2018

AP CollegeBoard

AP Physics C: Mechanics

Scoring Guidelines

General Notes About 2018 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

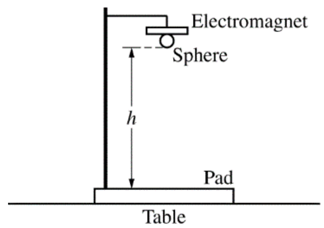
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points total **Distribution of points**



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) 2 points

While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because the measured time to fall the same distance will be larger, the acceleration must be less.		
Example: Because the time is larger and $a \propto 1/t^2$ *, then g must decrease.*		

Question 1 (continued)

Distribution of points

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

(b) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

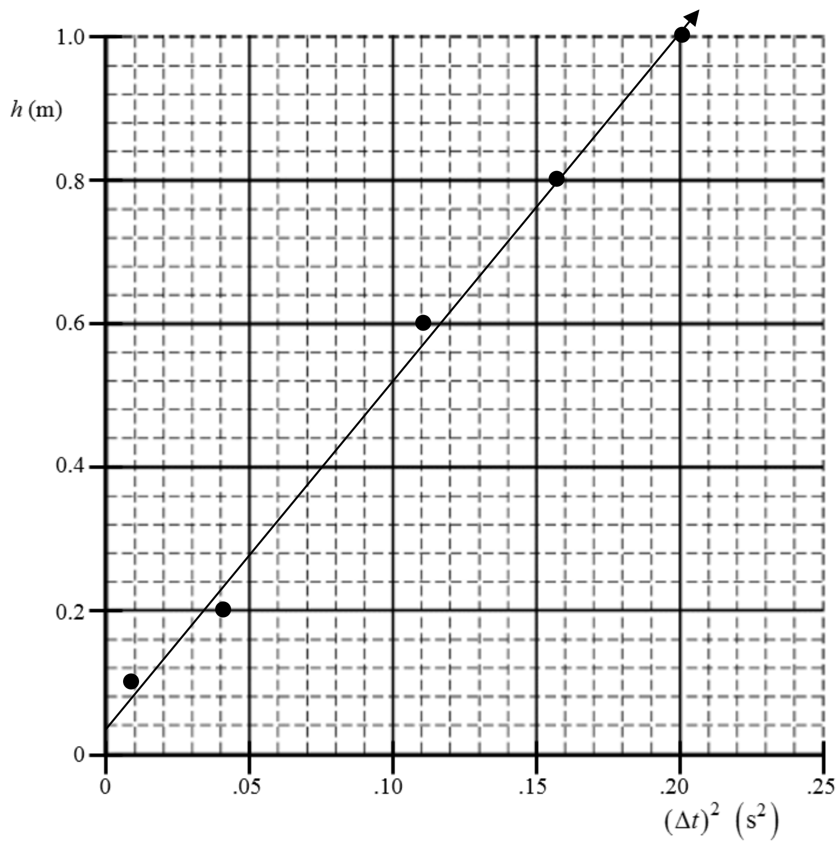
For correctly indicating two variables that will yield a straight line that could be used to determine a value for g		1 point
Example: Vertical Axis: h *Horizontal Axis:* $(\Delta t)^2$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination.		

Question 1 (continued)

Distribution of points

(c) 4 points

Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axis with variables and units consistent with part (b)		1 point
For correctly plotting data consistent with part (b)		1 point
For drawing a straight line consistent with the plotted data		1 point

Question 1 (continued)

Distribution of points

(d) 2 points

Using the straight line, calculate an experimental value for g.

For using points on the line rather than the data to calculate the slope		1 point
$\text{slope} = \dfrac{\Delta h}{\Delta(\Delta t)^2} = \dfrac{(0.80 - 0.20)\ \text{m}}{(0.160 - 0.039)\ \text{s}^2} = 4.96\ \text{m/s}^2$ (Linear regression $= 4.83\ \text{m/s}^2$)		
For correctly relating the slope to the acceleration due to gravity		1 point
$\text{slope} = \dfrac{1}{2}g \therefore g = 2 \times \text{slope} = 2 \times \left(4.96\ \text{m/s}^2\right)$		
$g = 9.92\ \text{m/s}^2$ (Linear regression $= 9.66\ \text{m/s}^2$)		

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. 1 point

Derive an expression for the velocity of the sphere as a function of time.

For correctly taking the time derivative of the given equation		1 point
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
$v(t) = \dfrac{dy}{dt} = 2At + B$		
Note: Credit is earned for substituting numbers: $v(t) = \left(11.5\ \text{m/s}^2\right)t - 0.524\ \text{m/s}$.		

ii. 2 points

Calculate the new experimental value for g.

For correctly relating the given equation to a correct kinematic equation		1 point
$\Delta y = y - y_0 = v_1 t + \dfrac{1}{2}at^2$		
$y = y_0 + v_1 t + \dfrac{1}{2}at^2 = At^2 + Bt + C$		
For correctly relating the equation to the value of g		1 point
$A = \dfrac{1}{2}a = \dfrac{1}{2}g$		
$g = 2A = (2)\left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		

Question 1 (continued)

Distribution of points

iii. 1 point

Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)(ii).

For correctly calculating the percent error		1 point
$\%\text{error} = \dfrac{\lvert\text{acc} - \text{exp}\rvert}{\text{acc}} \times 100\% = \dfrac{\left\lvert\left(11.5\ \text{m/s}^2\right) - \left(9.81\ \text{m/s}^2\right)\right\rvert}{\left(9.81\ \text{m/s}^2\right)} \times 100\%$		
$\%\text{error} = 17.2\%$		
Note: Credit is earned if percent error is expressed as positive or negative.		

iv. 2 points

Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

For relating the coefficients of the equation to the kinematic variables		1 point
$y = At^2 + Bt + C\ \therefore v_1 = B = -0.524\ \text{m/s}$		
$a = 2A = 2 \times \left(5.75\ \text{m/s}^2\right) = 11.5\ \text{m/s}^2$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using an appropriate kinematics equation to determine the velocity of the sphere		1 point
$v_2^2 = v_1^2 + 2a\Delta y$		
$v = \sqrt{v_1^2 + 2a\Delta y} = \sqrt{(-0.524\ \text{m/s})^2 + (2)\left(11.5\ \text{m/s}^2\right)(1.40\ \text{m} - 0.080\ \text{m})}$		
$v = 5.54\ \text{m/s}$		
Alternate solution:		*Alternate Points*
For correctly determining the time of fall for the sphere		*1 point*
$y = At^2 + Bt + C$		
$1.40 = \left(5.75\ \text{m/s}^2\right)t^2 + (-0.524\ \text{m/s})t + (0.080\ \text{m})$		
$5.75t^2 - 0.524t - 1.32 = 0$		
$t = 0.53\ \text{s or} - 0.44\ \text{s}\ \therefore t = 0.53\ \text{s}$		
For correctly using the equation from part (e)(i)		*1 point*
$v(t) = \left(11.5\ \text{m/s}^2\right)(0.53\ \text{s}) - 0.524\ \text{m/s}$		
$v = 5.54\ \text{m/s}$		

Question 1 (continued)

Distribution of points

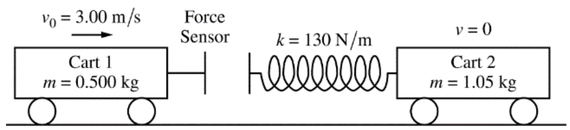
(e)

iv. (continued)

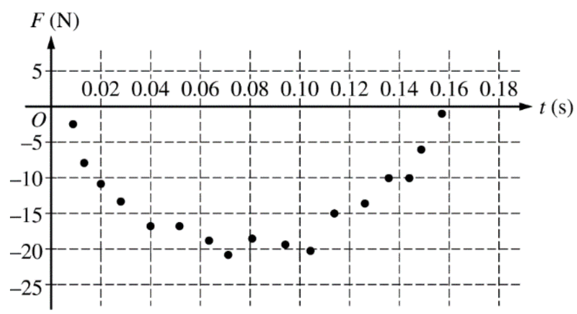
Alternate Solution — Conservation of Energy		
For relating the coefficients of the equation to the initial height and speed		1 point
$y = At^2 + Bt + C \therefore v_1 = B = -0.524\ \text{m/s}$		
$y_0 = C = 0.080\ \text{m}$		
For correctly using conservation of energy to determine the velocity of the sphere		1 point
$U_i + K_i = U_f + K_f$		
$mgh + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2$		
$v = \sqrt{2(11.5)(1.40 - 0.080) + (-0.524)^2} = 5.54\ \text{m/s}$		

Question 2

15 points total **Distribution of points**



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - \left(500\ \text{N/s}\right)t$.



(a) 3 points

Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

For using the given force equation in the integral to determine the impulse delivered to the cart		1 point
$J = \int F \bullet dt = \int 3200t^2 - 500t\ dt$		
For integrating the force using the correct limits or constant of integration		1 point
$J = \int_{t=0}^{t=0.16\text{ s}} \left(3200t^2 - 500t\right)dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_{t=0}^{t=0.16}$		
$J = \left(\left(\frac{3200}{3}\right)(0.16)^3 - (250)(0.16)^2\right) - 0$		
For a correct answer		1 point
$J = -2.03\ \text{N}\bullet\text{s}$		

Question 2 (continued)

Distribution of points

(b)

i. 1 point

Calculate the speed of cart 1 after the collision.

For correct substitution into the impulse-momentum equation of the answer from part (a) to determine the speed of cart 1		1 point
$J = m_1\left(v_{1f} - v_{1i}\right) \therefore v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{(-2.03 \text{ N•s})}{(0.500 \text{ kg})} + (3.00 \text{ m/s})$		
$v_{1f} = -1.06 \text{ m/s}$		

ii. 1 point

In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

For correctly selecting "Left"		1 point

(c)

i. 2 points

Calculate the speed of cart 2 after the collision.

For using a correct equation to determine the speed of cart 2		1 point
$-J = -m_1\left(v_{1f} - v_{1i}\right) = m_2 v_{2f} \therefore v_{2f} = \dfrac{-J}{m_2}$		
For correct substitution of the answer from part (a)		1 point
$v_{2f} = \dfrac{(2.03 \text{ N•s})}{(1.05 \text{ kg})}$		
$v_{2f} = 1.93 \text{ m/s}$		

Question 2 (continued)

Distribution of points

(c)

i. (continued)

Alternate solution		*Alternate Points*
For using the conservation of momentum to calculate the speed of cart 2 after the collision		*1 point*
$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \;\therefore\; v_{2f} = \dfrac{m_1 \left(v_{1i} - v_{1f} \right)}{m_2}$		
For correct substitution of the answer from part (b)(i)		*1 point*
$v_{2f} = \dfrac{(0.500\text{ kg})((3.00\text{ m/s}) - (-1.06\text{ m/s}))}{(1.05\text{ kg})} = 1.93\text{ m/s}$		

ii. 2 points

Show that the collision between the two carts is elastic.

For indicating that the initial and final kinetic energies must be equal		1 point
$K_{1i} = K_{1f} + K_{2f}$		
For correct substitutions of answers from parts (b)(i) and (c)(i) into the calculations of the initial and final kinetic energies		1 point
$\left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 = \left(\frac{1}{2}\right)(0.500\text{ kg})(-1.06\text{ m/s})^2 + \left(\frac{1}{2}\right)(1.05\text{ kg})(1.93\text{ m/s})^2$		
$2.25\text{ J} \approx 2.24\text{ J}$		

(d)

i. 2 points

Calculate the speed of the center of mass of the two-cart-spring system.

For using the equation for the conservation of momentum to calculate the speed for the center of mass of the system		1 point
$m_1 v_{1i} = (m_1 + m_2) v_{cm} \;\therefore\; v_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$		
For correct substitution into the equation above		1 point
$v_{cm} = \dfrac{(0.500\text{ kg})(3.00\text{ m/s})}{(0.500\text{ kg} + 1.05\text{ kg})} = 0.97\text{ m/s}$		

Question 2 (continued)

Distribution of points

(d) (continued)

ii. 3 points

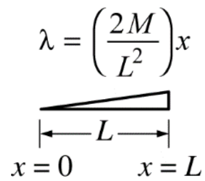
Calculate the maximum elastic potential energy stored in the spring.

For using conservation of energy to calculate the maximum elastic potential energy stored in the spring		1 point
$K_i = K_f + U_{Sf}$		
For using the speed of the center of mass of the system for kinetic energy		1 point
$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2)v_{cm}{}^2 + U_{Sf}$		
$U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}{}^2$		
For correct substitution into above equation		1 point
$U_S = \left(\frac{1}{2}\right)(0.500\text{ kg})(3.00\text{ m/s})^2 - \frac{1}{2}(0.500\text{ kg} + 1.05\text{ kg})(0.97\text{ m/s})^2$		
$U_S = 1.52\text{ J}$		
Alternate Solution:		*Alternate Points*
For correctly determining the magnitude of the maximum force exerted between the carts		1 point
Set $\frac{dF}{dt} = 0 = 6400t - 500 \therefore 6400t = 500 \therefore t = 0.078\text{ s}$		
$F_{MAX} = 3200(0.078)^2 - 500(0.078) = -19.5$		
$\lvert F_{MAX} \rvert = 19.5\text{ N}$		
Note: Can estimate from the graph, and accept the range of 20 N to 21 N.		
For calculating the maximum compression of the spring		1 point
$F_{MAX} = -kx_{MAX}$		
$x_{MAX} = -\frac{F_{MAX}}{k} = -\frac{(-19.5\text{ N})}{(130\text{ N/m})} = 0.15\text{ m}$		
Note: If estimating from the graph, accept the range from 0.15 m to 0.16 m.		
For substituting into an equation for the elastic potential energy at maximum spring compression		1 point
$U_S = \frac{1}{2}kx_{MAX}^2 = \left(\frac{1}{2}\right)(130\text{ N/m})(0.15\text{ m})^2$		
$U_S = 1.46\text{ J}$		
Note: If estimating from the graph, accept range from 1.46 J to 1.70 J.		

Units point: 1 point for correct units on all calculated answers

Question 3

15 points total **Distribution of points**



A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) 3 points

Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

For relating x to r properly in an integral to calculate the moment of inertia		1 point
$I = \int r^2 dm = \int x^2 dm$		
For correctly using the linear mass density to substitute into the equation above		1 point
$m = \int \lambda dx = \int \left(2M/L^2\right) x dx \therefore dm = \left(2M/L^2\right) x dx$		
$I = \int \left(2M/L^2\right) x^3 dx$		
For integrating using the correct limits or constant of integration		1 point
$I = \int_{x=0}^{x=L} \left(2M/L^2\right) x^3 dx = \left[\frac{\left(2M/L^2\right) x^4}{4}\right]_{x=0}^{x=L} = \frac{2M}{L^2}\left(\frac{L^4}{4} - 0\right) = ML^2/2$		

Question 3 (continued)

Distribution of points

Figure 1 Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) 2 points

Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

For setting the total rotational inertia for the hoop-rod system equal to the sum of the rotational inertias of the hoop and the three rods		1 point
$I = 3I_{rod} + I_{hoop}$		
$I = 3\left(ML^2/2\right) + ML^2$		
For a correct answer with work		1 point
$I = \frac{5}{2}ML^2$		

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

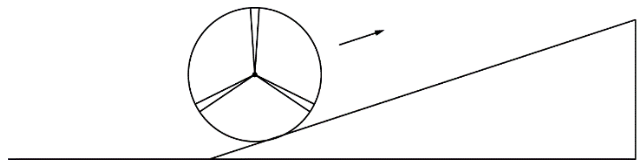
(c) 3 points

Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.

For using an appropriate equation to determine the final angular speed of the hoop		1 point
$\tau\Delta t = I(\omega_2 - \omega_1)$		
$Fr\Delta t = I\omega$		
$\omega = \dfrac{Fr\Delta t}{I}$		
For relating the lever arm to the length of the rod		1 point
$\omega = \dfrac{FL\Delta t}{I}$		
For correct substitution from part (b)		1 point
$\omega = \dfrac{FL\Delta t}{\left(\frac{5}{2}\right)ML^2} = \dfrac{2F\Delta t}{5ML}$		

Question 3 (continued)

Distribution of points

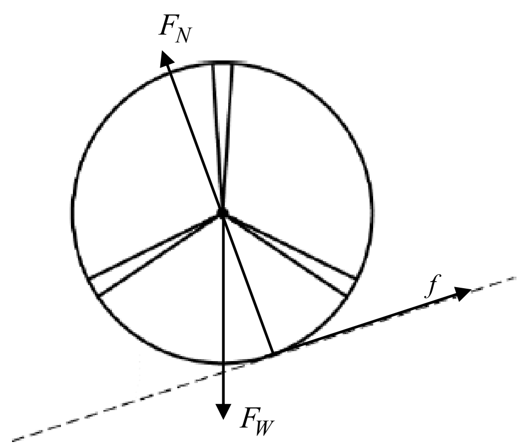


The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. 3 points

On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



For drawing the weight of the hoop-rod system in the correct direction		1 point
For drawing the normal force in the correct direction		1 point
For drawing the frictional force in the correct direction		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors or any vector has an incorrect point of exertion.		

Question 3 (continued)

Distribution of points

ii. 1 point

Justify your choice for the direction of each of the forces drawn in part (d)(i).

For a correct justification of the direction of the forces in part (d)(i)		1 point
Example: The normal force is always perpendicular to the surface, so it will be directed up and to the left. The gravitational force is always vertically downward. The friction will be opposite of the direction of rotation; therefore, it is directed up the incline.		

(e) 3 points

Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

For including both linear and rotational kinetic energy in an equation for the conservation of energy to determine the final height of the hoop		1 point
$K_1 = U_{g2}$		
$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$		
$H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}{mg}$		
For correct substitution of $v = L\omega$		1 point
$H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$		
$H = \dfrac{\frac{1}{2}\left(m_{tot}L^2 + I_{tot}\right)\omega^2}{m_{tot}g}$		
For correct substitution of inertias into energy equation		1 point
$H = \dfrac{\frac{1}{2}\left((3M + M)L^2 + I_{tot}\right)\omega^2}{(3M + M)g} = \dfrac{\left(4ML^2 + I_{tot}\right)\omega^2}{8Mg}$		