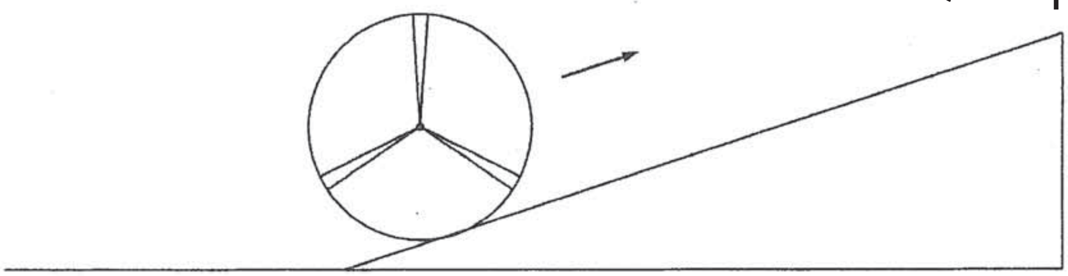
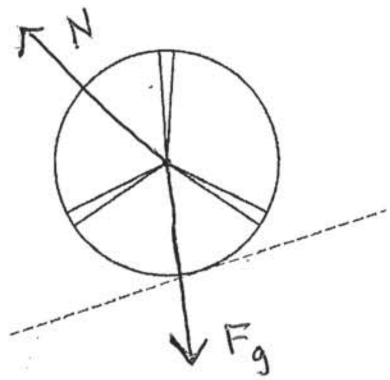


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

Normal Force is perpendicular to the surface

The Force of gravity is downwards towards the center of the Earth

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

Conservation of Rotational Kinetic Energy

$\frac{1}{2} I w^2 = mgh$

$\frac{1}{2} I_{tot} w^2 = mgh$ $\boxed{h = \frac{I_{tot} w^2}{2mg}}$

AP® PHYSICS C: MECHANICS
2018 SCORING COMMENTARY

Question 3

Overview

The responses to this question were expected to demonstrate the following:

- The ability to determine the moment of inertia of an object with varying mass density.
- An understanding of how to use the principle of superposition and the rotational inertia of basic objects to determine the total rotational inertia for complicated objects.
- The ability to determine the forces that are acting on the object as it rolls without slipping.
- An understanding of the conservation of energy as the hoop moves.

Sample: M Q3 A
Score: 12

All parts earned full credit. In part (a) x is related to r, dm is related to λdx, and the equation is integrated with correct limits, which earned 3 points. In part (b) the rotational inertias of the hoop and three rods are added together, and a correct answer is given, which earned 2 points. In part (c) an appropriate equation for angular speed is used, the lever arm is related to L, and a correct substitution for Δt is shown, which earned 3 points. In part (d)(i) all three forces are shown correctly with the friction correctly up the incline, so 3 points were earned. In part (d)(ii) the justification for all forces is correct, so 1 point was earned. In part (e) both linear and rotational kinetic energy are included in the equation, and the substitutions of $v = \omega L$ is shown, and the inertias (I_{tot} and $4M$) are correct, so 3 points were earned.

Sample: M Q3 B
Score: 10

Parts (a), (b), and (c) earned full credit for a total of 8 points. In part (d)(i) the weight is shown correctly, but the point of exertion for the normal force is incorrect, and friction is not shown, so 1 point was earned. In part (d)(ii) the justification for friction is incorrect, so no points were earned. In part (e) both linear and rotational kinetic energy are included in the equation, but the substitutions of $v = \omega L$ and $4M$ are not indicated, so 1 point was earned.

Sample: M Q3 C
Score: 3

In part (a) the relation of x to r and dm to λdx are not correct, and the integration is not set up correctly, so no points were earned. Part (b) does not include all three rods in the inertia equation, so no points were earned. In part (c) the substitutions from part (b) are made, but the correct equation is not shown, so 1 point was earned. In part (d)(i) the weight is correctly shown, but the normal forces do not have the correct point of exertion, and friction is not shown, so 1 point was earned. In part (d)(ii) the justification is correct for both forces drawn in part (d)(i), so 1 point was earned. In part (e) linear kinetic energy is not included in the equation, and $4M$ is not substituted, so no points were earned.